Empirically Estimated “True” PPP Indexes

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This paper uses basic-heading data constructed under the auspices of the Eurostat-OECD Purchasing Power Parity (PPP) Programme in conjunction with the underlying price data for Canada to estimate "true" PPP indexes for the OECD-24 countries based on country-specific parameterizations of a CES utility function. These indexes are then used to measure how closely particular axiomatic multilateral comparison formulae approximate "the truth" thereby giving an indication of the meaning and reliability of widely used PPP indexes such as those of the OECD and the Penn World Table.

Key Words: economic approaches to index-number theory; index numbers; multilateral comparisons; purchasing power parities.

JEL Classification Numbers: C32, C43, C81, D12, E31, O57.

1. Introduction

Two indexes of relative purchasing power, the Eltetö-Köves-Szulc (EKS) and the Geary-Khamis (GK), are now commonly used as the basis for determining real income and consumption levels among two or more countries. Since neither of these indexes is directly informed by an economic approach to the theory of international comparisons,[1] they cannot be said to yield "true" purchasing power parities (PPPs) in the sense of being numbers that correspond to the relative minimum costs of given (representative) levels of satisfaction in a bloc of countries. The basic limitation of all such indexes derives from the fact that the quantities of commodities that would be purchased in each country under any other country's prices are not directly observable.

There are three economic approaches to the measurement of true PPPs. The first seeks to establish upper and lower bounds on true measures in terms of observable price and quantity data. Konüs (1924, pp. 20–21) established such bounds under the assumption of a single representative consumer; Dowrick and Quiggin (1997) tightened these bounds by assuming

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[1] That is, one that "make[s] use of the assumption of optimizing behaviour on the part of economic agents" (Diewert, 1999, p. 20).

that the single consumer's preferences are homothetic; and Pollak (1971, p. 11) and Armstrong (2001a, Theorems 4 and 5) established looser bounds in the many-consumer case. The second approach, which originated in Konüs and Byushgens (1926), endeavours to show that particular axiomatic indexes[2] are exact for particular functional forms of a common linearly homogeneous utility function. Diewert (1976, 1981) provided the first thorough analysis of this approach in a bilateral context, and Diewert (1999) and Armstrong (2001a, Sec. 6) were the first to use it in a multilateral one. The third approach seeks to make direct numerical estimates of true PPPs based on specific assumptions about the nature of the underlying consumer preferences. Developed by Lloyd (1975) and Moulton (1996) in the context of intertemporal price-level comparisons, this approach has been applied only once before in an interspatial context. Kravis et al. (1982, pp. 366–74) estimated a linear expenditure system—under the assumption that "each person in each of the [relevant] countries [has] the same utility function" (p. 368)—"with the use of ICP[-1975] data on the four major groupings (food, clothing, shelter, and all other)" (p. 368) for thirty-four countries, and then used this system as the basis for calculating a U.S.-specific per capita consumption index.[3]

One shared feature of Lloyd's and Moulton's estimates is that the parameterization of the assumed functional form of the true price index is not achieved via an empirical estimation of the associated system of demands. Rather, independent estimates of the relevant parameter

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[2] That is, indexes that treat both prices and quantities as independent variables.

[3] Dowrick and Quiggin (1997) could also be seen as an exponent of the third approach since the bounds therein result from the implicit determination of a piecewise-linear function that represents a non-parametric approximation to the assumed homothetic utility function.

values were used. By contrast, the present paper employs a panel data set of prices and quantities to estimate country-specific demand systems and then uses these as the basis for calculating Lloyd-Moulton–type estimates of true PPPs.

Section 2 describes the model used to estimate the demand-system parameters and the associated true PPPs. Conceptual descriptions of the latter are provided in Section 3. The data set is described in Section 4, and the empirical results are presented and analyzed in Section 5. Section 6 reconciles these results with Diewert (1999), and Section 7 concludes.

2. The Estimation Model

Assume a representative household faced with a fixed budget $y \in \mathbb{R}_{++}$ and commodity prices $\mathbf{p} := (p_1, \ldots, p_m)^\top \in \mathbb{R}^m_{++}$ that chooses a consumption bundle $\mathbf{x} := (x_1, \ldots, x_m)^\top \in \mathbb{R}^m_{+} \setminus \{\mathbf{0}\}$ to maximize a constant-elasticity-of-substitution (CES) utility function

$$u(\mathbf{x}) = \left(\sum_{\ell} \alpha_\ell x_\ell^\beta \right)^{\frac{1}{\beta}} \quad , \tag{1}$$

where $\alpha_\ell \in \mathbb{R}_+$, $\sum_\ell \alpha_\ell = 1$, and $\beta \in (-\infty, 1]$. CES preferences are assumed because the paucity of detailed cross-county consumption data does not allow the estimation of more sophisticated function forms. The (necessary) Kuhn-Tucker (1951) conditions for the (representative) household's decision problem yield the consumer demand system

$$\omega_\ell = \frac{a_\ell p_\ell^b}{\sum_j a_j p_j^b} \quad , \ell = 1, \ldots, m, \tag{2}$$

where $\omega_\ell := p_\ell x_\ell / y$ is the ℓth expenditure share, $a_\ell := \alpha_\ell^{1-b}$, $\equiv 0$ if $x_\ell = 0$, and $b := \beta / (\beta - 1)$.

Suppose that all m expenditure shares and only (the first) m^* ($< m$) commodity prices are observed in each of the T periods for which data on the household's purchases are collected. If the commodity types that correspond to the missing prices are fairly representative of the basket used to calculate the consumer price index (P) for the country in which the household resides, it is reasonable to assume that each missing price is proportional to P; i.e.,

$$p_j = \gamma_j P \, , \, j = m^* + 1, \dots, m, \tag{3}$$

where $\gamma_j \in \mathbb{R}_{++}$ is the relevant factor of proportionality. Substituting for p_j in (2) using (3) yields

$$\begin{aligned} \omega_\ell &= \frac{a_\ell p_\ell^b}{\sum_{j=1}^{m^*} a_j p_j^b + \sum_{j=m^*+1}^{m} a_j \left(\gamma_j P\right)^b} \\ &= \frac{a_\ell p_\ell^b}{\sum_{j=1}^{m^*} a_j p_j^b + A P^b} \, , \end{aligned}$$

where $A := \sum_{j=m^*+1}^{m} a_j \gamma_j^b$. Since P is a suitable proxy for the $m - m^*$ missing prices, equations (2) with $m := m^* + 1$, $p_m := P$ and $a_m := A$ is an estimable system of share equations.

The most common stochastic assumption in relation to the estimation of a system of share equations such as (2) is to add to the right-hand side of each equation a normally distributed disturbance e_ℓ with the property that $E\left(\tilde{\mathbf{e}}\tilde{\mathbf{e}}^\top\right) = \tilde{\mathbf{\Omega}}$, where $\tilde{\mathbf{e}} := (e_1, \dots, e_m)^\top$ and $\tilde{\mathbf{\Omega}}$ is positive definite and constant across observations. Accordingly, the observed shares are

$$\omega_\ell = \frac{a_\ell p_\ell^b}{\sum_j a_j p_j^b} + e_\ell \, , \, \ell = 1, \dots, m. \tag{4}$$

The assumption that the share equations (4) have a constant disturbance covariance matrix $\tilde{\mathbf{\Omega}}$ seems reasonable because $\omega_\ell \in [0, 1]$ for all ℓ. Since $\sum_\ell \omega_\ell = 1$ and since the right-hand sides of (4) sum to one by construction, it must be the case that $\mathbf{1}_m^\top \tilde{\mathbf{e}} = 0$, where $\mathbf{1}_m$ denotes the m-dimensional column vector of ones. Consequently, $\mathbf{1}_m^\top \tilde{\mathbf{\Omega}} = \mathbf{0}$, which means that $\tilde{\mathbf{\Omega}}$ is singular. In turn, this means that the (joint) probability density for $\tilde{\mathbf{e}}$ may be expressed in terms of the probability density of any $m - 1$ of the e_ℓ.[4]

[4] See for example Barten (1969).

Arbitrarily dropping the mth share equation, the probability density for $\mathbf{e}^t := (e_1^t, \ldots, e_{m-1}^t)^\top$ with corresponding covariance matrix $\mathbf{\Omega}$ is given by

$$f\left(\mathbf{e}^t\right) = (2\pi)^{-\frac{1}{2}(m-1)} \left|\mathbf{\Omega}\right|^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}\mathbf{e}^{t\top}\mathbf{\Omega}^{-1}\mathbf{e}^t\right) \tag{5}$$

and the log(arithm) of the likelihood function for the sample of T (independent) observations is given as

$$L\left(\mathbf{a}, b, \mathbf{\Omega}\right) = -\frac{T}{2}\left(m-1\right)\ln 2\pi - \frac{T}{2}\ln\left|\mathbf{\Omega}\right| - \frac{1}{2}\sum_t \mathbf{e}^{t\top}\mathbf{\Omega}^{-1}\mathbf{e}^t\ , \tag{6}$$

where $\mathbf{a} := (a_1, \ldots, a_m)^\top$. The aforementioned paucity of data motivates the additional assumption of "diagonal heteroscedasticity"; i.e, $\mathbf{\Omega} = \operatorname{diag}\boldsymbol{\sigma}^2$, where $\boldsymbol{\sigma}^2 = \left(\sigma_1^2, \ldots, \sigma_{m-1}^2\right)^\top$. This is because an unrestricted covariance matrix would have too many parameters whereas a diagonal *homo*scedastic covariance matrix would be overly simplistic.

The first-order necessary conditions for maximizing $L\left(\mathbf{a}, b, \mathbf{\Omega}\right)$ given $\mathbf{a}$ and b yield

$$\hat{\sigma}_\ell^2 = \frac{1}{T}\sum_t \left(e_\ell^t\right)^2\ ,\ \ell = 1, \ldots, m-1, \tag{7}$$

which, upon substitution for $\boldsymbol{\sigma}^2$ in (6), yields

$$\tilde{L}(\mathbf{a}, b) := L(\mathbf{a}, b, \hat{\boldsymbol{\sigma}}^2(\mathbf{a}, b)) = -\frac{T}{2}\left[(m-1)\ln\left(2\pi\right) - m\right] - \frac{T}{2}\sum_\ell \ln(\hat{\sigma}_\ell^2)\ .$$

By (7), maximizing $\tilde{L}(\mathbf{a}, b)$ is equivalent to minimizing

$$\phi(\mathbf{a}, b) := \frac{1}{2}\sum_j\sum_t \left(e_j^t\right)^2 = \frac{1}{2}\sum_j\sum_t \left[\omega_j^t - \frac{a_j\left(p_j^t\right)^b}{\sum_k a_k\left(p_k^t\right)^b}\right]^2\ , \tag{8}$$

where the equality follows by (4).

Given a sample of T observations of m commodity prices and expenditure-shares for a particular country, the solution to the multidimensional nonlinear least-squares problem $\min_{\mathbf{a},b}\phi(\mathbf{a}, b)$ is a country-specific parameterization of the representative household's preferences. Obtaining such parameterizations for several countries enables the estimation of true PPP indexes relevant to those countries as a group. The next section of the paper provides descriptions of the two specific indexes of this sort that are calculated and compared with the corresponding EKS and GK indexes in Section 5.

3. True PPP Indexes

Consider a bloc comprising $n \geq 2$ countries, each of which has a representative household faced with a fixed budget $y_k \in \mathbb{R}_{++}$ and country-specific commodity prices $\mathbf{p}_k := (p_{k1}, \ldots, p_{km})^\top \in \mathbb{R}^m_{++}$, $k \in \{1, \ldots, n\} =: \mathcal{N}$. Subject to this constraint, household k chooses a consumption bundle $\mathbf{x}_k := (x_{k1}, \ldots, x_{km})^\top \in \mathbb{R}^m_+ \setminus \{\mathbf{0}\}$ to maximize a CES utility function with country-specific parameters.

In this context, the (Konüs-type) true PPP index for household k is the ratio of the minimum expenditure required to attain a particular utility level υ_k under the price regimes of any two countries in the bloc:

$$r_k(\mathbf{p}_i, \mathbf{p}_j, \upsilon_k) := \frac{c_k(\mathbf{p}_i, \upsilon_k)}{c_k(\mathbf{p}_j, \upsilon_k)} , \tag{9}$$

where

$$c_k(\mathbf{p}, \upsilon_k) := \min_{\mathbf{x} \in \mathbb{R}^m_+ \setminus \{\mathbf{0}\}} \left\{ \mathbf{p}^\top \mathbf{x} : u_k(\mathbf{x}) \geq \upsilon_k \right\} . \tag{10}$$

Since the functional form of u_k is CES,

$$c_k(\mathbf{p}, \upsilon_k) = \left(\sum_\ell a_{k\ell} p_\ell^{b_k} \right)^{\frac{1}{b_k}} \upsilon_k \tag{11}$$

and

$$r_k(\mathbf{p}_i, \mathbf{p}_j, \upsilon_k) = \left[\frac{\sum_\ell a_{k\ell} \left(p_{i\ell}\right)^{b_k}}{\sum_\ell a_{k\ell} \left(p_{j\ell}\right)^{b_k}} \right]^{\frac{1}{b_k}} . \tag{12}$$

The number $r_k(\mathbf{p}_i, \mathbf{p}_j, \upsilon_k)$ is the factor by which household k's nominal expenditure at country-i

prices must be deflated in order to make it equal to the same household's nominal expenditure at country-j prices. Since r_k is a ratio of minimum costs of the same utility level, it is transitive with respect to $\mathbf{p}_i$ and $\mathbf{p}_j$.

Clearly, r_k is country-specific. Since there is, in general, no good reason to choose one country's representative household over another's to represent the bloc as a whole, it is necessary to aggregate the r_ks into an index that in some sense reflects the preferences of all representative households in the bloc. A desirable requirement of such an index is that it preserve the transitivity property of r_k. One way of doing this is to take a household-share–weighted geometric mean of the country-specific PPP indexes to obtain the *multiplicative democratic PPP index for country i relative to country j*:

$$R_{MD}(\mathbf{p}_i, \mathbf{p}_j, \boldsymbol{\upsilon}, \mathbf{h}) := \prod_k \left[r_k(\mathbf{p}_i, \mathbf{p}_j, \upsilon_k)\right]^{\bar{h}_k} \ , \tag{13}$$

where $\bar{h}_k := h_k / \mathbf{1}_n^\top \mathbf{h}$ is the fraction of bloc households living in country k, $\mathbf{1}_n$ being the n-dimensional (column) vector of ones and $\mathbf{h} := (h_1, \ldots, h_n)^\top$ being the vector of household numbers. This index was originally defined by Diewert (1984) in an intertemporal context.

Another way to construct a transitive PPP index for a bloc of countries is due to Prais (1959) and Pollak (1980). The (*Prais-Pollak*) *plutocratic PPP index* is an expenditure-share–weighted arithmetic mean of the country-specific PPP indexes; i.e.,

$$R_{PP}(\mathbf{p}_i, \mathbf{p}_j, \boldsymbol{\upsilon}, \mathbf{h}) := \sum_k \alpha_k(\mathbf{p}_j, \boldsymbol{\upsilon}, \mathbf{h}, 1)\, r_k(\mathbf{p}_i, \mathbf{p}_j, \upsilon_k) \ , \tag{14}$$

where

$$\alpha_k(\mathbf{p}_j, \boldsymbol{\upsilon}, \mathbf{h}, 1) := \frac{h_k\, c_k(\mathbf{p}_j, \upsilon_k)}{\sum_l h_l\, c_l(\mathbf{p}_j, \upsilon_l)} \tag{15}$$

is country-k's share of (possibly hypothetical) bloc expenditure at prices $\mathbf{p}_j$ and utility levels

$\boldsymbol{\upsilon} := (\upsilon_1, \ldots, \upsilon_n)^\top$.

As shown in Armstrong (2001a), the multiplicative democratic and plutocratic PPP indexes are both members of the class of bloc-specific PPP indexes that are means-of-order-λ ($\in \mathbb{R}$) of the r_ks and have the same desirable properties of dimensionality, monotonicity, homogeneity and transitivity as does r_k. Hence, from the perspective of economic theory, there is an infinite number of ways to construct a suitable bloc-specific PPP index. The two possibilities described above have been singled out because of their relative familiarity and intuitive bases.

Since the underlying utility function is homothetic, r_k is independent of υ_k and is therefore "free of the objection that the true price index is arbitrary because it applies only to a particular level of utility" (Lloyd, 1975, p. 303).[5] As a household-weighted average of the r_ks, the multiplicative democratic PPP index inherits this property with respect to $\boldsymbol{\upsilon}$. The plutocratic PPP index does not, however, because α_k depends on $\boldsymbol{\upsilon}$. Consequently, it is necessary to choose a utility level for each household in order to calculate plutocratic PPPs.

The natural choice for υ_k is the value $u_k^t := u_k(\mathbf{x}_k^t)$ attained by household k when facing prices $\mathbf{p}_k^t$. This value is given by the associated indirect utility function as

$$v(\mathbf{p}_k^t, y_k^t) = \left[\sum_{\ell} a_{k\ell} \left(p_{k\ell}^t \right)^{b_k} \right]^{-\frac{1}{b_k}} y_k^t \, . \tag{16}$$

Hence, from (15) and (14),

$$\alpha_k(\mathbf{p}_j^t, \mathbf{u}^t, \mathbf{h}^t, 1) = \frac{h_k^t \, y_k^t \, / \, r_k(\mathbf{p}_k^t, \mathbf{p}_j^t, u_k^t)}{\sum_l h_l^t \, y_l^t \, / \, r_l(\mathbf{p}_l^t, \mathbf{p}_j^t, u_l^t)} \tag{17}$$

5 It is not, of course, free of the objection stemming from "150 years of empirical evidence ... that demand patterns are inconsistent with homotheticity" (Slesnick, 1998, p. 2111).

and

$$R_{PP}(\mathbf{p}_i^t, \mathbf{p}_j^t, \mathbf{u}^t, \mathbf{h}^t) = \sum_k \left\{ \sum_l \left[\frac{h_k^t\, y_k^t \,/\, r_k(\mathbf{p}_k^t, \mathbf{p}_i^t, u_k^t)}{h_l^t\, y_l^t \,/\, r_l(\mathbf{p}_l^t, \mathbf{p}_j^t, u_l^t)} \right]^{-1} \right\}^{-1} , \tag{18}$$

where $\mathbf{u}^t := (u_1^t, \ldots, u_n^t)^\top$. Since $h_k^t\, y_k^t$ is the nominal value of consumption expenditure in country k during year t, $h_k^t\, y_k^t \,/\, r_k(\mathbf{p}_k^t, \mathbf{p}_i^t, u_k^t)$ is the corresponding real value in country-i currency units and R_{PP} is a sort of harmonic mean of the possible ratios of such values of the same dimensionality.

4. The Data

The raw price and expenditure data used in the empirical work of the next section are those of the Eurostat-OECD PPP Programme. These data include the OECD-calculated PPPs and national expenditures pertaining to each of the first 157 basic headings[6] of the major aggregate called "Final Consumption of Resident Households"[7] for each of the OECD-24 countries during 1990, 1993 and 1996,[8] as well as the underlying detailed price data supplied to the OECD by Statistics Canada. The latter component was necessary because the basic-heading PPPs by themselves contain no information about the relative prices within a given country. This is due to the fact that the PPP for country k at basic heading ℓ is measured in terms of country-k currency

[6] In principle, a basic heading consists of a small group of similar well-defined goods or services. In practice, it is the lowest level of classification for which expenditures can be estimated. Consequently, an actual basic heading can cover a broader range of commodities than is theoretically desirable.

[7] The 158th basic heading in this aggregate, "net purchases abroad," was excluded in order to avoid the complications related to the possibility of negative national expenditures.

[8] The three most recent "benchmark" years for which PPP data were available at the time this research was undertaken. Descriptions of these data are given in OECD (1992/93, 1995/96, 1999).

units per U.S. dollar; i.e.,

$$\bar{p}_{kl} \equiv \frac{p_{k\ell}}{p_{US,\ell}} \, .$$

Given the basic-heading prices for Canada ($\mathbf{p}_{CA}$), however, the corresponding prices for each of the other twenty-three countries can be extracted from $(\bar{p}_{kl})$ as

$$p_{k\ell} = \frac{p_{CA,\ell}}{\bar{p}_{CA,\ell}} \, \bar{p}_{k\ell} \, .$$

Of the relevant 157 basic headings, Statistics Canada collected detailed price information pertinent to 131 in 1990, 135 in 1993, and 118 in 1996—114 of which are common to all three years. The prices associated with each of these 114 basic headings were aggregated into $\mathbf{p}_{CA}$ by taking unweighted geometric means. Since the missing basic headings are fairly evenly distributed among the major expenditure categories at the next highest level of aggregation, they were treated as a single good and then priced using the "all items" consumer price indexes reported in OECD (1989–97).[9]

The 115 observed expenditure shares for each representative household in each year were calculated directly from the raw expenditure data. The household numbers ($\mathbf{h}$) were furnished either directly or indirectly[10] by the United Nations (1993, 1997), UN-Habitat (2002) and the Australian Bureau of Statistics (1995).

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9 Since the detailed prices for each benchmark year of the Eurostat–OECD PPP Programme were collected by the associated national statistical services over a period of between two and three years, an unweighted average of the monthly (or, in the cases of Australia, New Zealand and Ireland, quarterly) Consumer Price Indexes over the relevant period was used as the missing-goods price for each country in each benchmark year.

10 Via geometric interpolation or extrapolation from non-benchmark–year household numbers.

5. Empirical Results

Given the data described in the preceding section, the objective function (8) of the multidimensional nonlinear least-squares minimization problem stated at the end of Section 2 can be re-written in terms of a vector of residuals of $(m-1)\,T = 342$ functions $\boldsymbol{\varepsilon}(\mathbf{a}, b) := [\varepsilon_1(\mathbf{a}, b), \ldots, \varepsilon_{342}(\mathbf{a}, b)]^\top$,

$$\varepsilon_{114(t-1)+j}(\mathbf{a}, b) := \omega_j^t - \frac{a_j \left(p_j^t\right)^b}{\sum_{k=1}^{115} a_k \left(p_k^t\right)^b} \ , \tag{19}$$

in $(m-1)+1 = 115$ parameters $a_1, \ldots, a_{114}, b$ under the normalization $a_{115} = 1$; i.e.,

$$\phi(\mathbf{a}, b) = \tfrac{1}{2} \left\|\boldsymbol{\varepsilon}(\mathbf{a}, b)\right\|^2 \ , \tag{20}$$

where $\|\cdot\|$ refers to the ℓ_2 vector norm. Written in this form, the problem is amenable to Moré's (1978) "robust and efficient implementation of a version of the Levenberg-Marquardt algorithm" (p. 106), which proceeds from an initial guess for $(\mathbf{a}, b)$ using the linearization

$$\|\boldsymbol{\varepsilon}(\mathbf{a} + \mathbf{da}, b + db)\| \approx \left\| \boldsymbol{\varepsilon}(\mathbf{a}, b) + D\boldsymbol{\varepsilon}(\mathbf{a}, b) \left[da_1, \ldots, da_{114}, db\right]^\top \right\| \ ,$$

where

$$D\boldsymbol{\varepsilon}(\mathbf{a}, b) := \begin{bmatrix} \frac{\partial \varepsilon_1(\mathbf{a},b)}{\partial a_1} & \cdots & \frac{\partial \varepsilon_1(\mathbf{a},b)}{\partial a_{114}} & \frac{\partial \varepsilon_1(\mathbf{a},b)}{\partial b} \\ \vdots & & \vdots & \vdots \\ \frac{\partial \varepsilon_{342}(\mathbf{a},b)}{\partial a_1} & \cdots & \frac{\partial \varepsilon_{342}(\mathbf{a},b)}{\partial a_{114}} & \frac{\partial \varepsilon_{342}(\mathbf{a},b)}{\partial b} \end{bmatrix}$$

is the relevant Jacobian matrix and $(\mathbf{da}, db)$, $da_{115} = 0$, is the proposed step.

For each OECD-24 country, the problem was run on one of the SunFire 6800 servers of the High Performance Computing Virtual Laboratory (HPCVL) at Carleton University in double precision and under the Unix C compiler with the MINPACK-based nonlinear least-squares fitting routines of the GNU Scientific Library (2002). From the initial guess $b = 0$ and

$$a_\ell = \frac{\bar{\omega}_\ell \,/\, \bar{p}_\ell^b}{\bar{\omega}_m \,/\, \bar{p}_m^b} \ \ , \ell = 1, \ldots, m,$$

where $\bar{\omega}_\ell := (\prod_t \omega_\ell^t)^{\frac{1}{T}}$ and $\bar{p}_\ell := (\prod_t p_\ell^t)^{\frac{1}{T}}$, each run converged rapidly to the associated global minimizer $(\mathbf{a}^*, b^*)$.[11] The b components of these minimizers are presented as elasticities of substitution $\sigma^* := 1 - b^*$ in Table 1 along with the four a components corresponding to the four highest mean expenditure shares,[12] the minimum residuals $\|\boldsymbol{\varepsilon}(\mathbf{a}^*, b^*)\|$, and the numbers of iterations to convergence. The residuals range from 0.014 to 0.110 with a median value of 0.038 indicating reasonably good fits to the data. The σ values range from 0.294 to 1.23 with a median value of 0.906 indicating moderate degrees of substitutability at the basic-heading level. The values of a_{60}^*, a_{59}^*, a_{85}^* and a_{113}^* range, respectively, from 0.029 to 5.04, from 0.015 to 1.46, from 0.001 to 3.94 and from 0.076 to 0.560. These wide ranges, in conjunction with those of the other estimated a values, imply a high degree of variation in tastes across the OECD-24 countries.

Tables 2, 3 and 4 present the multiplicative democratic and plutocratic PPPs (columns 4 and 5) calculated on the basis of the estimated preferences of the representative households using equations (13) and (14) for each OECD-24 country in each of the three considered benchmark years. The underlying household numbers (expressed as fractions of the OECD-24 total) and actual bloc expenditure shares at U.S. prices, $\alpha_k(\mathbf{p}_{US}^t, \mathbf{u}^t, \mathbf{h}^t, 1)$ given by equation (17), are also

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11 That this was so is strongly suggested by the fact that, for a given country, precisely the same solution emerged from different initial guesses, including one with $b = 10$.

12 $\frac{1}{72} \sum_t \sum_k \omega_{k,\ell}^t$ equal to 0.100, 0.043, 0.041 and 0.035, respectively, for $\ell = 60, 59, 85$ and 113, which correspond to "imputed rents of owner-occupiers," "rents of tenants," "passenger vehicles with diesel engines" and "restaurants and take-aways."

included (columns 2 and 3), as are the corresponding exchange rates (column 8) and EKS and GK PPPs (columns 6 and 7). Descriptions of the multilateral comparison formulae used to calculate these PPPs are given in Armstrong (2003, Sec. 4).

Lloyd (1975, Sec. II) used a percentage difference formula to calculate mis-specification biases between different parameterizations of a true (two-level CES) bilateral price index and a non-true (axiomatic) counterpart (the Laspeyres index). As shown in Armstrong (2001b, Sec. 2), analogous biases in the present multilateral context are most appropriately measured via the mean absolute log difference indicator

$$\Delta_{\rho,R} = \frac{\sum_i \sum_{j \neq i} \left| \ln \left(\frac{\rho^{ij}}{R^{ij}} \right) \right|}{n\,(n-1)}\,, \tag{21}$$

which gives the total bias that results from using the set of (n^2) PPPs $\{\rho^{ij}\}$ generated by an axiomatic index-number formula ρ in place of the corresponding set $\{R^{ij}\}$ generated by a true index-number formula R. Since $\Delta_{\rho,R}$ possesses the most important properties of ordinary distance, it is a reasonable and intuitive measure of the difference between alternative sets of PPPs. Since it is based on the normed, symmetric, and additive log difference indicator, it avoids the asymmetry and non-additivity problems of a measure based on percentage differences while behaving in approximately the same manner as such a measure when the ratios ρ^{ij}/R^{ij} are each close to one.

The total biases between the true and non-true PPPs in Tables 2, 3 and 4 are on the order of forty per cent; i.e., $\Delta_{\rho,R} \approx 0.4$.[13] By contrast, the total biases between the true PPPs and the

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[13] Specifically, the mean absolute log differences between the multiplicative democratic and EKS private final consumption PPPs in 1990, 1993 and 1996 are, respectively, 0.405, 0.388 and 0.385; the differences between the multiplicative democratic and GK PPPs are 0.374, 0.373 and 0.366; the differences between the plutocratic and EKS

exchange rates are on the order of fifty per cent. Thus, although these results lend support to the consensus view among experts that axiomatic PPPs are closer to "the truth" than are exchange rates, they also suggest that the former gap may be quite wide. Explaining how this could be so in the light of recent results obtained under the exact approach is the focus of the following section.

6. Reconciliation With Past Results

Diewert (1999, Sec. 2) introduced a multilateral counterpart to his bilateral concept of superlativeness[14] that follows from a "natural" exactness property. Specifically, under "the very strong assumption that a common linearly homogeneous [utility] function u exists across countries" (p. 20), the multilateral axiomatic system $\{\rho^{ij}(\mathbf{P}, \mathbf{X}, \mathbf{h})\}$ is exact for the differentiable unit expenditure function $\pi : \mathbb{R}^m_{++} \to \mathbb{R}$ dual to u if, for all $(i, j) \in \mathcal{N} \times \mathcal{N}$, $\mathbf{P} := (\mathbf{p}_1, \ldots, \mathbf{p}_n)^\top \in \mathbb{R}^{nm}_{++}$ and $\boldsymbol{\mu} := (\mu_1, \ldots, \mu_n)^\top \in \mathbb{R}^n_{++}$,

$$\rho^{ij}(\mathbf{P}, \mathbf{X}, \mathbf{h}) = \frac{\pi(\mathbf{p}_i)}{\pi(\mathbf{p}_j)} \tag{22}$$

when $\mathbf{X} := (\mathbf{x}_1, \ldots, \mathbf{x}_n)^\top$ is equal to the vector of Hicksian demands $[\nabla\pi(\mathbf{p}_1)\mu_1, \ldots, \nabla\pi(\mathbf{p}_n)\mu_n]^\top$. A system $\{\rho^{ij}(\mathbf{P}, \mathbf{X}, \mathbf{h})\}$ is deemed to be superlative if it is exact for a flexible functional form; i.e., for a π that can provide a second-order differential approximation to an *arbitrary*, twice continuously differentiable, linearly homogeneous unit expenditure function.

Since Diewert (1999, Prop. 8) has proven that the EKS system $\left\{\rho^{ij}_{EKS}(\mathbf{P}, \mathbf{X}, \mathbf{h})\right\}$ is superlative in reference to the homogeneous quadratic unit expenditure function

$$\pi(\mathbf{p}) = \left(\mathbf{p}^\top \mathbf{B} \mathbf{p}\right)^{\frac{1}{2}} \text{ with } \mathbf{B}^\top = \mathbf{B}, \tag{23}$$

PPPs are 0.434, 0.415 and 0.419; and the differences between the plutocratic and GK PPPs are 0.402, 0.400 and 0.400.

[14] See Diewert (1976, p. 117) or Diewert (1981, p. 185).

it may be argued that $\{\rho^{ij}_{EKS}(\mathbf{P}, \mathbf{X}, \mathbf{h})\}$ is consistent with underlying utility functions that allow for adequate substitution possibilities between commodities. Why, then, do the empirical results of the preceding section manifest such a large gap between the EKS PPPs and the corresponding true PPPs based on CES utility functions?[15] The answer to this question stems from Collier's (1999, p. 103) remark that "[f]or international ... comparisons we are pushing the methodological envelope when we insist on playing the game solely under the assumption of identical preferences." In contrast to the exact approach of Diewert (1999), the approach of the present paper admits the possibility of variation in tastes across countries. Since the results that follow from the latter approach suggest the existence of a high degree of such variation, a critical premiss of the former approach would appear to be false.

An indication of just how critical, in an empirical sense, the assumption of identical preferences is to the claim that EKS PPPs should be close to their true counterparts can be provided by a simple numerical example. Assume two countries ($n = 2$) each comprised of a single household ($h_k = 1$) that has Cobb-Douglas preferences ($\sigma_k = 1 \Leftrightarrow b_k = 0 \Leftrightarrow \beta_k = 0$) over two commodities ($m = 2$). Suppose that the budgets and expenditure shares are such that the two households choose the same consumption bundle in equilibrium; i.e.,

$$x_{i\ell}(\mathbf{p}_i, y_i) = x_{j\ell}(\mathbf{p}_j, y_j)$$

or, equivalently,

$$\frac{\alpha_{i\ell}}{\alpha_{j\ell}} \frac{y_i}{y_j} = \frac{p_{i\ell}}{p_{j\ell}} \, ,$$

where $\alpha_{k\ell}$ is the share of country-k expenditure on commodity ℓ. In this context, the multiplicative

[15] The difference of roughly four per cent between the EKS and GK PPPs is due to the fact that the former comprise a superlative system whereas the latter do not (Diewert, 1999, Prop. 4).

democratic and EKS PPPs for country 1 relative to country 2 are given as

$$\begin{aligned} R_{MD}^{12} &= \left[\frac{c_1(\mathbf{p}_1, u_1)}{c_1(\mathbf{p}_2, u_1)} \frac{c_2(\mathbf{p}_1, u_2)}{c_2(\mathbf{p}_2, u_2)}\right]^{\frac{1}{2}} \\ &= \left[\frac{y_1}{y_2} \frac{c_2(\mathbf{p}_1, u_2)}{c_1(\mathbf{p}_2, u_1)}\right]^{\frac{1}{2}} \end{aligned} \tag{24}$$

and

$$\rho_{EKS}^{12} = \left(\frac{\mathbf{p}_1^\intercal \mathbf{x}_1}{\mathbf{p}_2^\intercal \mathbf{x}_2}\right)^{\frac{1}{2}} \prod_k \left(\frac{\mathbf{p}_k^\intercal \mathbf{x}_2}{\mathbf{p}_2^\intercal \mathbf{x}_k} \frac{\mathbf{p}_1^\intercal \mathbf{x}_k}{\mathbf{p}_k^\intercal \mathbf{x}_1}\right)^{\frac{1}{2n}} \tag{25}$$

$$\begin{aligned} &= \left(\frac{y_1}{y_2}\right)^{\frac{1}{2}} \prod_k \left(\frac{\mathbf{p}_1^\intercal \mathbf{x}_1}{\mathbf{p}_2^\intercal \mathbf{x}_2}\right)^{\frac{1}{4}} \quad \text{since } \mathbf{x}_2 = \mathbf{x}_1 \\ &= \left(\frac{y_1}{y_2}\right)^{\frac{1}{2}} \left(\frac{y_1}{y_2}\right)^{\frac{1}{2}} \\ &= \frac{y_1}{y_2}\,, \end{aligned} \tag{26}$$

respectively. Supposing further that $(\alpha_{11}, \alpha_{21}) = (0.562, 0.02)$, $(y_1, y_2) = (2.28, 1.02)$ and $p_{12} = p_{22} = 1$, the situation of the two countries is as depicted in Figure 1 with each household consuming one unit of each commodity.

From (26) and (24), the ratio of the EKS and multiplicative democratic PPPs for country 1 relative to country 2 is

$$\frac{\rho_{EKS}^{12}}{R_{MD}^{12}} = \left[\frac{y_1/y_2}{c_2(\mathbf{p}_1, u_2)/c_1(\mathbf{p}_2, u_1)}\right]^{\frac{1}{2}} . \tag{27}$$

In terms of Figure 1, the right-hand side of this equation is the square root of the ratio of the x_2-intercepts of the actual country-1 and country-2 budget lines divided by the ratio of the x_2-intercepts of the hypothetical country-2 and country-1 budget lines that would just enable the attainment of each country's equilibrium indifference curve at the other country's prices.

Substituting for these intercepts using the rough estimates suggested by the graph yields

$$\frac{\rho_{EKS}^{12}}{R_{MD}^{12}} \approx \left[\frac{2.3/1}{1/0.2}\right]^{\frac{1}{2}} = 0.678 \ ,$$

which, upon substitution into (21), yields

$$\Delta_{EKS,MD} = \left|\ln\left(\frac{\rho_{EKS}^{12}}{R_{MD}^{12}}\right)\right| \approx |\ln 0.678| \approx 0.4 \ .$$

Thus a bias on the order of forty per cent from the use of $\{\rho_{EKS}^{ij}\}$ in place of $\{R_{MD}^{ij}\}$ can be due to differences in only the α parameters of underlying (country-specific) CES utility functions.

The welfare aspects of the preceding example are also of interest. By Armstrong (2001b, Theorem 4), the (real) bloc consumption shares associated with the multiplicative democratic and EKS PPP indexes are given as

$$S_{MD,1} = \left\{1 + \frac{y_2}{y_1}/R_{MD}^{21}\right\}^{-1} \approx \left\{1 + \frac{1}{2.3}\left[\frac{2.3}{1}\frac{1}{0.2}\right]^{\frac{1}{2}}\right\}^{-1} = 0.40 \ ,$$

$$S_{MD,2} = \left\{1 + \frac{y_1}{y_2}/R_{MD}^{12}\right\}^{-1} \approx \left\{1 + \frac{2.3}{1}\left[\frac{1}{2.3}\frac{0.2}{1}\right]^{\frac{1}{2}}\right\}^{-1} = 0.60$$

and

$$\sigma_{EKS,i} = \left\{1 + \frac{y_j}{y_i}/\rho_{EKS}^{ji}\right\}^{-1} = \left\{1 + \frac{y_j}{y_i}\frac{y_i}{y_j}\right\}^{-1} = 0.50 \ , \ j \neq i = 1,2.$$

In words, the country-2 household is fifty per cent better off than its country-1 counterpart according to the true index with equal household weights, and equally well off according to the axiomatic EKS index. The important point here is that if we did not know the preferences of either household, the fact that both consume an identical amount of each commodity in equilibrium would lead us to draw the same inference about relative well-being as does the EKS index. The welfare inference drawn by way of the multiplicative democratic index differs markedly from that of the EKS because the former is informed directly by household preferences whereas the latter is not.

7. Conclusion

There are infinitely-many ways to construct an economically suitable PPP index for a bloc of countries based on the preferences of the constituent households. The choice among such true bloc-specific indexes boils down to a value judgement about the weight that poor households should be given in relation to rich ones. Two possibilities are the multiplicative democratic PPP index, which weights each household equally, and the plutocratic PPP index, which weights each dollar of spending equally.

Since a true bloc-specific PPP index is an aggregation rule over the interspatial cost-of-living indexes of the relevant households, it depends on the preferences of those households. The accuracy of an *estimated* true bloc-specific PPP index, then, depends on the soundness of the underlying assumptions about the context of consumer choice and the functional form of preferences. Due to the paucity of relevant data, the present paper makes fairly strong assumptions in these respects: namely, that commodity prices are uniform throughout each country during each period; that a representative household exists for each country; and that each representative household has (possibly different) fixed CES preferences over 114 of the first 157 basic headings of the major aggregate called "Final Consumption of Resident Households" and a sub-index of the other 43.

On account of the limitations imposed by the data, it cannot be claimed that the multiplicative democratic and plutocratic PPP indexes estimated above are accurate. They are, however, readily comparable to the axiomatic EKS and GK PPP indexes calculated on the same basis. The large differences found between these true and non-true index numbers were shown to be due to the wide variation in tastes inferred from the data. Since it is unlikely that there will be sufficient additional data in the foreseeable future to achieve substantially better true PPP estimates, this result should serve to inform the interpretation of axiomatic PPPs rather than as an argument for replacing them in practice.

TABLE 1—NONLINEAR LEAST-SQUARES ESTIMATES

Country	σ^*	a_{60}^*	a_{59}^*	a_{85}^*	a_{113}^*	$\|\varepsilon(\mathbf{a}^*, b^*)\|$	Iter.
Canada	0.86641	0.83537	0.31109	0.18550	0.40519	0.021666	11
United States	0.89130	0.45877	0.17084	0.14029	0.26079	0.016280	11
Japan	0.95021	0.52916	0.15821	0.07911	0.15851	0.054321	10
Australia	1.02578	0.73247	0.23481	0.23994	0.10018	0.034828	8
New Zealand	1.03932	1.16117	0.27837	0.29063	0.22592	0.029961	6
Austria	1.03554	0.82552	0.34184	0.47806	0.55952	0.027009	6
Belgium	0.81755	0.14304	0.07921	0.04306	0.07557	0.030044	18
Denmark	1.00824	1.00599	0.54437	0.32507	0.21985	0.047550	9
Finland	0.83063	0.57992	0.15151	0.09348	0.32184	0.051478	15
France	0.85933	0.45583	0.18467	0.08795	0.21951	0.014321	12
Germany	0.60784	0.24584	0.23310	0.03524	0.22691	0.030332	19
Greece	0.40610	0.02911	0.01476	0.00067	0.13985	0.056910	27
Iceland	0.59671	0.18532	0.02634	0.01157	0.14126	0.058866	25
Ireland	0.78979	0.27773	0.06109	0.05623	0.15804	0.054320	16
Italy	0.95071	0.38773	0.11017	0.13172	0.25959	0.041470	8
Luxembourg	1.04536	1.10903	0.16844	1.03775	0.21318	0.035364	10
Netherlands	0.88028	0.31191	0.23271	0.07852	0.12616	0.023930	10
Norway	1.21657	1.01172	0.63899	1.26886	0.19088	0.109594	17
Portugal	1.02245	0.38034	0.08714	0.51022	0.47391	0.043889	6
Spain	0.92009	0.13629	0.10530	0.08489	0.20907	0.020660	9
Sweden	0.55666	0.36461	0.21671	0.01590	0.21170	0.043661	16
Switzerland	0.29374	0.09640	0.17156	0.00319	0.50229	0.063231	24
Turkey	1.23039	5.04023	1.46414	3.93954	0.23691	0.057426	19
United Kingdom	0.93935	0.41406	0.21924	0.19009	0.19966	0.034595	7

TABLE 2—PPPS FOR PRIVATE FINAL CONSUMPTION EXPENDITURE IN 1990 (NATIONAL CURRENCY PER U.S. DOLLAR)

Country	$h / \mathbf{1} \cdot \mathbf{h}$	α	MD	PP	EKS	GK	ER[a]
Canada	0.03313	0.02133	1.2556	1.2506	1.3452	1.3100	1.17
United States	0.31067	0.25093	1.0000	1.0000	1.0000	1.0000	1.00
Japan	0.13742	0.27225	82.07	76.99	206.68	186.50	145
Australia	0.01923	0.01126	1.3699	1.3617	1.4466	1.3881	1.28
New Zealand	0.00390	0.00187	1.5849	1.5761	1.6546	1.5811	1.68
Austria	0.01009	0.00669	8.671	8.355	14.309	14.017	11.3
Belgium	0.01326	0.02029	21.809	20.777	40.605	39.100	33.3
Denmark	0.00762	0.00397	6.4242	6.2272	9.8040	9.0791	6.17
Finland	0.00688	0.00342	4.6634	4.5460	6.8639	6.6568	3.83
France	0.07271	0.05688	4.7344	4.6043	6.7094	6.4839	5.43
Germany	0.09138	0.05649	1.7893	1.7698	2.0711	2.0052	1.61
Greece	0.01076	0.01613	65.99	62.21	140.61	131.70	158
Iceland	0.00031	0.00035	44.395	42.374	90.735	85.478	58.3
Ireland	0.00343	0.00125	0.6937	0.6964	0.6840	0.6802	0.603
Italy	0.06776	0.12951	451.9	421.8	1387.2	1331.7	1195
Luxembourg	0.00048	0.00049	20.772	19.798	37.103	35.986	33.3
Netherlands	0.01986	0.01157	1.8424	1.8160	2.1485	2.0257	1.82
Norway	0.00592	0.00256	6.957	6.754	10.671	10.139	6.26
Portugal	0.01057	0.00624	48.51	45.77	105.32	93.87	142.2
Spain	0.03889	0.06496	51.39	48.55	113.32	109.61	101.6
Sweden	0.01294	0.00679	6.1748	6.0027	9.4910	9.0175	5.92
Switzerland	0.00960	0.00706	1.8399	1.8225	2.2255	2.1961	1.38
Turkey	0.03780	0.01424	631.5	587.5	1597.9	1235.2	2613
United Kingdom	0.07538	0.03349	0.6578	0.6628	0.5990	0.5867	0.561

[a] *Source:* OECD (1992, Table 3.1).

TABLE 3—PPPS FOR PRIVATE FINAL CONSUMPTION EXPENDITURE
IN 1993 (NATIONAL CURRENCY PER U.S. DOLLAR)

Country	*h* / **1·h**	α	MD	PP	EKS	GK	ER[a]
Canada	0.03317	0.01980	1.4014	1.3962	1.2990	1.2714	1.2901
United States	0.30537	0.28611	1.0000	1.0000	1.0000	1.0000	1.0000
Japan	0.13405	0.24692	95.26	89.88	201.49	179.01	111.19
Australia	0.01975	0.01097	1.5522	1.5487	1.4036	1.3511	1.4706
New Zealand	0.00389	0.00187	1.7448	1.7359	1.5948	1.5452	1.8505
Austria	0.00975	0.00668	9.921	9.568	14.438	14.403	11.635
Belgium	0.01273	0.02012	24.278	23.171	39.898	38.705	34.561
Denmark	0.00734	0.00381	7.0633	6.8625	9.6156	9.4580	6.4847
Finland	0.00665	0.00299	5.3782	5.2396	6.9268	6.8959	5.7185
France	0.07052	0.05349	5.4271	5.2935	6.9028	6.6796	5.6650
Germany	0.10733	0.06654	2.0307	2.0103	2.1364	2.0832	1.6536
Greece	0.01060	0.02012	109.30	103.61	200.07	187.34	229.35
Iceland	0.00031	0.00035	52.283	49.815	93.121	90.030	67.600
Ireland	0.00338	0.00143	0.8106	0.8169	0.6940	0.6578	0.6831
Italy	0.06580	0.11885	584.2	547.9	1564.8	1537.2	1572.3
Luxembourg	0.00047	0.00059	23.608	22.548	38.051	36.789	34.561
Netherlands	0.01946	0.01197	2.0601	2.0379	2.1925	2.0322	1.8576
Norway	0.00576	0.00279	7.491	7.283	9.832	9.120	7.094
Portugal	0.01030	0.00683	69.34	65.77	131.10	124.78	160.86
Spain	0.03783	0.05936	67.16	63.70	125.41	122.18	127.34
Sweden	0.01255	0.00631	7.650	7.447	10.598	10.561	7.790
Switzerland	0.00942	0.00712	2.0836	2.0647	2.2506	2.2201	1.4775
Turkey	0.04054	0.01478	3194.2	3001.4	6680.6	5967.7	10985
United Kingdom	0.07303	0.03020	0.8443	0.8537	0.6710	0.6797	0.6661

[a] *Source:* OECD (1995, Table 3.1).

TABLE 4—PPPS FOR PRIVATE FINAL CONSUMPTION EXPENDITURE IN 1996 (NATIONAL CURRENCY PER U.S. DOLLAR)

Country	*h* / **1·h**	α	MD	PP	EKS	GK	ER[a]
Canada	0.03416	0.01989	1.4181	1.4135	1.2251	1.1971	1.3635
United States	0.30142	0.31161	1.0000	1.0000	1.0000	1.0000	1.0000
Japan	0.13564	0.25000	92.35	85.68	185.22	164.89	108.78
Australia	0.02106	0.01225	1.5773	1.5726	1.3829	1.3280	1.2779
New Zealand	0.00400	0.00206	1.7588	1.7449	1.5821	1.5157	1.4549
Austria	0.00971	0.00628	10.348	9.926	14.297	14.083	10.587
Belgium	0.01258	0.02002	25.506	24.088	39.380	38.371	30.968
Denmark	0.00727	0.00411	7.0548	6.8123	9.0103	8.5416	5.7994
Finland	0.00660	0.00298	5.6023	5.4357	6.7083	6.5491	4.5928
France	0.07031	0.04945	5.8657	5.6880	7.1442	6.9420	5.1167
Germany	0.10526	0.06586	2.1373	2.1157	2.0833	1.9943	1.5048
Greece	0.01092	0.02011	142.24	133.33	242.46	228.96	240.78
Iceland	0.00031	0.00037	55.481	52.341	89.531	86.446	66.711
Ireland	0.00344	0.00155	0.9145	0.9199	0.7236	0.6878	0.6253
Italy	0.06569	0.11055	673.4	622.1	1662.1	1627.8	1543.7
Luxembourg	0.00048	0.00057	25.145	23.733	38.859	37.563	30.968
Netherlands	0.01959	0.01164	2.2022	2.1762	2.1576	2.0030	1.6862
Norway	0.00577	0.00285	8.008	7.729	10.156	9.730	6.459
Portugal	0.01043	0.00685	80.37	75.17	141.22	132.86	154.27
Spain	0.03743	0.05905	75.18	70.40	131.07	121.95	126.67
Sweden	0.01251	0.00603	8.235	7.966	10.609	10.217	6.710
Switzerland	0.00950	0.00631	2.1388	2.1213	2.1287	2.0287	1.2356
Turkey	0.04317	0.00002	20703	19172	44139	37257	81405
United Kingdom	0.07274	0.02958	0.9140	0.9276	0.6964	0.6931	0.6413

[a] *Source:* OECD (1999, Table 1).

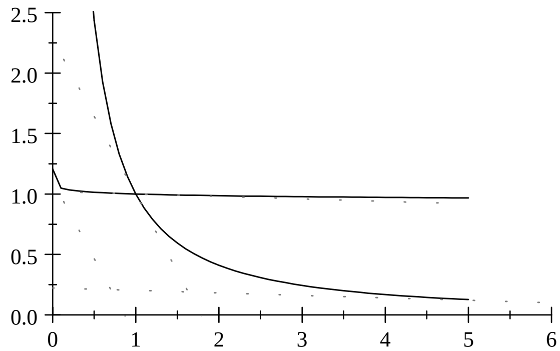

FIGURE 1. $(\alpha_{11}, \alpha_{21}) = (0.562, 0.02)$.

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