# Multiple solutions of direct kinematics of 3-RPR parallel manipulators

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A planar parallel 3-RPR parallel manipulator (Fig.1) consists of three anchor points $(A, B, C)$ in a base connected via three extensible legs $(r_1, r_2, r_3)$ to a triangular platform $(D, E, F)$. In the direct kinematics (DK) one has to compute the pose of the platform when the design of the manipulator (location of the base points and the shape of the moving platform) and the lengths of the legs are given. It is well known that this task allows six solutions (Fig.1 right, shows an example with six real solutions, i.e. geometrically, where the vertices of a given triangle are located on three given circles). When some of the solutions of the DK coincide the manipulator

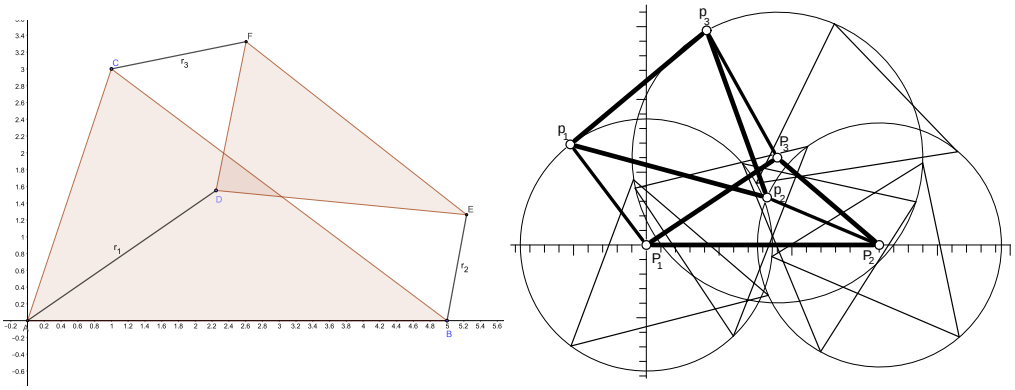


Figure 1: 3-RPR parallel manipulator Example with six real solutions of the DK

becomes singular. In the presentation it will be shown that multiple solutions of the direct kinematics are a new way of looking into the notions of "shakiness" of mechanisms or "flexibility" of pin jointed frameworks of rigid bars. For the first time general conditions for maximal (= 6) coinciding solutions will be given (Fig.1 (left) shows an example with six coinciding solutions). The configuration in Fig.1 does not look exceptional, but a 3-RPR parallel manipulator in such a configuration exhibits a free motion, which can be shown explicitly for example by adding small backlash in the revolute joints of one leg. From mechanical point of view such a design must be avoided. It will be discussed that multiple solutions of the DK are far beyond the classical singularity theory of a manipulator. They belong to so called constraint singularities which describe special situations in the configuration space of the manipulator (Fig.2).

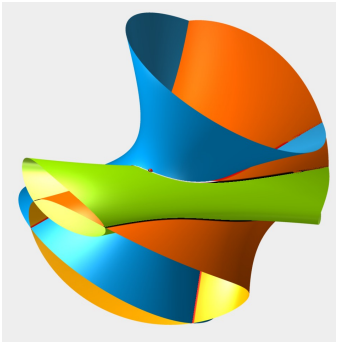

Figure 2: Six coinciding solutions of DK in configuration space

The discussion is done within the framework of algebraic geometry and polynomial equations, because multiple solutions are closed sets. We have therefore also to explain the advantages and limitations of this approach.