Applied Analysis Day

Date: Friday October 20, 2017
Location: Room HP4351, Herzberg Building, Carleton University, Ottawa

Objective: The objective of the workshop is to encourage scientific exchange within the applied analysis community in Ottawa and further afield. Some recent progress in the fields of partial differential equations, dynamical systems, calculus of variations, numerical analysis, scientific computing and mathematical modeling will be discussed. Graduate students and PostDocs in mathematics are encouraged to participate.

This workshop is a part of the celebration of the 25th anniversary of the Fields Institute.

Carleton UNIVERSITY

FIELDS 25TH ANNIVERSARY

Program

9-10 R. McCann (University of Toronto)
Title: Optimal transportation between unequal dimensions
Abstract: In the last few decades, the theory of optimal transportation has blossomed into a powerful tool for exploring applications both within and outside mathematics Its impact is felt in such far flung areas as geometry, analysis, dynamics, partial differential equations, economics, machine learning, weather prediction, and computer vision. The basic problem is to transport one probability density onto other, while minimizing a given cost c(x,y) per unit transported. In the vast majority of applications, the probability densities live on spaces with the same (finite) dimension. After briefly surveying a few highlights from this theory, we focus our attention on what can be said when the densities instead live on spaces with two different (yet finite) dimensions. Although the answer can still be characterized as the solution to a fully nonlinear differential equation, it now becomes badly nonlocal in general. Remarkably however, one can identify conditions under which the equation becomes local, elliptic, and amenable to further analysis.

10-10:30 A. Momeni (Carleton University)
Title: Critical points on convex sets vs critical points!
Abstract: A variational principle is introduced to provide a newformulation and resolution for several boundary value problems with a variational structure. This principle allows one to deal with problems well beyond the weakly compact structure. As a result, we studyseveral super-critical local and non-local partial differential equations.

10:30-11 Coffee break

11-11:30 E. Lorin (Carleton University)
Title: Analysis of convergence of Schwarz waveform relaxation methods for the Schroedinger equation
Abstract: The aim of this talk is to derive and numerically validate some asymptotic estimates of the convergence rate of Classical and Optimized Schwarz Waveform Relaxation (SWR) domain decomposition methods applied to the linear and nonlinear Schroedinger equations.

11:30-12 V. LeBlanc (University of Ottawa)
Title: Weak lattice symmetry-breaking and modulated travelling spiral waves
Abstract: Re-entrant spiral waves are observed in many different situations in nature, perhaps most importantly in excitable electrophysiological tissue where they are believed to be responsible for pathological conditions such as cardiac arrhythmias, epileptic seizures and hallucinations. Mathematically, spiral waves occur as solutions to systems of nonlinear reaction-diffusion partial differential equations (RDPDEs) which are frequently used as models for electrophysiological phenomena. Because of the invariance of these RDPDEs with respect to the Euclidean group SE(2) of planar translations and rotations, much progress has been made in understanding the dynamics and bifurcations of spiral waves using the theory of group-equivariant dynamical systems. In reality however, Euclidean symmetry is at best an approximation. Inhomogeneities and anisotropy in the medium of propagation of the waves break the Euclidean symmetry, and can lead to such phenomena as anchoring and drifting. In this talk, we study the effects on modulated travelling spiral waves near a small perturbation which breaks the continuous SE(2) symmetry,but preserves the symmetry of a regular square lattice.

12:13:30 Lunch

13:30-14 Y. Bourgault (University of Ottawa)
Title: How close can be the solutions of the monodomain and bidomain models?
Abstract: The bidomain model is usually considered as the most complete model to describe the propagation of the cardiac action potential (AP) at the organ level while properly accounting for the anisotropic conduction properties of the myocardium. A simpler version of the model, the monodomain model, is often used to reduce the computational cost, since the bidomain model is expensive to solve. There are instances where both models give solutions relatively close to each other. We will explain how the bidomain can be formulated as a monodomain model with a properly chosen conduction operator, and use this formulation of the bidomain model to assess the error in replacing the bidomain model by a monodomain model. We will present an optimal control problem to adjust the conductance in the monodomain model to recover the conduction speed of an AP wave. Using such control problem, we will show how to recover the solution of the bidomain model with the monodomain model. These concepts will be illustrated by numerical results. This work was conducted jointly with Prof. Yves Coudière, University of Bordeaux.

14-14:30 D. Amundsen (Carleton University)
Title: Transition of Resonant Response in Bounded Media
Abstract: Resonant behaviour in bounded domains under the influence of weakly periodic forcing is a well known and well studied phenomenon. After a transient phase, nonlinear effects induce saturation leading to steady, long time, behaviour. The qualitative nature of this response is generally governed by the nature of the underlying linear spectrum, leading to qualitatively distinct regimes with associated distinct scalings. While each regime is well understood in isolation, the connections between them are not. Through parametric variation of a general class of nonlinear PDEs we will explore this transition and analytic techniques to bridge the distinct response regimes which are in turn relevant in a broad range of underlying applications.

14:30:15 A. Novruzi (University of Ottawa)
Title: A first and second order convergence method in optimization problems
Abstract: I will present a new method for solving optimization problems. The method is of gradient type and simultaneously has a quadratic convergence in a certain sense. The method is developed in a abstract framework. It is very useful particularly in large computationally extensive problems.

15-15:30 Coffee break

15:30-16: L. Campbell (Carleton University)
*Title:*Nonlinear dynamics of vortex Rossby waves in a cyclonic vortex
Abstract: Observational analyses of hurricanes in the tropical atmosphere indicate the existence of spiral rainbands which propagate outwards from the eye and affect the structure and intensity of the hurricane. These disturbances may be described as vortex Rossby waves. It has been suggested that vortex Rossby waves may play a role in the eyewall replacement cycle observed in tropical cyclones in which concentric rings of high-intensity wind develop and propagate in towards the centre of the cyclone. I will present asymptotic analyses (joint work with L. Nikitina) and numerical simulations of wave-mean-flow interactions in a cyclonic vortex in a two-dimensional configuration on a beta-plane.

16:16-30: X. Wang (University of Ottawa)

Title: Complex patterns in a resource-consumer model with short reproduction season

Abstract: Spatial heterogeneous distribution of organisms is commonly seen in nature. The reasons underlying such heterogeneity motivate extensive studies. In this paper, we propose an impulsive reaction-diffusion equation model for a resource-consumer system to study pattern formation in a homogeneous landscape. We consider that the reproduction and dispersal of the consumer happen at distinct stages, and the reproduction season is short whereas the dispersal stage is long. Analytical results and numerical simulations show that pattern formation is possible for the system if ratio-dependent functional response is chosen. More importantly, pattern formation may occur if the consumer and the resource diffuse at a comparable scale, which is different from the continuous time analogy. Furthermore, numerical simulations show that low over-winter survival rate of the resource promotes the occurrence of pattern formation. This suggests that spatial patterns may emerge more easily due to the discontinuity in population densities.