

Rings and Fields Math3158A, Winter 2022

Instructor: Dr. Steven Wang, 4368HP
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Lectures: Tuesday, Thursday, 8:35am - 9:55pm, Online

Tutorials: Tuesday, 1:35pm - 3:25pm, Online.

Office hours: Tuesday 10:00am-11:00am.

Recommended Textbooks:

“*Algebra.*”, by Michael Artin, Prentice Hall, 2nd edition, 2010.

“*Galois Theory*”, by David A. Cox, Wiley, 2nd edition, 2012.

“*Algebra with Galois Theory*”, by Emil Artin, New York University, American Mathematical Society, 2007.

“*Field Theory and Galois Theory*”, by Steven H. Weintraub, Universitext, Springer Science & Business Media, Inc., 2006.

“*Fields and Galois Theory*”, by John M. Howie, Springer Undergraduate Mathematics Series 2006.

Prerequisites: MATH 2100 (Minimum Grade of C-), or permission of the School.

Course Objective: The purpose of this course is to introduce students Galois theory of fields extensions. List of topics include algebraic and transcendental elements, minimum polynomials, symbolic root adjunction, degree of a field extension, multiplicativity of degree, constructions with a compass and straightedge, homomorphism of field extensions, finite fields, Frobenius map, counting irreducible polynomials over \mathbb{Z}_p , function fields, transcendental extensions and bases, algebraically closed fields, normal extensions, separable extensions, splitting fields, primitive elements, characterization of normal fields, action of Galois group on roots, Galois correspondence, cyclotomic fields, Kummer extensions, solvability in radicals, Galois groups. Exact topics may be revised as the term progresses.

Evaluation: Quizzes (20%), Midterm (20%), Assignment (20%), and Final Examination (40%).

Tutorials: Tutorials begin on the week of January 24, 2022. In each tutorial you will be given a quiz with the exception of March 15. There will be 8 quizzes in total.

Midterm: The midterm will be held during the class on Thursday March 17, with a total of 20%.

Final Examination: This is a three hour closed-book exam scheduled by the University and will take place sometime during the examination period (April 14-28, 2022). Students wishing to see their examination papers must make an appointment within three weeks of the examination. This privilege is for you to learn where you went wrong and is not an opportunity to argue about the marking!

Calculators: You may use only simple non-programmable, non-graphing calculators for the tests and the final examination in this course. I reserve the right to disallow any calculators.

Academic Accommodation

You may need special arrangements to meet your academic obligations during the term. For an accommodation request the processes are as follows:

Pregnancy obligation: write to me with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details visit the Equity Services website: http://carleton.ca/equity/accommodation/student_guide.htm

Religious obligation: write to me with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details visit the Equity Services website: http://carleton.ca/equity/accommodation/student_guide.htm

Students with disabilities requiring academic accommodations: in this course must register with the Paul Menton Centre for Students with Disabilities

(PMC) for a formal evaluation of disability-related needs. Documented disabilities could include but are not limited to mobility/physical impairments, specific Learning Disabilities (LD), psychiatric/psychological disabilities, sensory disabilities, Attention Deficit Hyperactivity Disorder (ADHD), and chronic medical conditions. Registered PMC students are required to contact the PMC, 613-520-6608, every term to ensure that I receive your Letter of Accommodation, no later than two weeks before the first assignment is due or the first in-class test/midterm requiring accommodations. If you only require accommodations for your formally scheduled exam(s) in this course, please submit your request for accommodations to PMC by the last official day to withdraw from classes in each term. For more details visit the PMC website: http://www.carleton.ca/pmc/students/acad_accom.html

Math3158A Tentative lecture schedule—subject to change

Week	Dates	Topics
1	Jan. 10-14	Introduction, algebraic elements, transcendental elements and extension fields.
2	Jan. 17-21	algebraic elements and extension fields, adjoining roots, explicit realization of $F(\alpha)$; multiple roots, isomorphisms of field extension.
3	Jan. 24-28	degree of field extensions, algebraic extensions, compass and straightedge construction, doubling a cube and trisecting an angle,
4	Jan. 31-Feb. 4	finite fields; number of irreducible polynomials.
5	Feb. 7-11	transcendental extension, transcendence basis, algebraic closed fields.
6	Feb. 14-18	primitive elements and primitive element theorem, extensions of a homomorphism.
7	Feb. 21-25	Winter break
8	Feb. 28-Mar. 4	extension of a homomorphism, finite normal extension, uniqueness of splitting fields, Examples of Galois groups, splitting fields of irreducible polynomials.
9	Mar. 7 -11	Galois group of finite extension, finite normal extension, Galois correspondence theorem.
10	Mar. 14-18	Galois correspondence theorem, cyclotomic extensions.
11	Mar. 21-25	cyclotomic polynomials and regular n -gon, Kummer extensions.
12	Mar. 28-Apr. 1	Solvable groups and solvable extensions, solvable groups and radical extensions.
13	Apr. 4 -8	example of unsolvable quintic equation; # of real roots, Cardano's formula, Extensions of degree 4.
14	Apr. 11-12	Extensions of finite fields.