

# MATH 4907—MATRIX GROUPS AND GEOMETRY

Winter and Fall 2021

The two topics of this course are related, but have different “vibes”. Matrices are objects usually associated with algebra, especially with the manipulations that come along with multiplication. On the other hand, geometry is associated with the idea of space and lends itself to drawing pictures. The connection between the two ideas is that matrices can give a precise description of the symmetries of a geometry. These matrices correspond to transformations of a space that leave something essential about the space invariant.

In this course we will explore several kinds of geometries: Euclidean, hyperbolic, symplectic, projective, etc. Each of these geometries has a matrix group attached to it. The matrix groups say something about the symmetries of the geometry, but the geometry can also tell us things about the algebra of the matrix groups.

This course is intended to be an introduction to the way mathematical research works—making connections between seemingly unrelated concepts, digging up answers, sharing results, and getting hooked on the world of ideas.

**How it works:** The course runs over the winter and fall terms of 2021. Let me know if you are interested in participating. I will set up a cuLearn page for everyone who gets in touch with me.

There is no formal commitment on your part in the winter term. If you are enjoying the course and would like to take it for credit then you should write the take-home exam at the end of the winter term. If you are satisfied with your grade then you can register formally for the fall term under MATH 4907 as a 0.5 credit course.

We will meet weekly for about 1.5 hours either on Zoom or BBB. I’ve tentatively set the time for Fridays 10:30-noon. If necessary, we can change the time. The first meeting will be on January 15.

I will be assigning exercises in the first term, but will not be grading them. We will discuss and present them during the lectures.

In the second term the assessments will become more formal. There will be three graded assignments and another take-home exam at the end of term.

**Instructor:**

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**Lectures:** (Tentatively) Fridays 10:30-12:00

**Grading scheme:**

Attendance and participation	20%
Assignments	30%
Take-home exam 1	25%
Take-home exam 2	25 %

**Attendance and participation:** 10% of the final grade will be given for attendance and participation in each term. Students will occasionally be expected to present solutions to exercises during the lecture times.

**Assignments:** There will be three 2-week assignments, each worth 10% of the final grade. They will be posted on September 30, October 14, and November 4.

**Take-home exams:** There will be two take-home exams. Each exam will be given on the last day of classes and will be due on the final day of the examination period (April 14-27 in winter 2021).

**Proposed topics:**

- Groups: definition and familiar examples
- The general linear group  $GL(n, \mathbb{R})$  and the special linear group  $SL(n, \mathbb{R})$
- The orthogonal group  $O(n)$ , Euclidean space and symmetric bilinear forms
- The Cartan-Dieudonné Theorem on reflections
- Affine transformations
- The group  $O(m, n)$  and hyperbolic space
- Lorentz transformations,  $O(3, 1)$  and  $SL(2, \mathbb{C})$
- The unitary group  $U(n)$  and sesquilinear forms
- $U(2)$  and the ring of quaternions
- $U(3)$  and the ring of octonions
- The symplectic group  $Sp(n, \mathbb{R})$  and alternating bilinear forms
- Symplectic space, transvections
- Heisenberg groups
- Projective space and the projective general linear group  $PGL(n, \mathbb{R})$
- Similitude groups  $GSp(n, \mathbb{R})$ ,  $GSO(n, \mathbb{R})$
- Simple groups

**Academic accommodations:** The Paul Menton Centre for Students with Disabilities (PMC) provides services to students with Learning Disabilities (LD), psychiatric/mental health disabilities, Attention Deficit Hyperactivity Disorder (ADHD), Autism Spectrum Disorders (ASD), chronic medical conditions, and impairments in mobility, hearing, and vision. If you have a disability requiring academic accommodations in this course, please contact PMC at 613-520-6608 or [pmc@carleton.ca](mailto:pmc@carleton.ca) for a formal evaluation. If you are already registered with the PMC, contact your PMC coordinator to send me your Letter of Accommodation at the beginning of the term, and no later than two weeks before the first in-class scheduled test or exam requiring accommodation (if applicable). Requests made within two weeks will be reviewed on a case-by-case basis. After requesting accommodation from PMC, meet with me to ensure accommodation arrangements are made. Please consult the PMC website ([www.carleton.ca/pmc](http://www.carleton.ca/pmc)) for the deadline to request accommodations for the formally-scheduled exam (if applicable).