STAT 5709-W Probability Theory II
School of Mathematics and Statistics
Carleton University
Winter 2021

Instructor:	Dr. Yiqiang Q. Zhao
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Coursepage:	on cuLearn

Text: Patrick Billingsley, *Probability and Measure*, Anniversary Edition, John Wiley, 2012.

First meeting: 16:05 Tuesday January 12 (via zoom)

Lectures: Lecture notes and pre-recorded lectures will be posted on cuLearn (There is no synchronous teaching!)

Scheduled Lecture time: Tuesday and Thursday: 16:05 – 17:25 will be used for:
First meeting, Tuesday January 12 (mandatory)
Midterm, Tueday March 16 (mandatory)
Office hrs, every Thursday

Office Hours: At scheduled Thursday lecture time (via zoom):

Prerequisites: STAT 5708; or permission of the School

Marking Scheme: Assignments (3): 30% (due dates: February 11, March 4, and April 2, resp.)
Term test (1): 30% (75 min closed book)
                (Tuesday March 16 during the scheduled lecture time)
Final exam: 40% (3 hrs; closed book)

Important dates:

January 12 (Tuesday)	First meeting
February 16 – 19	Winter break, classes suspended
April 13 (Tuesday)	Last day of scheduled classes
April 16 – 27	Exam period

Email communication with instructor: According to Carleton policy under the Freedom of Information and Protection of Privacy Act (FIPPA), please use your Carleton Connect account for all course related email.

Announcements: You are responsible for keeping up with all information about the course communicated through the course web page at cuLearn, and e-mail.

A total of 3 assignments worth 10% each: Students should do independent work on the assignments. **Late assignments** will not be accepted unless a written request, describing the

reason why you could not complete the work on time, has been submitted to the instructor before the due date, and an arrangement made by the instructor.

A single PDF file for each assignment is required to be submitted to the course page through cuLearn before the deadline!

Term test (30%, synchronously on-line): You should write the test. No make up, early or late test will be arranged, except for medical reasons (a doctor's note must be presented), or situations in accordance with Carleton's accommodation policies. The missing test will be counted as zero.

A single PDF file is required to be submitted to the course page through cuLearn at the end of the test!

Final examination (40%, synchronously on-line): There is a three (3) hour closed book exam scheduled by the University during the final exam period. It is the responsibility of each student to be available at the time of the examination. In particular, no travel plans for the examination period in April should be made until the examination schedule is published.

A single PDF file is required to be submitted to the course page through cuLearn at the end of the exam!

Please note that tests and examinations in this course will use a remote proctoring service provided by Scheduling and Examination Services. You can find more information at:

`https://carleton.ca/ses/e-proctoring/`

Calculators: You may only use non-programmable, non-graphing calculators for the tests and the final examination in this course. No other electronic device is allowed, such as cell-phones, electronic dictionary, palm pilot, etc.

Paul Menton Centre: Students with disabilities requiring academic accommodations in this course must contact a coordinator at the Paul Menton Centre (PMC) for Students with Disabilities (500 University Centre, Tel: 613-520-6608) to complete the necessary Letters of Accommodation. After registering with the PMC, make an appointment to meet and discuss your needs with the instructor in order to make the necessary arrangements as early in the term as possible. Please notice the deadline for submitting completed forms to the Paul Menton Centre for formally scheduled exam accommodations.

Religious obligations: Students requiring accommodation on the basis of religious obligations should make a formal, written request for alternate dates and/or means of satisfying academic requirements. Such requests should be made within the first two weeks of the class or as soon as the need for accommodation is known to exist, but no later than two weeks before the compulsory event. Accommodation is to be worked out directly and on an individual basis between the student and the instructor. Accommodation is made in a way that ensures fairness and avoids academic disadvantage to the student. Please refer to the website of Equity Services for a list of holy days and Carleton's Academic Accommodation policies.

Pregnancy: Pregnant students requiring academic accommodation are encouraged to contact an Equity Advisor in Equity Services to complete a letter of accommodation. You should make an appointment with the instructor to discuss your needs at least two weeks prior to the first academic event in which it is anticipated that the accommodation will be required.

Course Outline:

1. **Section 25: Weak Convergence**

   - Weak convergence for distributions

   - Weak convergence for probability measures

   - Weak convergence for random variables

   - Convergence in probability

   - Fundamental theorems

2. **Section 26: Characteristic Functions**

   - Moments and derivatives

   - Independence

   - Inversion

   - Uniqueness theorem

   - Continuity theorem

3. **Section 27: Central Limit Theorem**

   - Lindeberg-Lévy theorem

   - Lindeberg condition and Lindeberg theorem

   - Lyapounov's condition and Lyapounov's theorem

4. **Section 32: Radon-Nikodym Theorem**

   - Hahn decomomposition

   - Absolute continuity and singularity

   - Radon-Nikodym theorem

5. **Section 33: Conditional Probability**

   - Discrete case

   - General case

   - Properties of the conditional probability

   - Conditional probability distributions

6. **Section 34: Conditional Expectation**

- Concepts

- Properties of conditional expectation

- Conditional distributions and expectations

7. **Section 35: Martingales**

- Martingales

- Submartingales

- Gambling

- Functions of martingales

- Stopping times

- Inequalities

- Optimal sampling theorem

- Convergence theorems

- Derivatives

- Likelihood ratios

- Reversed Martingales

- de Finetti's theorem

- Bayes estimation

- Central limit theorem