CARLETON UNIVERSITY

SCHOOL OF
MATHEMATICS AND STATISTICS

HONOURS PROJECT

TITLE: The Utility of Signaling in the Job Market

AUTHOR: Justin Leo

SUPERVISOR: Brett Stevens

DATE: May 6th 2019

MATH4905 Honours Math Project - Utility of Signaling in the Job Market

Justin Leo

May 7, 2019

1 Introduction

Discussion

Signaling Theory is a concept in Game Theory and Economics devlopped by Michael Spence during the 1970s. The idea is that in some markets, asymmetric information prevents the optimal exchange of goods and services from occuring. To avoid this problem, one party can send a signal to the other to influence the other's decisions.

This paper examines the game played between employees and employers in the job market, using university education as the only signal of information on which decisions are made. To analyze this game we will be expanding on Michael Spence's model in his paper "Job Market Signalling". In this model we will examine two different scenarios. Scenario one is a single employer and two types of employees, Scenario two is two-employer, with an infinite hiring capacity vs. two types of employees. The types of employers are type 1 and type 2 and the types of employees are type I(H) and type II(L). Type 1 employers prefer type I employees and we make no constraints on type 2 employers. Each employer is faced with a decision, whether to hire from university or high school. Employees have the option of going to university, or enter the job market with just a high school degree. In this model it is assumed that the education costs for type I employees are smaller than type IIs. We will also assume that type I employees are more productive than type II for jobs with type 1 employers.

The education cost difference is a fundamental mechanism of the game. Despite the fact that tuition would typically have the same dollar value, it is assumed that it would be harder and more time consuming for type L students compared to type H students to graduate on and achieve the same GPAs. We reflect this in the cost.

Employers know the frequencies of type H and L employees and determine wages by computing expected productivities, which is the sum of the frequency of each type of employee in the market multiplied by their expected productivity in the job.

Defining Notation

It is expected that there will be 5 possbile stable outcomes. (i) Each type goes to school(ii) some Is go to university, some Is go to high school, all IIs stay. (iii) all Is go to university, all IIs stay in high school, (iv) all Is go to university, some IIs stay in high school, some IIs go to university. (v) all Is go to university, all IIs go to university. There are no other possible stable equilibria as they will either contradict conditions we set in the game or break down into one of the 5 previous cases (this is explained in more detail in the single employer case).

We will denote the frequncies as π. π_H, π_L are the frequencies for total type I(H) and II(L) employees. $\pi_{HU}, \pi_{HS}, \pi_{LU}, \pi_{LS}$ are frequencies for the type of employees at university and school. $\pi_{GU}, \pi_{GS}, \pi_{BU}, \pi_{BS}$ are frequencies for the amount of employees from university/high school that each employer hires.

Note that: $\pi_{HU} + \pi_{HS} = \pi_H$, $\pi_{LU} + \pi_{LS} = \pi_L$, $\pi_{GU} + \pi_{GS} = \pi_G$, $\pi_{BU} + \pi_{BS} = \pi_B$

We define P_{mn} for m=H,L and n=G,B as the expected productivities of each employee according to each employer, below is the productivity matrix:

Productivity Matrix	Firm Type G	Firm Type B
Employee Type H	P_{HG}	P_{HB}
Employee Type L	P_{LG}	P_{LB}

This is the education cost matrix comparing the costs of signalling for each type of employee:

Education Cost Matrix	University	School
Employee Type H	C_H	0
Employee Type L	C_L	0

We derive the wages in the single employer case using the weighted average of each frequency times the employer's expectation of that employee type's productivity. Note that we are assuming perfect competition in the labour market and infinite capacity.

$$W_u = \frac{(\pi_{HU})}{(\pi_{HU}+\pi_{LU})}P_H + \frac{(\pi_{LU})}{(\pi_{HU}+\pi_{LU})}P_L$$

$$W_s = \frac{(\pi_{HS})}{(\pi_{HS}+\pi_{LS})}P_H + \frac{(\pi_{LS})}{(\pi_{HS}+\pi_{LS})}P_L$$

The wages in the two employer case are (note that $\pi_{HU} + \pi_{LU} = \pi_U$ and $\pi_{HS} + \pi_{LS} = \pi_S$):

$$W_{UG} = \frac{\pi_{HU}}{\pi_U}P_{HG} + \frac{\pi_{LU}}{\pi_U}P_{LG}$$

$$W_{UB} = \frac{\pi_{HU}}{\pi_U}P_{HB} + \frac{\pi_{LU}}{\pi_U}P_{LB}$$

$$W_{SG} = \frac{\pi_{HS}}{\pi_S}P_{HG} + \frac{\pi_{LS}}{\pi_S}P_{LG}$$

$$W_{SB} = \frac{\pi_{HS}}{\pi_S}P_{HB} + \frac{\pi_{LS}}{\pi_S}P_{LB}$$

Utility for each employer is defined as:

$$U_G = (\frac{\pi_{GU}\pi_{HU}}{\pi_U} + \frac{\pi_{GS}\pi_{HS}}{\pi_S})P_{HG} + (\frac{\pi_{GU}\pi_{LU}}{\pi_U} + \frac{\pi_{GS}\pi_{LS}}{\pi_S})P_{LG} - \pi_{GU}W_u - \pi_{GS}W_s$$

$$U_B = (\frac{\pi_{BU}\pi_{HU}}{\pi_U} + \frac{\pi_{BS}\pi_{HS}}{\pi_S})P_{HB} + (\frac{\pi_{BU}\pi_{LU}}{\pi_U} + \frac{\pi_{BS}\pi_{LS}}{\pi_S})P_{LB} - \pi_{BU}W_u - \pi_{BS}W_s$$

Profit for each employee is defined as:

$$V_H = \frac{\pi_{HU}}{\pi_H}(W_u - C_H) + \frac{\pi_{HS}}{\pi_H}(W_s)$$

$$V_L = \frac{\pi_{LU}}{\pi_L}(W_u - C_L) + \frac{\pi_{LS}}{\pi_L}(W_s)$$

$$V_{HU} = W_u - C_H, V_{LU} = W_u - C_L, V_{HS} = W_s, V_{LS} = W_s$$

Both firms offer equal Wu and Ws values and pass on all productivity to employees (in other words $V_G = V_B = 0$) in the infinite hiring capacity case. [1]

To understand the payoffs and decisions the players face, we need to understand how they will behave under different productivity possibilities. There are four productivity values and two different types of each employee/employer. This implies that there are 4! combinations of orderings of the productivity values. However we have imposed a constraint to reflect the preferences of type 1 employers, that is $P_{HG} > P_{LG}$; so 12 of the 24 possible orderings are discarded. Of the remaining 12, we will see that 6 end up behaving in the same way for both single and double employer cases. The remaining 6 are analyzed individually in the double employer case.

1.1 List of assumptions and conditions that should hold:

Type I employees have lower cost of education compared to type II and are therefore able to signal at a lower cost. ($C_L \geq C_H$) Type I employees are better at type 1 jobs than type II ($P_{HG} > P_{LG}$) [2]

In the case where the employers have infinite hiring capacities, the game takes place in a perfectly competitive market (explicit employer payoffs = 0), and employees always choose the highest salary they are offered. [1]

These are some conditions that need to hold:

Total supply of employees from each education type must equal the total amount hired by employers (market clearing condition):

$\frac{\pi_{HU}}{\pi_U}\pi_H + \frac{\pi_{LU}}{\pi_U}\pi_L = \frac{\pi_{HU}}{\pi_U}\pi_H + \frac{\pi_{BU}}{\pi_U}\pi_B$

$\frac{\pi_{HS}}{\pi_H}\pi_H + \frac{\pi_{LS}}{\pi_S}\pi_L = \frac{\pi_{GS}}{\pi_S}\pi_G + \frac{\pi_{BS}}{\pi_S}\pi_B$

Proposition 1.1: If there are type H employees choosing to go to university and signal, this strategy must have at least as large of an expected return compared to not signalling:

$\pi_{HU} > 0 \Rightarrow max(W_{ug}, W_{ub}) - C_H \geq max(W_{sg}, W_{sb})$ (1)

Similarly:

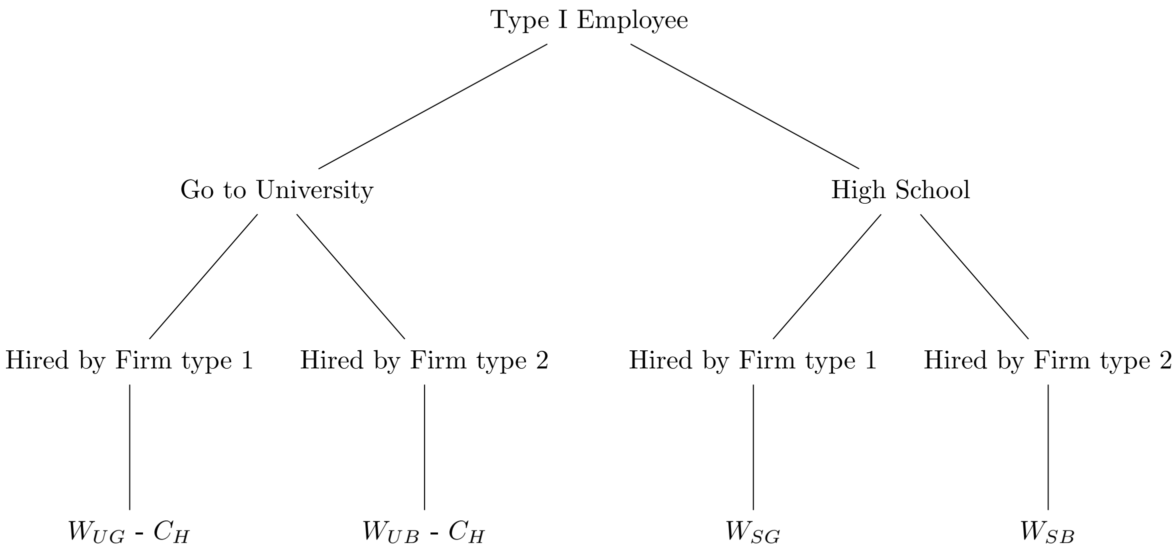
$\pi_{HS} > 0 \Rightarrow max(W_{UG}, W_{UB}) - C_H \leq max(W_{SG}, W_{SB})$ (2)

$\pi_{LU} > 0 \Rightarrow max(W_{UG}, W_{UB}) - C_L \geq max(W_{SG}, W_{SB})$ (3)

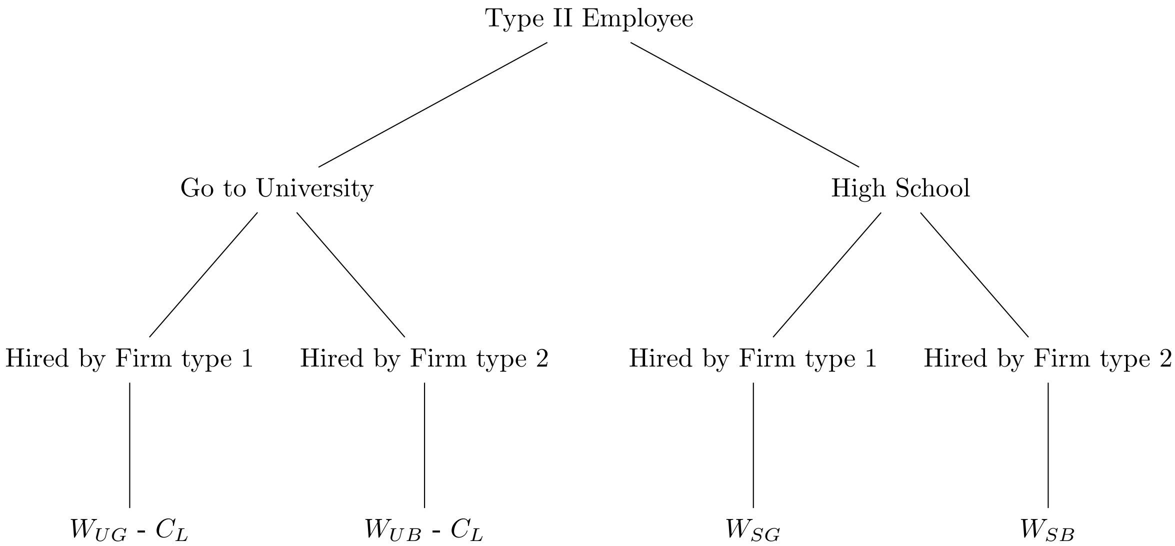
$\pi_{LS} > 0 \Rightarrow max(W_{UG}, W_{UB}) - C_L \leq max(W_{SG}, W_{SB})$ (4)

1.2 Game Trees

This is the game tree of a type I employee's perspective:



This is the game tree from a type II employee's perspective:



Lemma 1.1: Any scenario with $\pi_{HS} > 0$ and $\pi_{LU} > 0$ is not a Nash Equilibrium.

proof:

Let $\pi_{HS} > 0$, then the payoff for type H employees that go to school must be at least as good as good as the payoff to go to university. That is, $W_s \geq W_u - C_H$. Suppose $\pi_{LU} > 0$, then the payoff for type Ls to go to university must be at least as good as not, i.e. $W_s \leq W_u - C_L$. Combining the two inequalities leads to $C_L \leq C_H$, which is a contradiction by our assumption.

By Lemma 1.1, there can only be 5 scenarios that can produce a Nash Equilibrium.

Proposition 1.2: kX + (1-k)Y $\leq X \iff Y \leq X \iff kX+(1-k)Y \geq Y$ where $0 \leq k \leq 1$

The above proposition will come in handy when analyzing the stability of each nash equilibrium. By Proposition 1.2, any convex combination of two values X and Y is less than X provided that Y is less than or equal to X.

Remark: In any case where $\pi_{ps} = 0$, we will suppose a tiny fraction of Ps choose the alternative strategy. This will allow us to compute the wages and utilities of each when nobody goes to university for example. We will use this technique to analyze stability.

2 Single Employer

When there is only one employer, the decision employees face simplifies to a comparison of the wage offered for university graduates vs. non graduates. Employees will still choose the max wage offered to them.

2.1 The 5 possible outcomes are:

1) Pooling S: equilibrium where both types go to School
2) Mix H: All type L don't go to university, some type H go to university
3) Seperating Equilibrium: type H go to university, type L do not
4) Mix L: All type H settle at go to university, some type L go to university
5) Pooling U: equilibrium where both types go to university

We now discuss the conditions under which the equilibria exist and are stable.

1) Pooling S Equilibrium

$\pi_{HS} = \pi_H, \pi_{LS} = \pi_L, \pi_{HU} = \pi_{LU} = 0$

(a) $\pi_{HS} > 0 \Rightarrow W_u - C_H \leq W_s$ by (2)
(b) $\pi_{LS} > 0 \Rightarrow W_u - C_L \leq W_s$ by (4)

Because $\pi_{HU} = \pi_{LU} = 0$, $W_s = \pi_H P_H + \pi_L P_L$. To compute W_u, we suppose a small fraction of employees go to university, by Lemma 1.1, there can only be type Is doing so and thus $W_u = P_H$. We conclude $P_H - P_L \leq \frac{C_H}{\pi_L}$ and $P_H - P_L \leq \frac{C_L}{\pi_L}$ from plugging in the values for W_u and W_s into the equations (a) and (b):

$P_H - C_H \leq \pi_I P_H + \pi_L P_L \Leftrightarrow P_H(\pi_L) - P_L(\pi_L) \leq C_H \Leftrightarrow P_H - P_L \leq \frac{C_H}{\pi_L} \Leftrightarrow P_H - P_L \leq \frac{C_L}{\pi_L}$

To analyze stability suppose ϵ amount of Hs go to university, W_u is unchanged but now there is a change in W_s:

$$\Delta W_s = \frac{\pi_{HU-\epsilon}}{1-\epsilon} P_H + \frac{\pi_L}{1-\epsilon} P_L - \pi_H P_H + \pi_L P_L$$

$$= \frac{1}{1-\epsilon}(\pi_H P_H - \epsilon P_H + \pi_L P_L - (\pi_H P_H - \epsilon \pi_H P_H + \pi_L P_L - \epsilon \pi_L P_L))$$

$$= \frac{\epsilon \pi_L}{1-\epsilon}(P_L - P_H)$$

by assumption $P_L - P_H$ is negative, so the change in W_s is negative.

If $P_H - P_L < \frac{C_H}{\pi_L}$ any type Hs who attend university have the incentive to drop out because the increase in W_u is not significant enough to reverse in the inequality in (a), therefore is stable. If $P_H - P_L = \frac{C_H}{\pi_L}$, then as more Hs go to university W_s decreases (giving more incentive for more type Hs to go to university) and thus this equilibrium is not stable.

2) Mixed H Equilibrium

$\pi_{HU} > 0, \pi_{HS} > 0, \pi_{LU} = 0, \pi_{LS} = \pi_L$

(a) $\pi_{HU} > 0 \Rightarrow W_u - C_H \geq W_s$ by (1)

(b) $\pi_{HS} > 0 \Rightarrow W_u - C_H \leq W_s$ (2)

(c) $\pi_{LS} > 0 \Rightarrow W_u - C_L \leq W_s$ (4)

Because $\pi_{LU} = 0, W_u = P_H$ and $W_s = \frac{\pi_{HS}}{\pi_S} P_H + \frac{\pi_L}{\pi_S} P_L$. Combining (a) and (b) we get that $W_u - C_H = W_s \Rightarrow \pi_{HS} = \frac{\pi_L(P_H - P_L - CH)}{C_H}$. $0 \leq \pi_{IS} \leq \pi_I \Rightarrow P_H - P_L \geq C_H$ and $P_H - P_L \leq \frac{C_H}{\pi_L}$.

By Lemma 1.1, if there is a change in strategy it can only be type H employees choosing to switch strategy. When $P_H - P_L = C_H$, if π_{HU} increases (as in suppose ϵ amount of Hs go to university like in pool S) than W_s decreases, W_u is unchanged, and incentive is for type Hs to continue to leave for university (W_u unchanged, C_H unchanged, W_s has now decreased, the equality $W_u - C_H = W_s$ now favors the left hand side), thus this is unstable. When $C_H < P_H - P_L < \frac{C_H}{\pi_L}$ and $P_H - P_L = \frac{C_H}{\pi_L}$, as in the pooling s case, supposing a small amount of Hs go to university will decrease W_s, again since $W_u - C_H = W_s$ initially, the left hand side is now greater and the incentive will be for more Hs to follow, this is unstable.

3) Separating Equilibrium

$\pi_{HU} = \pi_H, \pi_{LU} = \pi_{HS} = 0, \pi_{LS} = \pi_L$

Since $\pi_{LU} = \pi_{HS} = 0, W_u = P_H$ and $W_s = P_L$

By (1) and (4): $W_u - C_H \geq W_s \geq W_u - C_L$. When we substitute the values for W_u and W_s we get:

$C_H \leq P_H - P_L \leq C_L$

We have that $P_H - C_H \geq P_L \geq P_H - C_L$. Let $P_H - P_L = C_H$ ($W_u - C_H = W_s$*) and suppose ϵ Hs drop out of university, W_u is unchanged but W_s now becomes: $W_s = \frac{\epsilon}{\epsilon + \pi_L} P_H + \frac{\pi_L}{\epsilon + \pi_L} P_L$. W_s increases slightly by proposition 1.2, and now the equality * favours the righthand side, giving incentive for more Hs to follow suit. Thus this is unstable. When $C_H < P_H - P_L < C_L$, and a small amount of Hs leave university, Ws increases slightly, but because such a tiny fraction of Hs switched, the change is not enough to offset the inequality given by (1) ($W_u - C_H > W_s$), so the incentive for Hs is to go back to university. Likewise taking a tiny fraction of Ls and supposing they go to university will not offset the inequality given by (4), this is a stable case. When $P_H - P_L = C_L$, if ϵ Hs leave university W_s increases, but $W_s = W_u - C_L \leq W_u - C_H$, so stable. If ϵ Ls leave university W_u decreases, so Ls have incentive not to go to university, therefore is also stable.

4) Mixed L Equilibrium

$\pi_H = \pi_{HU} > 0$, $\pi_{LU} > 0$, $\pi_{HS} = 0$, $\pi_{LS} > 0$

$W_u = \frac{\pi_H}{\pi_H + \pi_{LU}} P_H + \frac{\pi_{LU}}{\pi_H + \pi_{LU}} P_L$ and because $\pi_{HS} = 0$, $W_s = P_L$.

(a) $W_u - C_H \geq W_s$ By (1)
(b) $W_u - C_L \geq W_s$ by (3)
(c) $W_u - C_L \leq W_s$ by (4) thus $Wu - C_L = W_s$

From (a) and (c) and substituting in the values for Wu and Ws, we get that $\pi_{LU} = \frac{\pi_H(P_H - P_L - C_L}{C_L}$, because $0 \leq \pi_{LU} \leq \pi_H$ we have that $C_L \leq P_H - P_L \leq C_L/\pi_H$

When $C_L = P_H - P_L$, by Lemma 1.1 and the fact that $\pi_{HU} > 0$, if a small amount of employees change strategy it must be type Ls. Suppose a small amount of type Ls leave university instead:

$\Delta\pi_{LU} = \epsilon$
$\Delta W_u = \frac{\pi_H(P_L - P_H)\epsilon}{\pi_u(\pi_u\epsilon)}$ which has opposite sign to ϵ so equilibrium is stable.

When $C_L < P_H - P_L < C_L/\pi_H$ *, following the same process as before will increase W_s and decrease W_u, but the change is not significant enough to reverse the inequality *, and so the incentive is for Hs to go back, therefore stable. When $P_H - P_L = C_L/\pi_H$, this case is equivalent to $C_L < P_H - P_L$ $(C_L/\pi_H > C_L)$, and so is stable.

5) Pooling U Equilibrium

$\pi_{HU} = \pi_H, \pi_{LU} = \pi_L, \pi_{HS} = \pi_{LS} = 0$

(a) $\pi_{HU} > 0 \Rightarrow W_u - C_H \geq W_s$ by (1)
(b) $\pi_{LU} > 0 \Rightarrow W_u - C_L \geq W_s$ by (3)

$W_u = \pi_H P_H + \pi_L P_L$ and $W_s = P_L$, plugging in the values for W_u and W_s:

$\pi_H P_H + \pi_L P_L - C_L \geq P_H \Rightarrow P_H(\pi_H) + P_L(\pi_L - 1) \geq C_L \Rightarrow P_H - P_L \geq C_L/\pi_H$

Once again by Lemma 1.1 Ls are the only type that can change strategy. When $P_H - P_L = C_L/\pi_H$ the incentive is for all types to go to university $(W_u - C_L > W_s)$. If we suppose a small amount of type Ls leave, then the change in W_u is positive. So the incentive for Ls is to go back, this is stable. When $P_H - P_L > C_L/\pi_H$, then $P_H - P_L > C_L$ which is stable by case 4 mixed L equilibrium.

To summarize: the outcomes with a single employer depend on the magnitude of the productivity difference relative to $C_H, C_L, C_H/\pi_L, C_L/\pi_H$. Since $C_L \geq C_H$, $C_H < C_H/\pi_L$ and $C_L < C_L/\pi_H$, there are only 3 possible orderings of $C_H, C_L, C_H/\pi_L, C_L/\pi_H$: $C_H < \frac{C_H}{\pi_L} < C_L < \frac{C_L}{\pi_H}$, $C_H < C_L < \frac{C_L}{\pi_H} < \frac{C_H}{\pi_L}$ and $C_H < C_L < \frac{C_H}{\pi_L} < \frac{C_L}{\pi_H}$.

2.2 Stability Analysis of Each Outcome Under Single Employer Under Varying Education Costs Combinations

When $C_H < \frac{C_H}{\pi_L} < C_L < \frac{C_L}{\pi_H}$, there is only one possible equilibrium in 4 out the 5 ranges of P_H, P_L, and thus is the simplest case.

If the difference in productivities between employees ($P_H - P_L$) is less than C_H, the only possible outcome is a pooling S (see necessary conditions above). If this difference is between $\frac{C_H}{\pi_L}$ the only possible outcome is a separating equilibrium, between C_L and $\frac{C_L}{\pi_H}$ mix 2 is the only outcome, and if $(P_H - P_L)$ is greater than $\frac{C_H}{\pi_L}$ then pooling u is the only possible outcome. When the difference is between C_H and $\frac{C_H}{\pi_L}$ there are two possible outcomes, pooling s and separating equilibrium. We will check whether both are stable in this scenario. First, suppose pooling s, that is $\pi_{HU} = \pi_{LU} = 0$ and suppose that an ϵ amount of Hs go to U instead: W_s is now given as: $W_s = \frac{\pi_H - \epsilon}{\pi_H + \pi_L - \epsilon} P_H + \frac{\pi_H}{\pi_H + \pi_L - \epsilon} P_L$ using some algebra to rearrange the expression we get: $W_s = [\pi_H P_H + \pi_L P_L - \epsilon P_H](1 + \epsilon + \epsilon^2 + \epsilon^3 + ...)$. We know that $\pi_H P_H + \pi_L P_L$ is greater than $P_H - C_H$, and since ϵ is a very small amount, we get the expression: $\pi_H P_H + \pi_L P_L - \epsilon(P_H - P_L)\pi_L > P_H - C_H \implies \frac{C_H}{\pi_L} > (P_H - P_L)(1 + \epsilon)$. This falls in line with our initial condition and thus is a stable case. Instead now suppose separating equilibrium, $\pi_{HS} = \pi_{LU} = 0$. Let epsilon amount of Hs leave university: $W_s = \frac{\pi_H + \epsilon}{\pi_H + \pi_L + \epsilon} P_H + \frac{\pi_L}{\pi_H + \pi_L + \epsilon} P_L$ W_s gets closer to P_H and thus increases. We get an initial inequality: $P_H - C_H > \frac{\epsilon}{\pi_L + \epsilon} P_H + \frac{\pi_L}{\pi_L + \epsilon} P_L$. Once again rearranging we get $(P_H - P_L)(1 + \frac{\epsilon}{\pi_L}) > C_H$, which implies that this case is stable.

Now let's look at the case where $C_H < C_L < \frac{C_L}{\pi_H} < \frac{C_H}{\pi_L}$, here there are multiple potential equilibria in 3 out of the 5 scenarios, and so is the most complex case.

Below C_H is pooling s, between C_H and C_L are pooling s and separating equilibriums (both stable from previous analysis), between C_L and $\frac{C_L}{\pi_H}$ are mix 2 and pooling s, between $\frac{C_L}{\pi_H}$ and $\frac{C_H}{\pi_L}$ are both pooling equilibriums. To verify that both outcomes between C_L and $\frac{C_L}{\pi_L}$ are stable, let's first suppose a pooling s equilibrium (recall that $\pi_{HU} = \pi_{LU} = 0$ in this outcome), and let a small amount of type Hs go to university. Similarly to previous cases, W_s will get close to P_L and decrease. With that decrease, there is more incentive for type Ls and any remaining Hs to go to university as well, however once again we have seen previously that when Ls move to university this change drives the wages down. Beyond a certain point, there will be no more incentive for both types to leave to university, and more incentive to change back, thus this is a stable outcome. Now instead suppose a mixed 2 strategy, that is $\pi_{HS} = 0, W_s = P_L$, and let some Hs leave university. W_s gets closer to P_H and increases while W_u simultaneously decreases. This gives incentive for more type Hs to switch, driving W_u down further, in which case type Ls will also seek to switch. Eventually type Hs will switch back to university and some type Ls will switch to university up to the point where university wage is still greater than non university wage, thus we end up back at a mixed 2 strategy. Between $\frac{C_L}{\pi_H}, \frac{C_H}{\pi_L}$ there are two possible strategies, pooling s and pooling u. If pooling s, then some type Hs leaving to university can only happen if there is incentive for both types to go to university, since mixed strategies and separating equilibriums are not possible here. Therefore if we take a small amount of type Hs and suppose they choose to go to university instead, then the remaining type Hs and type Ls will choose to go to university, or the Hs that left must choose to go back, resulting in a pooling s, or pooling u outcome. Conversely if we start at a pooling u equilibrium and suppose some type Hs leave university, then all employees will leave and the result will be a pooling s outcome, or the type Hs that left must come back, resulting in a pooling u outcome, since there are no stable mixed strategies here.

When $C_H < C_L < \frac{C_H}{\pi_L} < \frac{C_L}{\pi_H}$, if the difference in productivities is less than C_H, pooling S is the only outcome, between C_H and C_L pooling S and seperating equilibrium are the only outcomes (both are stable using the exact same analysis from the previous case), between C_L and $\frac{C_H}{\pi_L}$ is a mix 2 strategy, between $\frac{C_H}{\pi_L}$ and $\frac{C_L}{\pi_H}$ there are 2 possible outcomes, mix 2 and pooling S (both are stable)

3 Two Employer ∞ - Capacity

In the two employer infinite capacity case, there are now two types of employers '1,2'. Employees can now be hired by different types of employers, but the same principles from the single employer case still apply: each employee chooses the highest wage offered to them. Which wage is greater depends on the relative ordering of the productivities. There are 4 productivty values and so there are 4! = 24 orderings. Note that any ordering where both productivities for one employer are greater than the other e.g. $P_{HG} \geq P_{LG} \geq P_{LB} \geq P_{HB}$ or $P_{HG} \geq P_{HB} \geq P_{LG} \geq P_{LB}$ reduce to the single employer case because we can determine directly through the productivity values which wage offered is better relative to the other. So the remaining 6 orderings to analyze are:

(1) $P_{HG} \geq P_{HB} \geq P_{LB} \geq P_{LG}$
(2) $P_{HG} \geq P_{LB} \geq P_{HB} \geq P_{LG}$
(3) $P_{HB} \geq P_{HG} \geq P_{LG} \geq P_{LB}$
(4) $P_{HG} \geq P_{LB} \geq P_{LG} \geq P_{HB}$
(5) $P_{LB} \geq P_{HG} \geq P_{HB} \geq P_{LG}$
(6) $P_{LB} \geq P_{HG} \geq P_{LG} \geq P_{HB}$

The necessary conditions for each stable outcome under 2 employers are displayed below:

1) Pooling S Equilibrium: We have that: $W_{SG} = \pi_H P_{HG} + \pi_L P_{LG}$, $W_{SB} = \pi_H P_{HB} + \pi_L P_{LB}$, $W_{UG} = P_{HG}$, $W_{UB} = P_{LG}$ by the necessary conditions of pooling S (reference single employer proportion values for pooling s). For pooling S to be true, both types of employees must prefer choosing school over university $max(W_{UG}, W_{UB}) - C_H \leq max(W_{SG}, W_{SB})$. Since the wages for going to university offered by each employer are just the productivity values, we get: $P_{HG} - C_H \leq max(W_{SG}, W_{SB})$ (recall that by assumption $P_{HG} \geq P_{LG}$).

2) Mixed H Equilibrium: From the single employer proportions for mix H we have that: $\pi_{HU} > 0, \pi_{HS} > 0$, $\pi_{LU} = 0, \pi_{LS} = \pi_L$, so $W_{UG} = P_{LG}, W_{UB} = P_{LB}$, W_{SG} and W_{SB} are formulated the usual way but with $\pi_H = \pi_{HU}$ and $\pi_L = \pi_{LU}$. In mix H all type Ls choose school, so $max(W_{UG}, W_{UB}) - C_H \leq max(W_{SG}, W_{SB})$ and type Hs are split between university and school, implying $max(W_{UG}, W_{UB}) - C_H = max(W_{SG}, W_{SB}) = max(P_{LG}, P_{LB})$. Since Hs choose both options, $max(W_{UG}, W_{UB}) - C_H = max(W_{SG}, W_{SB}) \Rightarrow P_{HG} - C_H = max(W_{SG}, W_{SB})$ and $max(W_{SG}, W_{SB}) \geq max(W_{UG}, W_{UB} - C_L$.

3) Separating Equilibrium: Recall that the proportions for a separating equilibrium are $\pi_{HS} = \pi_{LU} = 0, \pi_{LU} = \pi_L$, $\pi_{LS} = \pi_L$, that is all type Hs go to university and all type Ls stay in school. Thus we have that $W_{UG} = P_{HG}$, $W_{UB} = P_{HB}$, $W_{SG} = P_{LG}$ and $W_{SB} = P_{LB}$. The necessary condition is therefore given by: $max(P_{HG}, P_{HB}) - C_H \geq max(P_{LG}, P_{LB}) \geq$

$max(P_{HG}, P_{HB}) - C_L$.

4) Mixed L Equilibrium: Mix L equilibrium is the scenario where type Ls go to both university and school and type Hs go to university only: $\pi_{HU} = \pi_H, \pi_{HS} = 0$, $\pi_{LU} > 0, \pi_{LS} > 0$. Therefore $W_{SG} = P_{LG}, W_{SB} = P_{HB}$ and the wages for school are formulated as usual. Since Hs choose both options, $max(W_{UG}, W_{UB}) - C_H = max(W_{SG}, W_{SB}) \Rightarrow P_{HG} - C_H = max(W_{SG}, W_{SB})$ and $max(W_{SG}, W_{SB}) \leq max(W_{UG}, W_{UB} - C_H$.

5) Pooling U Equilibrium: In the pooling U scenario, both types of employees choose to go to university instead of school: $\pi_{HS} = \pi_{LS} = 0, \pi_{HU} = \pi_H, \pi_{LU} = \pi_L$. Since $C_L \geq C_H$ we need only the condition: $max(W_{UG}, W_{UB}) - C_L \geq max(W_{SG}, W_{SB})$

3.1 Stability Analysis of Each Outcome

We will begin to analyze the two employer cases. First, let us analyze the different cases for the expected productivities of each type of employee, according to each employer. Note that there are 12 possbile total orderings of $P_{HG}, P_{HB}, P_{LG}, P_{LB}$ (recall that we must have $P_{HG} > P_{LG}$ due to the initial expectations of the type 1 employer), and 6 of these orderings reduce to the single employer case where either firm type 1 or 2 is strictly preferred by all employees, so we will consider only the remaining 6.

Points of equilibrium for each combination

The stability of equilibrium points depends only on the relative ordering of P_{LB} and P_{HB}. Orderings (1), (4), (5), (6) will behave in the same way and (2) and (3) will each behave uniquely. The explanation for why that is is provided with the diagrams which follow.

(1) $P_{HG} \geq P_{HB} \geq P_{LB} \geq P_{LG}$

(2) $P_{HG} \geq P_{LB} \geq P_{HB} \geq P_{LG}$

(3) $P_{HB} \geq P_{HG} \geq P_{LG} \geq P_{LB}$

(4) $P_{HG} \geq P_{LB} \geq P_{LG} \geq P_{HB}$

(5) $P_{LB} \geq P_{HG} \geq P_{HB} \geq P_{LG}$

(6) $P_{LB} \geq P_{HG} \geq P_{LG} \geq P_{HB}$

Without knowing the exact values of the wages, we can still plot their curves as we know from the ordering of the productivities the slopes and intercepts of the utility curves. Note that it does not have to be true that W_{SG} is greater than or less than W_{SB}, we choose this ordering arbitrarily for now.

(i) $W_{UG} = \frac{\pi_L - \pi_{LU}}{\pi_L} P_{HG} + \frac{\pi_{LU}}{\pi_L}(\pi_H P_{HG} + \pi_L P_{LG})$

(ii) $W_{UB} = \frac{\pi_L - \pi_{LU}}{\pi_L} P_{HB} + \frac{\pi_{LU}}{\pi_L}(\pi_H P_{HG} + \pi_L P_{LG})$

(iii) $W_{SG} = \frac{\pi_H - \pi_{HU}}{\pi_S} P_{HG} + \frac{\pi_{LS}}{\pi_S} P_{LG}$

(iv) $W_{S2} = \frac{\pi_H - \pi_{HU}}{\pi_S} P_{HB} + \frac{\pi_{LS}}{\pi_S} P_{LB}$

We first determine the value of the π_{LU} that gives identical wages for both employees. To do this we will analyze two cases:

$0 \leq \pi_{HU} \leq \pi_H$
$0 \leq \pi_{LU} \leq \pi_L$

Case: $0 \leq \pi_{HU} \leq \pi_H$

There are three utility curves that are relevant, U_{HU}, U_{HS} and U_{LS} which are functions of π_{HU}.

$U_{HU} = \max(P_{HG}, P_{HB}) - C_H$ is a line of slope 0.

$U_{HS} = U_{LS} = \max(W_{SG}, W_{SB})$

To determine $\max(W_{SG}, W_{SB})$, we compute the values W_{SG}, W_{SB} when $\pi_{HU} = 0$, when $\pi_{HU} = \pi_H$ and also determine the value of π_{HU} at which the curves W_{SG}, W_{SB} cross if any.

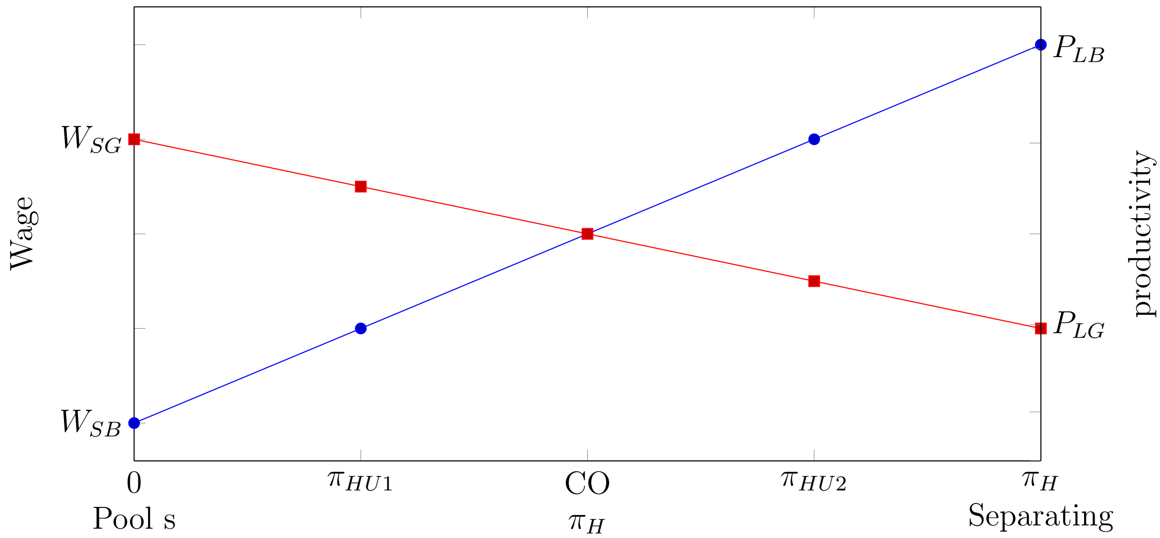
$W_{SG(0)} = \frac{\pi_H}{\pi_S} P_{HG} + \frac{\pi_{LS}}{\pi_S} P_{LG}$
$W_{SB(0)} = \frac{\pi_H}{\pi_S} P_{HB} + \frac{\pi_{LS}}{\pi_S} P_{LB}$
$W_{SG(\pi_H)} = \frac{\pi_{LS}}{\pi_S} P_{LG}$
$W_{SB(\pi_H)} = \frac{\pi_{LS}}{\pi_S} P_{LB}$

We know that the curve W_{SG} is always decreasing.

If the W_{SB} is also decreasing ($P_{HB} > P_{LB}$) then a change in π_{HU} by ϵ changes W_S in the opposite direction thus the only equilibria are at the endpoints of interval: either $\pi_{HU} = 0$ (pool s) or $\pi_{HU} = \pi_U$ (Separating). Pool s is stable if $max(W_{SG}(0), W_{SB}(0)) > \max(P_{HG}, P_{HB}) - C_H$ and unstable if $max(W_{SG}(0), W_{SB}(0)) \leq \max(P_{HG}, P_{HB}) - C_H$. Separating is stable if $\max(P_{HG}, P_{HB}) - C_H > max(P_{LG}, P_{LB})$. and unstable if $\max(P_{HG}, P_{HB}) - C_H \leq max(P_{LG}, P_{LB})$.

If W_{SB} is increasing ($P_{HB} < P_{LB}$, then there are no stable equilibria to the left of CO_{HU} and the the right of 0. the pool s stability is same as previous paragraph. To determine if there is a stable equilibria with $CO_{HU} < \pi_{HU} < \pi_H$ We know in this range that $\max(W_{SG}, W_{SB}) = W_{SB}$, so we compute the value of π_{HU} for which $W_{SB} = \max(P_{HG}, P_{HB}) - C_H$ (the Mix I equilibrium point). If this value is between CO_{HU} and π_H then it is a stable equilibrium, if not then it isn't. Separating equilibrium is stable $\max(P_{HG}, P_{HB}) - C_H > P_{LB}$ and unstable otherwise.

Pool S and Separating Diagram



Using (iii) and (iv) above, we can solve for the crossover point in this diagram by setting π_{HU} to CO_{HU} and setting each equation equal to eachother and solving for CO_{HU}.

$$W_{SG} = \frac{\pi_H - CO_{HU}}{\pi_S} P_{HG} + \frac{\pi_{LS}}{\pi_S} P_{LG} = \frac{\pi_H - CO_{HU}}{\pi_S} P_{HB} + \frac{\pi_{LS}}{\pi_S} P_{LB} = W_{SB}$$

$$\Rightarrow \frac{CO_{HU}(P_{HB} - P_{HG})}{\pi_S} + \frac{\pi_H(P_{HG} - P_{HB})}{\pi_S} + \frac{\pi_{LS}}{\pi_S}(P_{LG} - P_{LB}) = 0$$

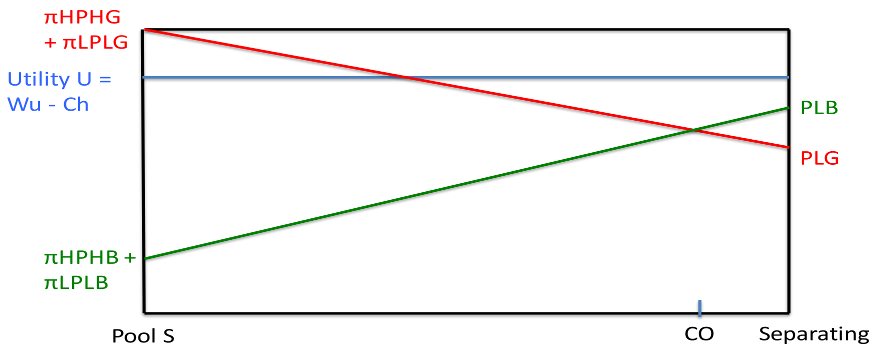
$$\Rightarrow CO_{HU} = \frac{\pi_{LS}(P_{LB}-P_{LG})+\pi_H(P_{HB}-P_{HG})}{(P_{HB}-P_{HG})}$$

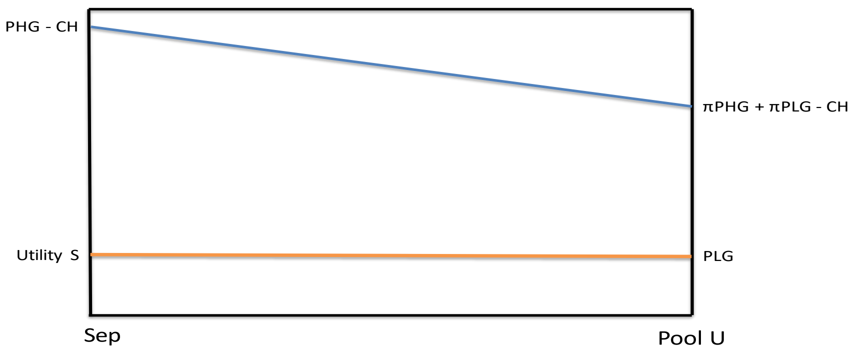
For $\pi_{HU} < CO_{HU}, max(W_{SG}, W_{SB}) = W_{SG}$, when ϵ more Hs go to university, W_{SG} decreases and gives incentive for more Hs to do the same. When ϵ Hs leave university, W_{SG} increases and gives incentive for more Hs to do the same. Thus, there are no stable equilibrium with $\pi_{HU} < CO_{HU}$

For $\pi_{HU} > CO_{HU}, max(W_{SG}, W_{SB}) = W_{SB}$, when ϵ Hs go to university, W_{SB} increases and gives incentive for Hs to go back. When ϵ Hs leave university, W_{SB} decreases and gives incentive for Hs to go back. Thus, this is a stable equilbrium point.

There is also the possibility that the wage would be different along right side y - axis in the diagram below, or along the left side y - axis in the diagram above. In this case there would not be an overlaping crossover point between the two curves in the figure. However, since one curve is continuously increasing and the other decreasing, we know that these curves will evetually intersect, this crossover point can either be outside of the curve to the right or crossover at a negative value for π_H. However, since π_{LU} is a population, it cannot be negative, and so any equilibrium occuring here cannot be possible. Likewise, to the right of π_L would imply that there are more L players going to university then there are Ls in total, which is also not possible for a stable equilibrium point.

Recall that profit V = $W_u - C_H$. When W_{SG} is above V then the outcome is pooling S, since the wage offered for going to school exceeds that of going to university. When it crosses V, both pooling S and Separating equilbrium are possible, and when it is below Utility U only separating is possible. W_{SB} can: cross W_u and W_{SG}, cross W_u but not W_{SG}, not cross W_u but cross W_{SG} or cross neither.





Case: $0 \le \pi_{LU} \le \pi_L$:

There are 3 utility curves, U_{HU}, U_{LU} and U_{LS} which are functions of π_{LU}.

$V_{LS} = max(P_{LG}, P_{LB})$ is line of slope 0

$V_{LU} = max(W_{UG}(\pi_{LU}), W_{UB}(\pi_{LU})) - C_L$

$V_{HU} = V_{LU} + C_L - C_H$ is not relevant here because type I employees don't make choices in this region of graph. So we need to understand curve V_{LU}. We compute the endpoints:

$W_{UG}(0) - C_L, W_{UB}(0) - C_L$

and

$W_{UG}(\pi_L) - C_L, W_{UB}(\pi_L) - C_L$
and compute the value of π_{LU} where the the two curves meet.

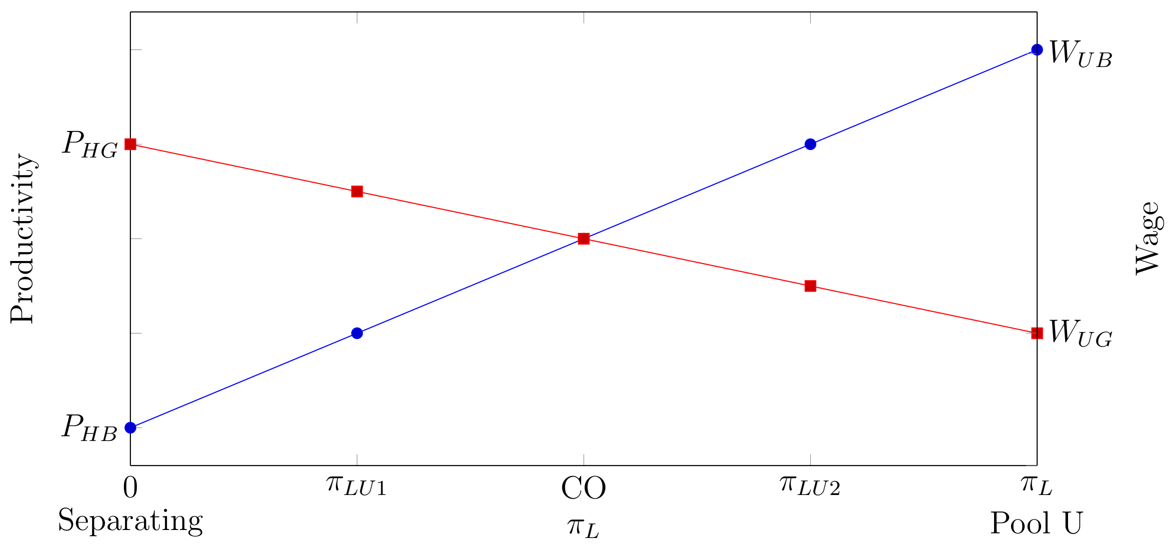
$W_{UG(0)} = \frac{\pi_L}{\pi_L} P_{HG} - C_L = P_{HG} - C_L$

$W_{UB(0)} = \frac{\pi_L}{\pi_L} P_{HB} - C_L = P_{HB} - C_L$

$W_{UG(\pi_L)} = \frac{\pi_L}{\pi_L} (\pi_H P_{HG} + \pi_L P_{LG}) - C_L = (\pi_H P_{HG} + \pi_L P_{LG}) - C_L$

$W_{U2(\pi_L)} = \frac{\pi_L}{\pi_L} (\pi_H P_{HG} + \pi_L P_{LG}) - C_L = (\pi_H P_{HG} + \pi_L P_{LG}) - C_L$

Separating and Pool U Diagram



Deriving the crossover point in this diagram:

$W_{UG} = \frac{CO_{LU}}{\pi_L}(\pi_H P_{HG} + \pi_L P_{LG} - P_{HG}) + P_{HG} = \frac{CO_{LU}}{\pi_L}(\pi_H P_{HB} + \pi_L P_{LB} - P_{HB}) + P_{HB} = W_{UB}$

$CO_{LU} = \frac{\pi_H(P_{HB}-P_{HG})}{\pi_H(P_{HG}-P_{HB})+\pi_L(P_{LG}-P_{LB})-(P_{HG}-P_{LB})}$

If $W_{UG} - C_L$ is decreasing ($P_{HB} > P_{LB}$), then any equilibria in interior ($0 < \pi_{BU} < \pi_B$) are stable.

If $W_{UG} - C_L$ is increasing ($P_{HB} < P_{LG}$) then any equilbria left of $CO_{\pi_{LU}}$ are stable.

If there is a stable equilibrium in interior it is to left of $CO_{\pi_{LU}}$ and in this region $max(W_{UG}, W_{UB}) = W_{UG}$.

For $\pi_{LU} < CO_{LU}(\pi_{LU1}), max(W_{UG}, W_{UB}) = W_{UG}$, when ϵ Ls go to university, W_{UG} decreases and gives incentive to go back. When ϵ Ls leave university, W_{UG} increases and gives incentive to go back. Thus, this is a stable equilbrium point.

For $\pi_{LU} > CO_{LU}$ $(\pi_{LU2}), max(W_{UG}, W_{UB}) = W_{UB}$, when ϵ Ls go to university, W_{UB} increases and gives incentive for Ls to do the same. When ϵ Ls leave university, W_{UB} decreases and gives incentive for Ls to do the same. Thus, this is an unstable equilbrium point.

In the case where both wage lines are negatively sloped, it is possbile under a different wage ordering that there will be no point of intersection between the lines. In that case, one employer is striclty preferred to the other as the max of both wages will always be the higher line, and thus reduces to the single employer case.

If W_{SB} and W_{SG} are flipped relative to each other: then at EQ less than CO, $max(W_{SG}, W_{SB}) = W_{SG}$, when an ϵ amount of Hs go to university, W_{SG} decreases and gives incentive for more Hs to do the same. When ϵ Hs leave university, W_{SG} increases and gives incentive for more Hs to do the same. Thus, this is an unstable equilbrium point. At EQ greater than CO, $max(W_{SG}, W_{SB}) = W_{SB}$, when ϵ Hs go to university, W_{SB} decreases and gives incentive for more Hs to do the same. When ϵ Hs leave university, W_{SB} increases and gives incentive for more Hs to do same. Thus, this is an unstable equilbrium point.

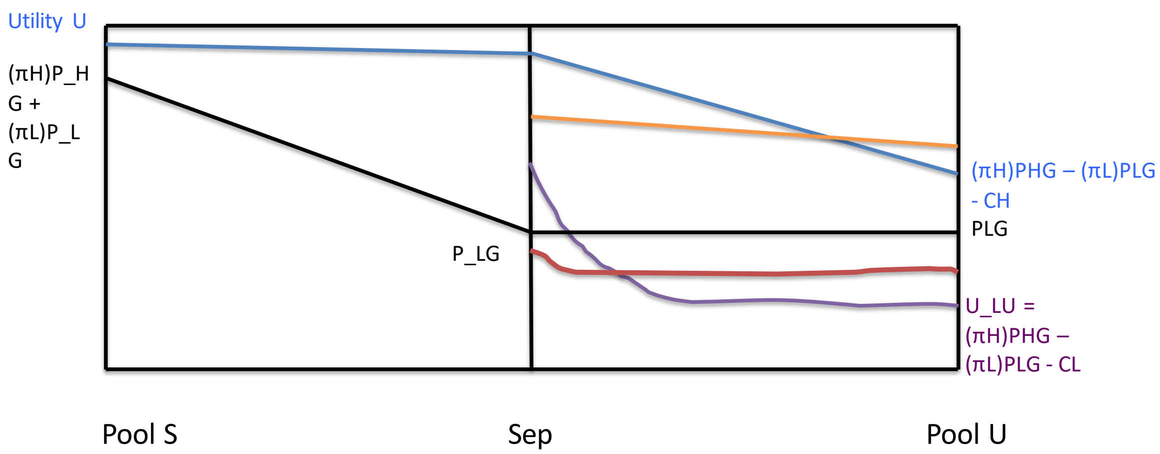
Likewise when we flip P_{HG} and P_{HB} relative to each other: at EQ less than CO, $max(W_{UG}, W_{UB}) = W_{UG}$, when ϵ Ls go to university, W_{UG} decreases and gives incentive to go back. When ϵ Ls leave university, W_{UG} increases and gives incentive to go back. Thus, this is a stable equilbrium point. At

EQ greater than CO, $max(W_{UG}, W_{UB}) = W_{UB}$, when ϵ Ls go to university, W_{UB} increases and gives incentive to go back. When ϵ Ls leave university, W_{UB} decreases and gives incentive to go back. Thus, this is an unstable equilbrium point.

Despite the different relative orderings of P_{HB} and P_{HG}, the behaviour of each equilibrium is the same. Thus, we can bundle (2) and (3) together as one group and the other 4 orderings as another group. From this we can deduce that the large scale behaviour of the equilbria depend only on the relative ordering of P_{LB} and P_{HB} since P_{LG} and P_{HG} are always in the same relative ordering. The relative ordering of P_{LB} and P_{HB} gives us the slope of the wage for type 2 employer, which is what affects the behaviour of the equilbria points. At EQ less than CO $(\pi_{LU1}), max(W_{UG}, W_{UB}) = W_{UG}$, when ϵ Ls go to university, W_{UG} decreases and gives incentive to go back. When ϵ Ls leave university, W_{UG} increases and gives incentive to go back. Thus, this is a stable equilbrium point. At EQ greater than CO $(\pi_{LU2}), max(W_{UG}, W_{UB}) = W_{UB}$, when ϵ Ls go to university, W_{UB} increases and gives incentive to go back. When ϵ Ls leave university, W_{UB} decreases and gives incentive to go back. Thus, this is an unstable equilbrium point.

We can now combine the pool S, sep and pool U diagrams into one to summarize the results.

Combining Pool S/Sep/Pool U



4 Conclusion

Spence's model analyzed a simple case with 1 employer and found that the cost of the signal was what determined whether the employee signaled. We have analyzed two employment scenarios: a modification of Spence's model and a 2 employer scenario, with a view to understanding our central question: when can a university degree be used as a signal to employers. We are most interested in the cases when the separating or mixed equiliria exist because these indicate the utility of the signal. With one employer the relative value between $P_H - P_L$ compared with C_H, C_H/π_L, C_L and C_L/π_H determine which equilibria can exist and are stable. When $P_H - P_L < \frac{C_H}{\pi_L}$ pooling S is stable, when $P_H - P_L = C_L$ mix L and separating are stable, when $C_H < P_H - P_L < C_L$ separating is stable, when $C_H < P_H - P_L < C_L/\pi_H$ mix L is stable, when $P_H - P_L = C_L/\pi_H$ mix L and pool U are stable and finally when $P_H - P_L > C_L/\pi_H$, pool U is stable. In the two employer case with infinite hiring capacity the stability of equilibrium points is dependent on the ordering of the productivity values according to the B employer, P_{LB} and P_{HB}, which gave the slope of this employer's wage. We found that only pool s, pool u, separating and mix H are stable, and that there was only utility in the signal for both under mix H and pool U.

4.1 References

[1] William Branch, "Perfect Competition", http://www.williambranch.org/perfect-competition/, [Online; accessed 7-January-2019]

[2] Michael Spence, "Job Market Signaling", 1973, https://pdfs.semanticscholar.org/2d89/1415c5f4faa5d1adf4492c01fc596231353e.pdf, [Online; accessed 7-January-2019]