

A Recurrent Auto-Encoder Neural Network Model for Multi-Channel Ship Pose Estimation to Facilitate Automated Vertical Landings of Aircrafts

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Abstract

The ability to accurately predict a ship's future roll, pitch, and heave motions from measured motions is vital for many maritime applications. Without accurate and reliable predictions autonomous systems may make choices that could cause damage to themselves, equipment on the ship's deck, or harm the ship's crew. Current state-of-the-art methods for predicting ship pose rely on either analytical methods, which make approximations on underlying dynamics of a ship's motion, or on older data driven methods. To facilitate the automated or autonomous vertical landing of an aircraft on the deck of a maritime vessel the ability to accurately predict the ship's future roll, pitch, and heave motions is vital. Current state-of-the-art methods for predicting ship pose rely on either analytical methods, which make approximations on underlying dynamics of a ship's motion, or on older data driven methods. The work presented in this paper uses a modern Recurrent Neural Network (RNN) architecture constructed using Gated Recurrent Unit (GRU) cells and an auto-encoder architecture in order to predict a ship's motion. The proposed GRU Autoencoder model is compared against the more common feed forward Neural Network (NN) non-linear autoregressive exogenous (NARX) model. Both NN models are tested for robustness by studying the impact that noise, sea state, and ship model has on the overall performance. It was found that the GRU Autoencoder model outperforms the NN NARX model in almost all scenarios and was more robust. Additionally, guidelines for creating a training dataset that will create more robust prediction models are also presented.

I. INTRODUCTION

Ships in the open ocean environment are expected to be able to operate in a wide range of ocean conditions. Ocean waves will induce pitching, rolling, and heaving motions on the ship which will negatively impact on-board operations. The ability to make quick and accurate predictions to the ships motion is important for tasks such as safely operating shipboard cranes [1] or vertical landings on the deck [2]. The roll, pitch, and heave motions can be measured using one or more sensors in order to construct time signals which a signal prediction model can use to estimate future ship motion values.

This work seeks to improve the state-of-the-art for ship motion prediction methods applied for automating Uninhabited Aerial Vehicle (UAV) vertical landings on ship decks. During flight, the UAV tracking phases, flight control and navigation are well studied [2][3]. However, the final descent and landing on moving platforms is still an unsolved problem due to the inherent difficulties of operation in this kind of scenario [4]. The work herein examines methods to improve vertical landings on a moving vessel. The use of mounted LIDAR sensors allows for the control system of the UAV to be independent of the ship it is landing from, removing hardware requirements from vessels and allowing for use on multiple ships. The UAV's landing system would rely solely on its own sensors and signal prediction model to perform a safe landing. The goal of the current work is to predict the roll, pitch, and heave of a ship to facilitate a safe landing of a UAV [2][3]. Broadly speaking, ship motion is a highly coupled stochastic non-linear system; the six-degrees of freedom of the ship motion have non-trivial dependencies on the stochastic ocean waves dynamics, the ship hull structure, and the relative orientation to the sea [5].

Analytical prediction models use approximations of the underlying description of the ships motion. By making an approximation, an analytical model may be applied to a wide variety of cases at the potential cost of performance in a specific scenario of interest. Fast Fourier Transform (FFT) methods are a typical analytical method that aim to estimate ship motion as the sum of sine waves and have been used to predict various ship motions [2][3][6][7]. Autoregressive (AR) models are a stochastic method that use recent measured history to predict a step into the future. AR models have been successfully implemented for predicting ship roll, pitch, and heave motion [8]. Other methods such as Prony analysis [9] and variant ellipsoid methods [4] have also been applied to predicting ship motion.

While the models listed above have all found success there is potential for improvement in data driven models which use measured or simulated data in order to create a prediction model tailored to a specific problem. Montáns et al. [10] published a review of data driven approaches in a wide range of engineering applications and noted that computational power and data availability are increasing and will create opportunities for new approaches to existing problems. Neural Networks (NN) are a form of data driven prediction models which have been applied to ship motion prediction and are becoming increasingly common. NNs have been used to predict ship and roll motions [11][12][13] as well the ship heave motion [14][15].

30 Feed forward NNs are adept at performing predictive tasks, but were mainly developed in the 1980s and there has been
31 significant advances to NNs since. Recurrent Neural Networks (RNNs) [16][17] are an adaptation of NNs that use recurring
32 connections to the same sets of weights to understand and make use of ordered data such as time signals, text, speech, and
33 music. Both the Elmann and Jordan RNN architectures have been applied to predicting ship motion [18][19]. The Elmann
34 RNN has been shown to outperform analytical models constructed with Kalman filters [19] and, when provided with sufficient
35 datasets, they also outperform the feed forward NNs [20][21][22].

36 A historical draw back with the Jordan and Elmann RNNs was a vanishing gradient problem that would prevent training
37 RNNs using longer sequences of data [23]. This problem was address by Hochreiter et al. with the creation of the Long
38 Short-Term Memory (LSTM) RNN cell structure [24]. By introducing two internal structures, referred to as gates, to the RNN
39 cell that is responsible for determine what information from previous timesteps is significant, the LSTM does not suffer from
40 the vanishing gradient problem, allowing for longer sequences to be used. Recently, the LSTM has been applied successfully
41 for predicting ship motion [25].

42 The Gated Recurrent Unit (GRU) is an adaptation of the LSTM that has less weights that need to be trained when compared
43 to the LSTM, making it easier to train and quicker to calculate [26]. It has been shown that GRU models outperform the
44 Jordan RNN and perform on par with the LSTM [27]. The GRU models have been applied to predicting ship roll, pitch, and
45 heave motions; where they were found to be comparable to the LSTM, though the GRU model was quicker to train [28].

46 Further development on the RNN structure was done by dividing the RNN into two substructures that each resemble a
47 full RNN [29]. The first substructure, named the encoder, takes the input signal and compacted the information into only the
48 significant features that describe it, as determined by training. The second substructure, named the decoder, takes the results
49 from the encoder and decompressed it to create the prediction. The combination of the encoder and decoder substructures,
50 referred to as an autoencoder, improved English to French when compared to other machine translation methods. Additionally,
51 the autoencoder could handle long sequences well and has found success at predicting vehicle trajectories [30]. The GRU
52 autoencoder structure utilizes the strengths of the GRU RNN for sequence learning and prediction as well as the advantages
53 of the divided autoencoder structure, and thus can be used to create a model which improves on the state-of-the-art for ship
54 motion prediction.

55 To the authors of this works best knowledge, the autoencoder RNN architecture has not been applied to the task of predicting
56 ship motion. Additionally, most applications use NARX structures which instead of predicting full, multi-channeled signals for
57 future ship motions. Data has also been limited when creating NN based models, with most datasets coming from a single set
58 of measurements. Prior work has commented that the impact of environmental factors, such as wave height and wind speed,

59 should be considered when constructing NN based predictive models [13]. However, limit testing for NN models has also been
60 minimal, with few authors including the impact of noise, varying sea state, and ship model in their analysis.

61 The work presented in this paper seeks to improve and contribute to the state-of-the-art by applying an auto-encoder RNN
62 structure with GRU cells to make multivariate predictions of future roll, pitch, and heave motion based on recently measured
63 data. The models presented in this work are constructed to only make use of instantaneous roll, pitch, and heave measurements
64 that would reflect those obtained from UAV mounted LIDAR sensors so that the models can be later implemented into an
65 automatic landing system. The data sets used in this work are generated from a combination of 21 different simulations. A
66 comparison with a single step-ahead NN NARX model, which is more common in literature, is performed to demonstrate
67 the improved performance of the GRU autoencoder [11][12][13][14][15][18][28]. Lastly, the impacts and measurement noise,
68 varying sea state, and changing ship models are considered in order to limit test the models and evaluate their ability to
69 generalize performance under a range of application focused extremes with the intent of creating a single, once-trained NN
70 model capable of handling a wide range of scenarios.

71 The remainder of this paper is as follows: Section II introduces the construction of the datasets used in this work and
72 represent the “problem” to be solved while Section III presents the formulation of three prediction models used to examine
73 the problem and highlight their baseline performance. Section IV will show studies of the impact of noise, varying sea states,
74 and changing ship models.

75 II. DATA CONSTRUCTION: SHIPMO3D SIMULATIONS

76 Neural Network models must be trained against sets of data which should be large and varied enough in order to fully
77 describe the application scenario the model will perform in. If the training data is limited in some way, such as by being
78 small or having data that are too similar, the performance of the NN will reflect the limited data and not the generalize well.
79 Furthermore, attempting to use a NN model with data from a set unlike the one it was trained on is expected to negatively
80 impact performance, potentially in ways with effects that cannot be easily predicted. Small data sets have been a limiting
81 factor in developing NN predictive models for predicting ship motion. The work presented in this paper uses simulations from
82 the validated ship motion modelling software ShipMo3D [5][31] to construct the training and validation datasets. Within the
83 current study, LIDAR or similar measurements are not used as an input to the proposed system. Previous work has developed
84 a method to determine the world frame ship motion when using a UAV mounted LIDAR system [3][2]. Thus, the current work
85 uses the world frame ship motion simulations to assess the proposed system while the case studies, section IV, provide insight
86 on external factors which impact measurement.

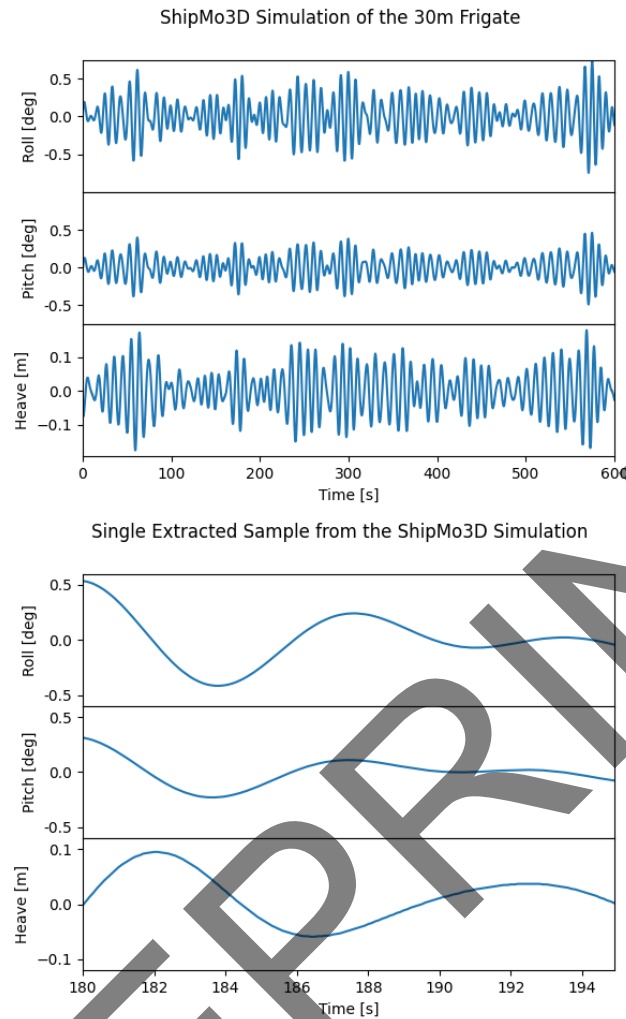


Fig. 1. **[top]** The roll (top row), pitch (middle row), and heave (bottom row) ship motions sampled from the ShipMo3D simulation where the ship was travelling at 10 kt with an angle of attack of 120 deg. **[bottom]** A typical 15 sample that is extracted from the simulation that would be used as a single input for a prediction model.

87 Each simulation use a 30 metre vessel and sea state 2 ocean conditions, ship speeds of 6 kn, 8 kn, and 10 kn and headings
 88 of 0, 30, 60, 90, 120, 150, and 180 degrees were used to create 21 simulations. The various combinations of ship speeds and
 89 headings provide a better representation of the general relationship of waves on the ship. Each simulation is ran for a total of
 90 6 minutes and is sampled at 10 Hz.

91 Fig. 1(top) shows a full ShipMo3D simulation where the ship was travelling at 10 kt with a heading of 120 deg. Fig.
 92 1(bottom) shows a 15 s sample that would be use to construct a typical input for the prediction models. While ShipMo3D
 93 provided the full kinematics of the ship, only the pitch, roll, and heave motions are extracted for use in this work. Additionally,
 94 while this work is limited to roll, pitch, and heave measurements as the input and target signals any measurable signals, such
 95 as the rates and accelerations, may be used as inputs or targets. The models presented in this work will require that all input
 96 channels share the same sampling frequency and that all target channels share the same sampling frequency.

97 The individual channels are normalized separately in order to have zero mean and a variation of 1. The normalization is
 98 done for two reasons, firstly, normalizing features inputted into a Neural Network increases the numeric stability of training
 99 and improves the speed of convergence towards a minimum, and secondly, without the normalization the channels may take
 100 values on different scales. Normalizing the data so that each channel is on the same scale prevents errors in any one channel
 101 from dominating the others and adding bias to the training process. The data signals are normalized using

$$\hat{x} = \frac{x - \mu}{\sqrt{\sigma^2}}, \quad (1)$$

102 where x is original data point, μ and σ^2 are the mean and variance calculated across all data points in all simulations, and
 103 \hat{x} is the new normalized data point. The mean values, across all 21 simulations, are near zero for roll, pitch, and heave. Prior
 104 to normalization the roll variance is $\sigma_{roll}^2 = 0.624 \text{ deg}^2$, the pitch variance is $\sigma_{pitch}^2 = 0.0785 \text{ deg}^2$, and the heave variance
 105 is $\sigma_{heave}^2 = 4.784 \times 10^{-3} \text{ m}^2 \text{ deg}^2$. After normalization the simulation data will have a mean of zero and a variance of one
 106 when constructing the individual data sample pairs for training and testing.

107 The input signal duration is chosen to be 15 s in order to ensure the measurements contain at least one peak wave period
 108 for up to sea state six [32], which is the highest sea state a UAV may attempt a vertical landing. The target signal duration is
 109 chosen to be 5 s, which is a typical descent for the UAV that allows for time to abort dangerous landings [2]. In most cases a
 110 NNs which is trained on more data will outperform NNs trained on less. In order to create the largest dataset possible, every
 111 continuous 20 s interval is extracted from the 21 simulations and used to construct input and target data pairs. In total, 121 800
 112 input-target data pairs are created. The data pairs are shuffled randomly and split into a training data set consisting of 80% of
 113 the total data, 97 440 data pairs, and a testing data set consisting of the remaining 20% of the total data, 24 360 data pairs.
 114 If the data were not shuffled a sampling bias towards a single simulation could negatively impact the batch gradient descent
 115 training process, leading to an under performing predictive model.

116 The training data set is used to optimize the NN weights against a Mean Squared Error (MSE) loss function and validation
 117 data set is used to evaluate performance. Using the validation data set a comparison can be made between prior NN NARX
 118 based models and the proposed GRU autoencoder model.

119 III. SHIP MOTION PREDICTION MODELS

120 This section presents the structure of the modern RNN architecture, constructed for making multi-channel sequence-to-
 121 sequence predictions using an autoencoder structure and Gated Recurrent Unit (GRU) cell structure. The proposed GRU
 122 Autoencoder model is also compared to a more common feed forward Neural Network (NN) non-linear autoregressive

123 exogenous (NARX) model. All models were trained and tested using the Tensorflow [33] framework; however, any similar
 124 NN software toolkit would work.

125 As the data used has a zero mean the NNs presented in this work do not include bias elements. While it is possible to
 126 include a bias in NN and allow training to conclude that the bias element must be zero in order to have this property, it would
 127 be inefficient when compared to applying prior knowledge and forcing the bias to be zero.

128 A. Neural Network NARX model

129 Feed-forward Neural Networks (NNs) are data driven models that are capable of learning important features from a set of
 130 data in order to make accurate predictions. NNs can be used to construct nonlinear autoregressive exogenous (NARX) models
 131 which are commonly used in ship motion prediction models. Different NN NARX models can be constructed by varying
 132 hyperparameters such as the number of steps ahead, input values, and target values. Figure 2 shows the NN NARX model used
 133 in this work; the NN NARX uses roll $r(t)$, pitch $p(t)$, and heave $h(t)$ motions, represented by the combined value $x(t)$ and
 134 flattened into a single vector, as the inputs. The NN has two sets of weights, one W_1 in a dense hidden layer with a rectified
 135 linear unit (ReLU) activation function and a second W_2 in the dense output layer. The total number of trainable parameters in
 136 the network is equal to sum of the sizes of W_1 and W_2 . As the input and output vector sizes are determined by the data the
 137 only way to control the total number of trainable parameters is to adjust the number of hidden neurons, which is the columns
 138 and row size if W_1 and W_2 respectively.

139 Only the measured values from the timesteps between t_0 , the starting time of the sampling window, and t_s , the length of the
 140 sampling window are included in the input vector. The final output is a vector containing the roll, pitch, and heave motions of
 141 the ship at the next timestep. The oldest measurements in the input vector are removed and the new prediction is appended to
 142 the input vector in order to construct the next input. Iterating the prediction, removal, and appending process creates the NN
 143 NARX model which can be used to construct indefinitely long predictions.

144 The NN NARX model is trained using batch gradient descent with a batch size of 32 and a learning rate of 1×10^{-4} . The
 145 chosen loss function is the Mean Squared Error (MSE) of normalized roll, pitch, and heave motions. Training is considered
 146 sufficient when the change in training loss is less than 1×10^{-4} . The NN NARX model trains quickly as it only computes 3
 147 values. However, since it is only predicting the ship motion at a single time step it may propagate errors when constructing
 148 long term predictions. The architecture of the NN NARX model does not consider the order of the inputs and therefore it may
 149 not be the ideal architecture for handling time dependent signals, such as ship motion. While the NN NARX model has been
 150 successful at predicting ship motion [11][12][13][14][15][18][28] the GRU Autoencoder model, which is designed to consider
 151 ordered inputs such as time signals, is expected to improve performance. Furthermore, the GRU Autoencoder trains against its

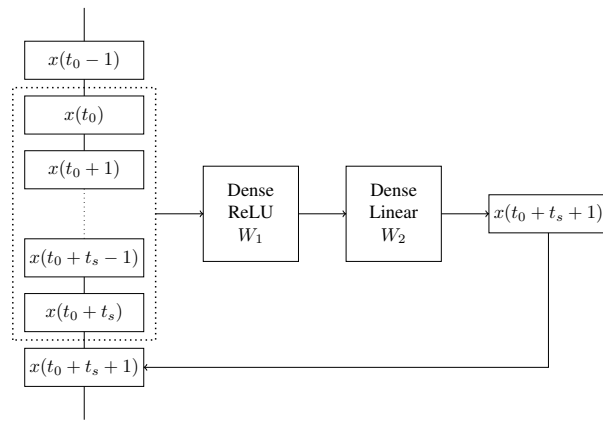


Fig. 2. A visual that describes the information flow in the NN NARX model. The inputs $x(t)$ consist of the roll $r(t)$, pitch $p(t)$, and heave $h(t)$ motions. Each prediction is made using information from the sampling time between t_0 and $t_0 + t_s$. After each prediction the oldest measurements are removed from the input vector and the predictions are appended to the input vector. Iterating the prediction and cycling allows for long term predictions using the single model.

152 ability to make full signal predictions which naturally reduces error from the predictions that the NN NARX would introduce.

153 It is believed that the GRU Autoencoder model will outperform the NN NARX model by a significant amount.

154 B. GRU Autoencoder

155 Recurrent Neural Networks (RNNs) are a class of Neural Networks that are structured to consider the order of inputs.
 156 RNNs are built from one or more cell structures that are provided inputs in a sequential manner and produce outputs at each
 157 step. They are meant to handle sequences of information, such as the ship motion time signals. Unlike the NN NARX model
 158 in Sec. III-A an RNN can be structured to create sequence outputs, such as full time signal predictions of ship motion.

159 RNN cells take up to two inputs, the current time signal and the prior cell state, and provides up to two outputs, the current
 160 prediction and the current cell state, for each time step. At each time step the information being passed forward in time,
 161 referred to as the cell state, will be updated using the previous cell state. Figure 3 shows the autoencoder RNN model, which
 162 segments the RNN into two components. The first component is the encoder, which will take in the signal inputs x_t and cell
 163 states at each time step, pass the cell states forward, but not return any signal. The second component is the decoder, where
 164 the cells will only take prior cell states as their inputs, pass the cell states forward, and return a signal at the current time step.
 165 The encoder and decoder use separate sets of weights and allow training to let the data determine which components of the
 166 inputs are most important and how those components form the predictions x_{t+n} . The cell states are used to pass information
 167 forward through time and are marked by arrows in Figure 3

168 Each of the cells shown in Fig. 3 represent a recurrent unit cell which defines the type of RNN that is being used. The older
 169 Elman and Jordan RNNs created cell states that would directly pass information forward with each time step. However, the
 170 GRU cell modifies the cell state before passing the information forward. The GRU cell structure shown in Fig. 4 is made with

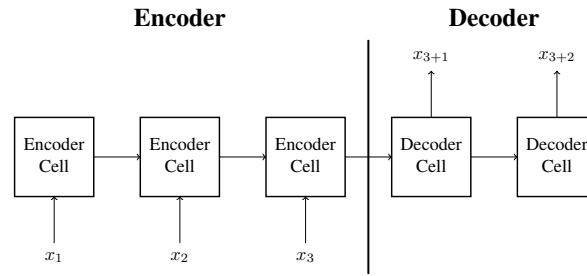


Fig. 3. The structure of an Autoencoder RNN model that uses three timesteps in its input sequence and calculates two timesteps in its output sequence. The Encoder is responsible only for determining what is important from the inputs x_t and does not produce predictions at any timestep. The Decoder takes the important information from the encoder and uses it to produce the outputs x_{t+n} and does not consider any input information directly.

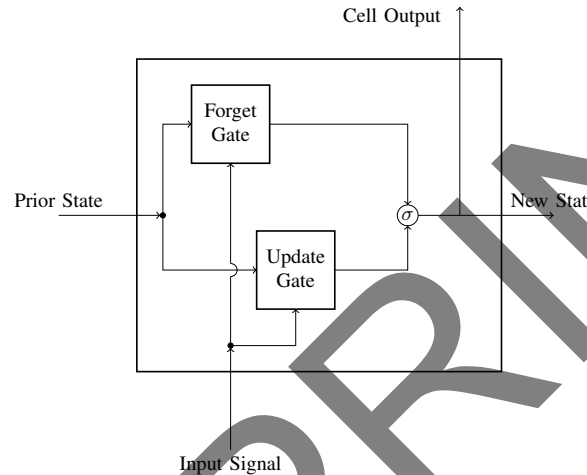


Fig. 4. A high-level visualization of how a GRU cell is structured. Both the current input and prior state are used in both the Forget and Update gates. Each gate has its own weights and will produce independent values which are used as inputs for a sub-function, marked by σ . The result is then passed forward as a new state and as the cell output for that timestep. Each of the Encoder and Decoder boxes in Fig. 3 are this cell structure.

171 two gate structures that each contain a set of trainable weights. The forget gate determines what information from the prior
 172 cell state is significant at the current time step and suppresses the information that is not relevant. The update gate determines
 173 what information from the current input should be emphasized in order to calculate the new cell state. The outputs of both
 174 gates are combined together and passed through a sigmoid activation function in order to create the new cell state.

175
 176 The GRU Autoencoder was trained using nearly the same methodology as the NN NARX model was in Sec. III-A. While
 177 the NN NARX model was trained for its ability to predict only the most immediate timestep, consisting of 3 values for roll,
 178 pitch, and heave, the GRU Autoencoder model was trained for its ability to predict full signals for each motion over a period
 179 of 5s at 10 Hz for a total of 150 values. The GRU Autoencoder model will almost certainly contain more trainable weights
 180 than the NN NARX model and as a consequence, will take longer to train; however, once trained it will perform significantly
 181 better.

182

183 C. Model Comparison

184 The performance of each of the two models described above were compared against each other by the Mean Squared Error
185 (MSE). As described in Sec. II the three signal channels are normalized separately and are dimensionless. The MSE value is
186 therefore also dimensionless and is calculated as the MSE of all channels. Since the objective is to make full signal predictions
187 the MSE is calculated over all timesteps in the target signals. The GRU Autoencoder model inherently makes full signal
188 predictions and the NN NARX model is iterated until the full output signal has been constructed.

189 As data driven models, the NN NARX model and the GRU Autoencoder model has a large number of parameters to select in
190 order to optimize predictions against the training dataset. Both the NN architecture and the number of parameters will impact
191 the performance of the models. A set of 30 sizes of weights were tested for the NN NARX and 15 layer sizes were tested for
192 the GRU Autoencoder. All other hyperparameters are held constant as their impact on performance would be minimal when
193 compared to the difference between the performance of the two architectures and the primary focus of the current study is to
194 evaluate the advantages of the modern GRU Autoencoder NN structure. The MSE on the testing dataset is show in Fig. 5;
195 the red circles show the testing MSE for the NN NARX models and the blue squares shows the testing MSE for the GRU
196 Autoencoder models.

197 For the NN NARX there is an upwards trend in the testing MSE as the number of trainable parameters increases, indicating
198 that the NN NARX model has a tendency to over-fit its parameters. The best performing NN NARX model had a layer size of
199 16, corresponding to 7248 trainable parameters, and a testing MSE value of 0.3747. The worst performing NN NARX model
200 had a layer size of 232, corresponding to 105 096 trainable parameters, and a testing MSE value of 2.687.

201 The GRU autoencoder models did not suffer from over-fitting as the NN NARX models did and showed a slight decreasing
202 trend in testing MSE values as the number of trainable parameters increases. The best performing GRU Autoencoder model
203 had a layer size of 120, corresponding to 88 920 trainable weights, and a testing MSE value of 0.0495. The worst performing
204 GRU Autoencoder model had a layer size of 24, corresponding to 3960 trainable parameters, and a testing MSE value of
205 0.1515.

206 All of the GRU Autoencoder models outperformed all of the NN NARX models, which is attributed to the data driven nature
207 of the GRU Autoencoder model allowing to learn the ship dynamics from the data directly, the architecture of the model which
208 considers time ordered inputs, and being trained against making full time signals instead of single timesteps. The proposed
209 GRU Autoencoder is arguably the better performing model for predicting ship motion.

210 For the remainder of the work presented in this paper the best performing NN NARX model with layer size of 16 and
211 a corresponding 7248 trainable parameters is chosen to evaluate performance. The GRU Autoencoder model with layer size

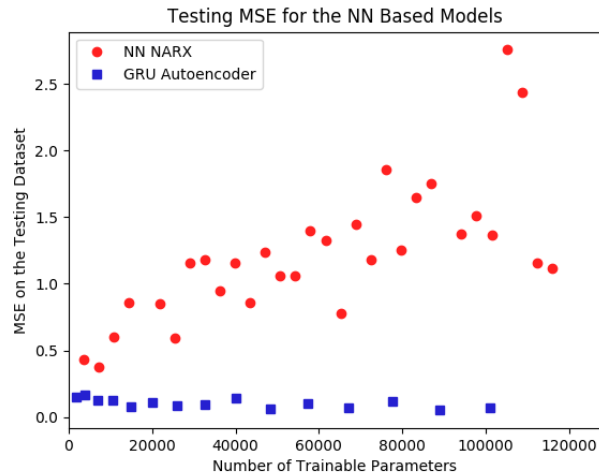


Fig. 5. The testing MSE values for various weight sizes sampled for the NN NARX model, red circles, and the GRU Autoencoder, blue squares. Every GRU Autoencoder models trained out performed every NN NARX model trained regardless of the number of trainable weights.

212 of 32, corresponding to 6816 trainable parameters and a testing MSE of 0.1243 is used to evaluate the GRU Autoencoder
 213 models. The increase in performance by adjusting the number of parameters in the GRU Autoencoder is notably less than the
 214 difference in performance between the GRU Autoencoder models and NN NARX models. In order to examine the performance
 215 differences between the two models is due to the choice of architecture, the GRU Autoencoder model with the closest number
 216 of trainable parameters to the best NN NARX model is chosen to represent the GRU Autoencoder models.

217 A sample prediction of the NN models is shown in Fig. 6. The solid black lines mark the input signals that the predictive
 218 models are provided and the solid green lines are the target signal that the models are attempting to reproduce. The red
 219 dashed line is the prediction made by the NN NARX model and the blue dash-dotted line is the prediction made by the GRU
 220 Autoencoder. Both the NN NARX and GRU Autoencoder models are able to make good predictions on the roll channel with
 221 the GRU Autoencoder predicting most of the channel to high accuracy and the NN NARX only losing accuracy after the
 222 second local minimum. The pitch and heave channels show the advantage of the GRU Autoencoder model over the NN NARX
 223 model. In both channels the NN NARX makes a large deviation from the target signal while the GRU Autoencoder remains
 224 much closer. As the GRU Autoencoder model is trained to value all time steps equally it is common for minor continuity
 225 issues to appear in the first few timesteps of the prediction, as seen in the pitch signal. While the GRU Autoencoder corrects
 226 the error from the discontinuity it must be considered for applications using real-time predictions.

227 The data used for training and testing purposes will likely not reflect the data observed in application. Varying factors such
 228 as noise in the signal measurement, changing sea states, and different ship models must be expected. As changing the nature of
 229 the data will impact the data driven models it is important to understand how applying the GRU Autoencoder model different
 230 situations affects performance.

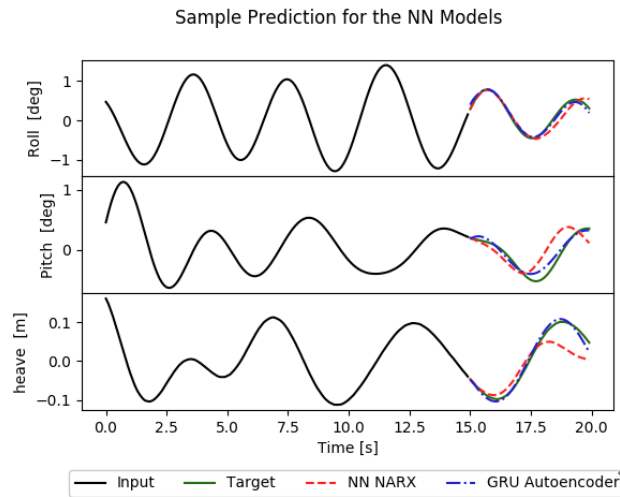


Fig. 6. A typical sample prediction for the NN NARX, the red dashed line, and the GRU Autoencoder, the blue dash-dotted line. The input signal provided to the models is shown as the solid black line and the models are attempting to predict the future motion shown by the solid green line.

IV. CASE STUDIES

The data measured in application scenarios is rarely identical to the data acquired *a priori*. Measurement noise, varying sea state levels, and different ship model dynamics can be expected and should be accounted for when implementing a ship motion prediction routine. As the NN NARX and GRU Autoencoder models require training against prior data to function it is critical to understand the impact of how these models react when presented with different data. While transfer learning methods may be applied to account for these differences the process of retraining during application can be computationally expensive and time consuming. This section presents a study of how noise, sea state, and ship model impact the performance of the NN NARX and GRU Autoencoder models and aims to assist in constructing guidelines for creating a model which can generalize a wide range of scenarios natively.

A. The Impact of Noise

The data described in Section II does not contain noise and so may not reflect in-situ data well as sensor data is rarely noise free. Thus, we seek to understand how the NN models presented handle noise similar to real sensor data. Noise is added to the simulated sea state 2 30 m vessel input signals by applying random, normally distributed values with zero mean and a variance of σ^2 . The noise levels are set to be equal across each of the normalized motion channels and cover a range up to $\sigma^2 = 1.0$, which correspond to pitch, roll, and heave variances of 0.079 deg^2 , 0.624 deg^2 , and $4.78 \times 10^{-3} \text{ cm}^2$ respectively.

Figure 7 shows how the NN models perform as the noise level increases, as judged by the MSE value on the full testing dataset. The solid blue line shows the best performing NN NARX model and the dash-dotted red line shows the selected GRU Autoencoder model.

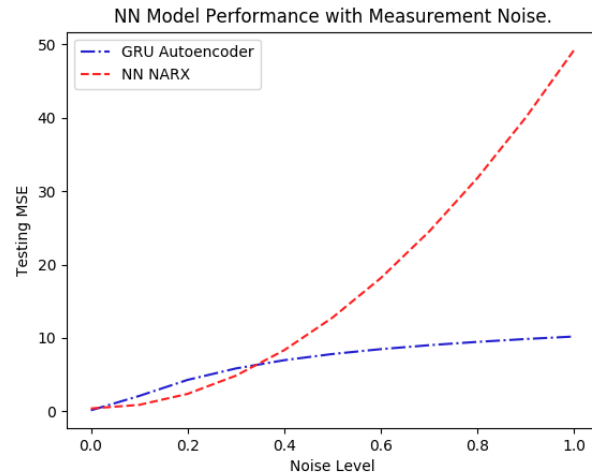


Fig. 7. The MSE values for the NN NARX model, shown by the dashed red line, and the GRU Autoencoder model, shown by the blue dash-dotted line, for various levels of noise in the testing inputs. As both models were trained without noise neither model is able to handle noise well.

249 Figure 7 shows how the testing MSE value varies when they modes are presented with noisy inputs; noting that the models
 250 in the figure are trained with noise free data. The NN NARX model, red dashed, and the GRU Autoencoder, blue dash-dotted,
 251 both have increasing testing MSE values as the noise level increases. For low levels of noise, below 0.35, the NN NARX has
 252 a lower MSE when compared to the proposed GRU Autoencoder model. The advantage of the NN NARX is likely due to the
 253 iterative calculation being able to correct well for small levels of noise. Unlike the NN NARX, whose MSE value increases
 254 exponentially as the noise increases beyond 0.35, the GRU Autoencoder MSE increases logarithmically. The GRU Autoencoders
 255 better performance on inputs with higher noise is due to training against full signals instead of individual timesteps, which
 256 allows the GRU Autoencoder to extract the underlying motion in the measured noise while making predictions.

257 For roll, pitch, and heave Fig.8 and 9 show typical sample for how the NN models perform when presented with a lower
 258 noise level of 0.2 and a higher noise level of 1.0 respectively. The true input signal is marked by the solid black line and
 259 the measured signal, which contains noise, is marked by the solid grey line. As seen in Fig. 8 the presence of noise has a
 260 dramatic affect on the performance of both NN models. In each of the roll, pitch, and heave motions the NN NARX model
 261 propagated the noise into its predictions. The GRU Autoencoder did not show noise like behaviour in its predictions, but as
 262 shown in Fig. 7, the quality of predictions is lower than the NN NARX. With the higher noise level in Fig. 9 the NN NARX has
 263 difficulties preventing itself from propagating errors, which are noted by the large overshoots in each of the motion channels. In
 264 comparison, the GRU Autoencoder in Fig. 9 stays close to the target motions, although with unsatisfactory prediction quality.

265 As NN models are data driven their prediction capabilities will be dependant on how much the training dataset matches the
 266 data measured in application. By including noise in the training data a NN model can learn to separate the underlying motion
 267 when making predictions. Fig. 10 and Fig. 11 plots the testing MSE of the NN models after being trained on data with a

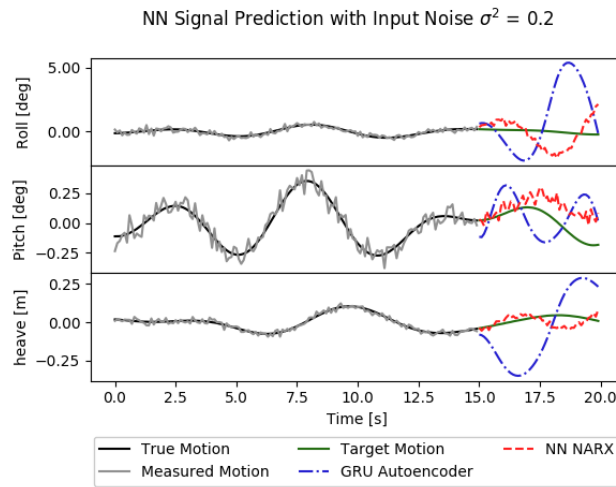


Fig. 8. A typical sample of the prediction made from the NN models, trained without noise, on inputs containing noise with a normalized variance of 0.2, which is a noise level where the NN NARX is expected to outperform the GRU Autoencoder, as shown in Fig. 7.

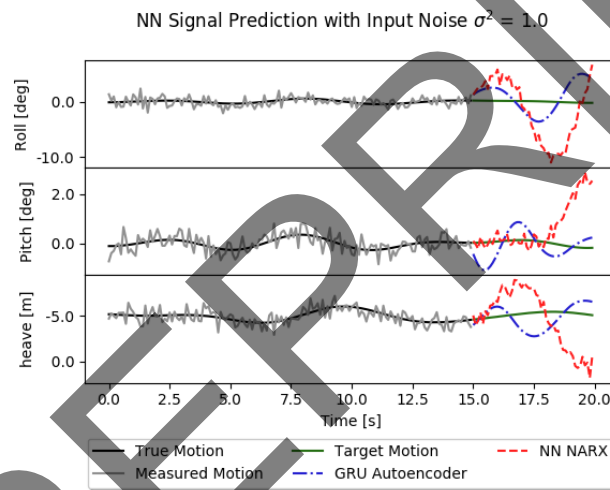


Fig. 9. A typical sample of the prediction made from the NN models, trained without noise, on inputs containing noise with a normalized variance of 1. At higher noise levels, such as shown here, the GRU Autoencoder is expected to outperform the NN NARX, as shown in Fig. 7.

268 noise level of $\sigma^2 = 0.2$ and a high noise level of $\sigma^2 = 1.0$ respectively. The red dashed line marks the performance of the
 269 NN NARX, the blue dash-dotted line marked the performance of the GRU Autoencoder. The characteristic of exponentially
 270 increasing testing MSE value for the NN NARX is visible and logarithmically increasing testing MSE for the GRU Autoencoder
 271 can be seen. The results of Fig. 10 and Fig. 11 the NN NARX does not outperform the GRU Autoencoder regardless of the
 272 noise level, indicating that the GRU Autoencoder architecture and training is better able to extract the underlying motions.

273 Table I shows the testing MSE values for the NN models when trained and tested on various noise levels. The NN NARX
 274 and GRU Autoencoder models performed better when trained with noise than when trained without. When trained without
 275 noise and presented with data that contained no noise the NN NARX and GRU Autoencoder models had MSE values of
 276 0.3774 and 0.1242, respectively. When the training data had a noise level of $\sigma^2 = 0.2$ and the testing data contained no noise,

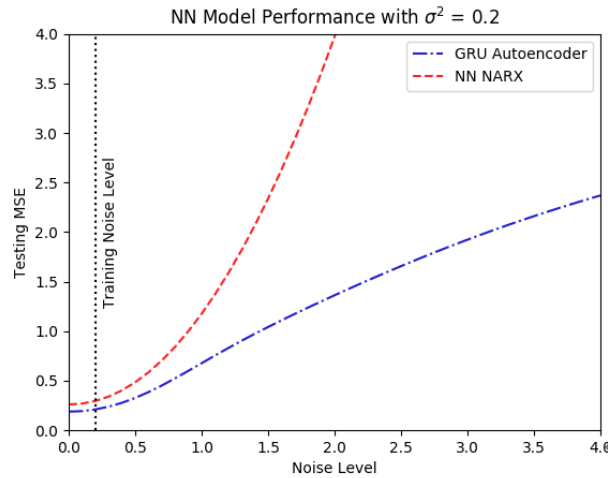


Fig. 10. The testing MSE values for the NN models after being retrained with data containing noise with normalized variance of 0.2. The NN NARX model, shown by blue cross, keeps its exponentially increasing trend. The GRU Autoencoder model, shown by red crosses, keeps its logarithmically increasing trend, though leveling off at a much higher noise level than when not trained with noise. Both NN models perform better than when not trained with noise and the GRU Autoencoder outperforms the NN NARX at all noise levels.

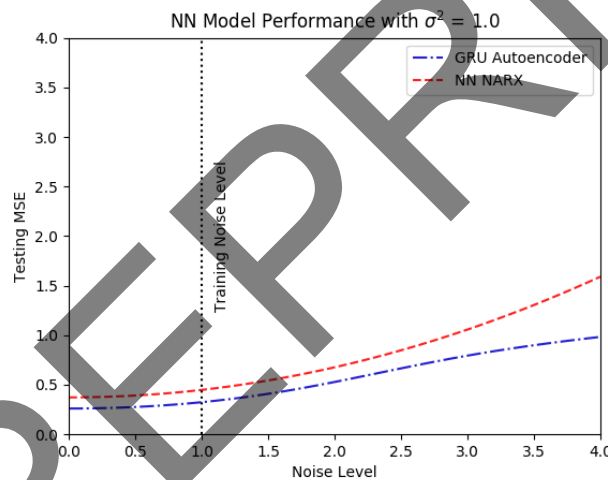


Fig. 11. The testing MSE values for the NN models after being retrained with data containing noise with normalized variance of 1.0. The NN NARX model, shown by blue cross, keeps its exponentially increasing trend. The GRU Autoencoder model, shown by red crosses, keeps its logarithmically increasing trend, though leveling off at a significantly higher noise level than when not trained with noise. Both NN models perform better than when trained with little to no noise and the GRU Autoencoder outperforms the NN NARX at all noise levels by a wide margin.

277 the NN NARX and GRU Autoencoder models had testing MSE values of 0.2612 and 0.1885, respectively. The noise trained
 278 NN NARX demonstrated an improvement, even when not evaluating noisy inputs. In comparison the GRU Autoencoder had
 279 a decrease in performance.

280 When trained with data containing a noise level of $\sigma^2 = 1.0$ and tested on data with the same level of noise, the NN NARX
 281 and GRU Autoencoder had MSE values of 0.4483 and 0.3243 respectively. Compared to the models that were not trained on
 282 noise, the improvement in performance is two orders of magnitude.

283 When presented with testing data that contained no noise the MSE values for the NN NARX and GRU Autoencoder that
 284 were trained on data with a noise level of $\sigma^2 = 1.0$ were 0.3729 and 0.2606 respectively. When presented with testing data

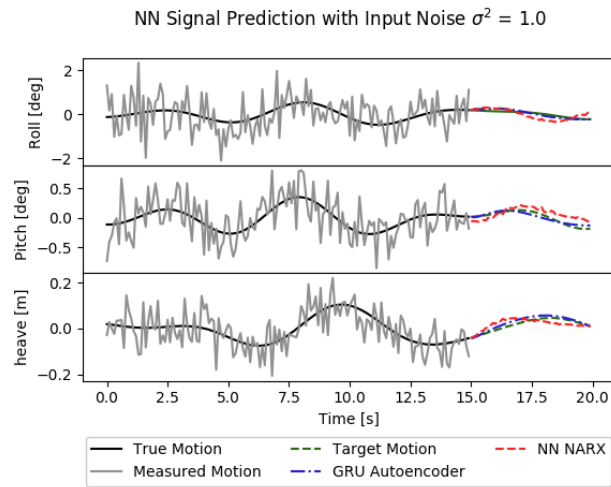


Fig. 12. A typical sample of the prediction made from the NN models, trained with noise with a normalized variance of 1.0 on inputs containing noise at the same level. Both the NN NARX models are able to extract the underlying motions and make predictions despite the high level of noise present in the inputs.

TABLE I

THE TESTING MSE VALUES FOR THE NN NARX AND GRU AUTOENCODER MODELS WHEN TRAINED AND TESTED ON VARIOUS LEVELS OF NOISE. TRAINING THE MODELS WITH NOISE LOWERS PERFORMANCE ON TESTING SAMPLES WITHOUT NOISE BUT GREATLY INCREASES PERFORMANCE ON TESTING SAMPLES WITH NOISE.

Model	Training Noise Level σ^2	Testing Noise Level σ^2		
		0	0.2	1.0
GRU Autoencoder	0	0.1242	4.274	10.19
	0.2	0.1885	0.2109	0.6776
	1.0	0.2606	0.2629	0.3243
NN NARX	0	0.3774	2.367	49.28
	0.2	0.2612	0.2960	1.176
	1.0	0.3729	0.3759	0.4483

285 that contained a noise level of $\sigma^2 = 0.2$ the high noise trained NN NARX and GRU Autoencoder models had testing MSE
 286 values of 0.3759 and 0.2629, respectively. When presented with testing data that contained a noise level of $\sigma^2 = 1.0$ the high
 287 noise trained NN NARX and GRU Autoencoder models had testing MSE values of 0.4483 and 0.3243, respectively.

288 The same sample from Fig. 9 is shown again in Fig. 12, but with predictions made from NN models that have been trained
 289 on the input noise level of $\sigma^2 = 1.0$ and the NN models perform significantly better when trained with noise when compared
 290 to when they are trained without noise, as was the case in Fig. 9.

291 The NN model behaviour when using datasets that contain noise indicate that NN based motion prediction models should
 292 include noise in training. Furthermore, the noise included should be at the same level, or above, what would be encountered in
 293 application. When trained with noise the GRU Autoencoder model outperformed the more commonly used NN NARX model,
 294 showing a clear advantage of the proposed prediction model. For the current application, the results show that the common
 295 practice of pre-filtering the input signals is not necessary as the NN models are able account for the noise in the data if properly
 296 trained.

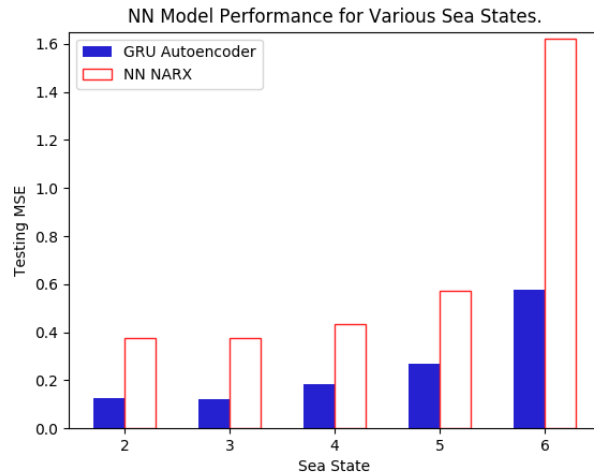


Fig. 13. The NN models are both trained on sea state 2 data and the testing MSE for sea states 2 through 6 are shown. The GRU Autoencoder model, shown as solid blue in the left column, outperforms the NN NARX, shown as hollow red bars in the right hand column, for all sea states. As expected, the more complicated sea state 6 waves are the hardest to predict for both NN models.

297 B. The Impact of Sea State

298 The NN models presented in Sec. III were trained and tested using data from sea state 2 simulations. In practical applications
 299 various sea states will be experienced by a system. Figure 13 shows the testing MSE values for sea states 2 through 6 for the
 300 two NN models. As suggested by Fig. 13, the GRU Autoencoder model outperforms the NN NARX model in all sea states.
 301 At sea state 2 the NN NARX had a testing MSE of 0.3774 and the GRU Autoencoder had a testing MSE of 0.1242. At sea
 302 state 6 the NN NARX had a testing MSE of 1.620 and the GRU Autoencoder had a testing MSE of 0.5774.

303 The overall impact of increasing sea state was not as significant as introducing noise. Increasing the sea state makes the
 304 signal inputs more complicated by changing both the range of possible amplitudes and adding additional underlying modes. The
 305 reason that increasing sea state did not impact the models performance as much as adding noise is due to of the normalization
 306 of Eq. (1). The normalization brings range of possible values in higher sea states to within a specified, dimensionless range,
 307 that is similar among all sea states. By normalizing the data the only increase in signal complexity comes from the increase
 308 in underlying modes.

309 From Fig. 13 when the NN models are trained on only the simpler sea state 2 data they are not able to handle the more
 310 complicated sea state 6 data. The NN models can be set up to handle multiple sea states simultaneously by including multiple
 311 sea states in the training data. Fig. 14 shows the NN NARX, in red hollow bars, and GRU Autoencoder, in blue solid bars,
 312 models after they were retrained to include all data from sea states 2 through 6. The NN NARX model had an average testing
 313 MSE value of 1.136, with the highest value of 1.315 occurring for the sea state 6 data and the lowest value of 1.052 occurring
 314 for the sea state 4 data. The GRU Autoencoder model had a significantly lower average testing MSE of 0.1306, with the

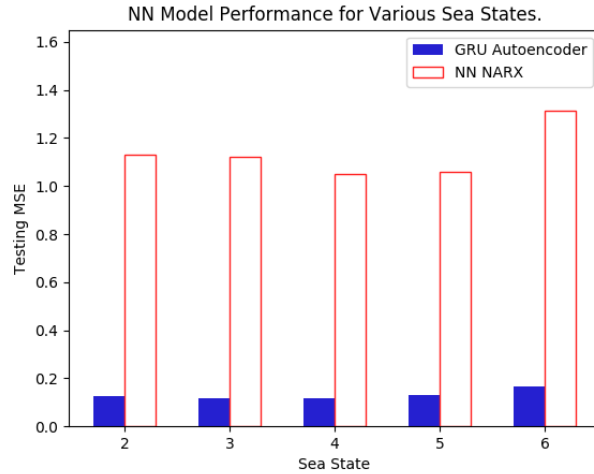


Fig. 14. The NN models after being retrained on data that contains equal portions from each sea state 2 through 6. The GRU Autoencoder model, shown as solid blue in the left column, outperforms the NN NARX, shown as hollow red bars in the right hand column, for all sea states. Unlike when trained on only one sea state, both NN models perform roughly equally for all sea states.

315 highest value of 0.1674 occurring for the sea state 6 data and the lowest value of 0.1153 occurring for the sea state 2 data.

316 By including the additional sea state data in training both NN models were able to perform consistently for all of the sea
 317 states that were included in training. However, the NN NARX performance significantly decreased on sea states 2 through 5
 318 and marginally increased on sea state 6. In comparison the GRU Autoencoder saw marginal increases in performance for the
 319 low sea states while also gaining large increases on the higher sea states. The GRU Autoencoder demonstrates a significant
 320 advantage of the NN NARX model for predicting across varying sea states, which agrees with the results presented in Fig. 5.
 321 As the data used for training is normalized applications will require some amount of initialization in order to measure enough
 322 data to calculate the variance that should be used to normalize the measured data. Once the variance is calculated the NN
 323 models will be ready for use.

324 C. The Impact of Ship Model

325 The NN models from Sec. III were trained using data from simulations of a 30m. vessel. Since applications may require
 326 multiple or different ship models which could have significantly different dynamics it is important to understand how changing
 327 ship models will impact the NN models.

328 Using ShipMo3D a second dataset was created using a 100m frigate. As the 100m frigate is much larger than the 30m vessel
 329 and is not expected to move much in sea state 2 the NN NARX and GRU Autoencoder models are retrained using data from
 330 the 30m vessel in sea state 4. Table II shows the testing MSE values for the NN NARX model and the GRU Autoencoder
 331 model for the 30m vessel used to create the training data and the new data from the 100m vessel, all at sea state 4. The NN
 332 NARX had testing MSE values of 0.5912 and 3.976 for the 30m and 100m vessels respectively while the GRU Autoencoder

Model	Training Data	Vessel Size	
		30m	100m
GRU Autoencoder	30m only	0.1458	0.9483
	30m & 100m	0.1524	0.01156
NN NARX	30m only	0.5912	3.976
	30m & 100m	0.3782	0.1489

TABLE II

THE NN MODELS ARE TRAINED ON DATA FROM SIMULATIONS OF A 30M VESSEL. THE GRU AUTOENCODER MODEL, SHOWN IN RED AS THE LEFT COLUMN, OUT PERFORMS THE NN NARX, SHOWN IN BLUE AS THE RIGHT HAND COLUMN, FOR ALL SEA STATES. AS IS EXPECTED, BOTH NN MODELS PERFORM BETTER ON DATA THAT REFLECTS THE TRAINING DATA THAN ON DATA THAT DOES NOT.

333 had testing MSE values of 0.1458 and 0.9483 for the 30m and 100m vessels respectively. As demonstrated in Sec. III the
 334 GRU Autoencoder model continues to outperform the NN NARX model. The results match expectations and the NN models
 335 perform better on the 30m vessel than the 100m vessel.

336 Another set of NN models were trained using data from both ship model datasets as part of the training and the results
 337 are also shown in Table II. The NN NARX had testing MSE values of 0.3782 and 0.1489 for the 30m and 100m vessels
 338 respectively. The GRU Autoencoder model outperformed the NN NARX with testing MSE values of 0.1524 and 0.01156 on
 339 the 30m and 100m vessels respectively. Unlike in Sec. IV-A and Sec. IV-B the NN models did not generalize in a way that
 340 balanced performance between the training datasets. Instead, training optimized the loss by focusing on optimizing predictions
 341 from the slower moving 100m vessel. However only the GRU Autoencoders ability to predict the 30m vessel motions was
 342 lowered, indicated by the slight rise of 0.0066 in testing MSE. The NN NARX improved on its ability to predict ship motion,
 343 indicated by its decrease in testing MSE of 0.2130 and 3.827 for the 30m and 100m vessels, respectively. The GRU Autoencoder
 344 saw a decrease in testing MSE of 0.9368 when predicting the 100m vessel.

345 Overall, the GRU Autoencoder performance remains superior performing model in comparison to the NN NARX model.
 346 However, as suggested by the near equal 30m vessel testing MSE values in Table II, there may be no advantage to training
 347 using multiple ship models unless the application requires it.

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