PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

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ABSTRACT

This short paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to the size of the market. Despite his view to the contrary, firms in Chamberlin's monopolistic competition are price takers, even though each firm's product is differentiated and has no perfect substitutes. There is a difference between perfect competition with product homogeneity and perfect competition with differentiated products, however. Advertising can pay off with differentiated products because products have separate identities and price depends on quality, even though firms are price takers for any given quality.

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PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

This short paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to the size of the market. Under the conditions given by Chamberlin [1965] in his classic treatise on monopolistic competition, firms would be price takers and perfect competition would prevail, although with a key exception relating to advertising. Similar results have been derived before—for example, Fradera [1986] and Rosen [1974]—but the approach here is simpler, shorter, and freer of restrictive assumptions. It focuses on the key issue of demand elasticity. Despite the widespread view in economics that monopolistic competitors face downward-sloping demand and produce with excess capacity and sub-optimal firm size, the existence of many imperfect substitutes for a product is enough to turn its supplier into a price taker.

To show this, I first note that monopolistic competition implies many firms in an industry—the result of free entry and exit and large market size relative to the output that minimizes average cost for any firm. Each firm supplies a single product, and as in perfect competition, has a small share of industry output that is treated as if it were zero in equilibrium. Firms maximize profit and reach a Nash equilibrium, in which no firm can gain profit by changing its price if prices of other firms remain unchanged. Marginal costs and prices are positive for firms that survive. Firms supply products that are close but not perfect substitutes, the factor that distinguishes monopolistic from perfect competition.

Let X be a differentiated product in an industry called the X industry that operates under monopolistic competition. Let P_x and x be the price and quantity of X and ε_x be the own-price elasticity of demand for X. Let d denote 'change in,' and suppose that P_x changes by dP_x, with all other prices in the economy remaining constant. If dP_x/P_x is numerically small, ε_x approximately equals dx/x divided by $-(dP_x/P_x)$, where dx is the change in quantity demanded of X. Let I^* be the income of the economy in which the X industry operates and E_x be the expenditure on all products that are neither substitutes for nor complements with X. If $I = I^* - E_x$, I is the sum of expenditures on X and on products that are either substitutes for or complements with X. The economy is assumed to be large enough that I^* is independent of changes in P_x and, by definition, E_x is unaffected by such changes. Thus I remains constant when P_x changes.

However, a change in P_x does generate changes in $P_x x$ and in each other output in I. Let $S_x = P_x x/I$ be the share of X in I, and when P_x changes with other prices constant, let ε_{Ax} be the share-weighted average cross-price elasticity of demand over the products that are substitutes for and complements with X. Each product's cross-price elasticity equals the percentage change in its quantity divided by the percentage change in P_x. The shares in question are the shares in I of expenditures on each substitute and complement, and the sum of these shares equals $(1 - S_x)$, while the sum of each share times that product's cross-price elasticity equals $(1 - S_x)\varepsilon_{Ax}$. Straightforward calculation gives $S_x(1 - \varepsilon_x) + (1 - S_x)\varepsilon_{Ax} = 0$, since a change in P_x does not affect I. Re-arranging this gives:

$$\varepsilon_x = 1 + [(1 - S_x)/S_x]\varepsilon_{Ax}. \quad (1).$$

Suppose that only a few firms are competing in the X industry initially and earning positive economic profits, but that these profits attract further entry until equilibrium is reached with zero economic profits. In the process, S_x tends to zero, and $(1 - S_x)/S_x$ tends to infinity. At the end of the paper, we shall ask how small S_x has to be in practice for a firm to be a price taker. Now the task is to show that as S_x tends to zero, ε_x tends to infinity. Here the key is to show that as S_x tends to zero, either ε_{Ax} remains bounded above zero or ε_x tends to infinity. However, if ε_{Ax} remains bounded above zero as S_x tends to zero—that is, if there exists a $B > 0$ such that $\varepsilon_{Ax} \geq B$—it is clear that ε_x also tends to infinity.

To show this, let I_x be total expenditure on the X industry—or total X industry output value—and let I_{nx} be the total expenditure on substitutes for and complements with X that are not in the X industry. Thus $I = I_x + I_{nx}$. If $S^x{}_x = P_x x/I_x$ is the share of X in I_x, then $S^x{}_x = S_x(I/I_x) \geq S_x$, with strict inequality holding when $I_{nx} > 0$. Therefore if $S^x{}_x$ tends to zero, the same will be true of S_x. Here ε_{Ax} is a sum of terms, each of which equals the share of a product in I times that product's cross-price elasticity—the percentage change in its output divided by the percentage change in P_x—divided by $(1 - S_x)$. We can write ε_{Ax} as $\varepsilon_{Ax} = \varepsilon^x{}_{Ax} + \varepsilon^{nx}{}_{Ax}$ where $\varepsilon^x{}_{Ax}$ gives this sum over all products in the X industry except X, and $\varepsilon^{nx}{}_{Ax}$ gives this sum over products that are substitutes for or complements with X, but which are not in the X industry. As entry occurs, consider what happens to the percentage change in I_x resulting from a small percentage decrease in P_x, with prices of other products again held constant. The decrease in P_x raises the demand for X and for products that are complementary with X and lowers the demand for products that are substitutes for X.

Moreover, either ε_x tends to infinity as S^x_x and S_x tend to zero—in which case dx/x becomes unbounded relative to $-(dP_x/P_x)$—or ε_x remains finite, in which case dx/x remains bounded relative to $-(dP_x/P_x)$. I shall show that assuming the latter leads to a contradiction, if we also assume that the increase in demand for products that are complementary with X remains bounded relative to dx and therefore relative to x and to P_xx as S^x_x and S_x tend to zero. This implies that dI_{nx}/P_xx remains bounded above and that dI_x/P_xx remains bounded below since $0 = dI = dI_x + dI_{nx}$. I shall argue below that the alternative strains credulity. Unless dx/x becomes unbounded relative to $-(dP_x/P_x)$, dI_x/P_xx will also be bounded above since the fall in P_x causes the quantities of substitute products to fall. That is, if $dx/x \leq -A(dP_x/P_x)$ for some finite and positive A, $dI_x/P_xx < (1 - A)(dP_x/P_x)$.

Thus we have:

$$dI_x/I_x = S^x_x[dI_x/P_xx], \qquad (2).$$

and it follows that dI_x/I_x tends to zero as S^x_x and S_x tend to zero. When dI_x/I_x tends to zero, dI_{nx}/I_{nx} tends to zero as well since in the limit:

$$0 = dI/I = (I_x/I)(dI_x/I_x) + (I_{nx}/I)(dI_{nx}/I_{nx}) = (I_{nx}/I)(dI_{nx}/I_{nx}). \qquad (3).$$

However, (dI_{nx}/I_{nx}) divided by (dP_x/P_x) tends to $\varepsilon^{nx}_{Ax}(I/I_{nx})$. Therefore, as S^x_x and S_x tend to zero, ε^{nx}_{Ax} also tends to zero if dx/x remains bounded relative to $-(dP_x/P_x)$. As a result, ε_{Ax} tends to ε^x_{Ax}.

The final step is to show that ε_x tends to infinity when ε_{Ax} tends to ε^x_{Ax}. This is equivalent to saying that dx/x could not remain bounded relative to $-(dP_x/P_x)$. If X did have a perfect substitute, a fall in P_x, with no change in the price of the substitute, would reduce the demand for this substitute to zero, since buyers will not buy a product at a higher price when it is available at a lower price. This result depends only on the two products being perfect substitutes and is independent of how many other substitutes for X there are. The same is true when there are only imperfect substitutes for X. If X and another product are partial, but close substitutes, their cross-price elasticities will again measure this closeness, regardless of how many other substitutes there are for these products. A fall in P_x will transfer a significant part of the demand for a close substitute to X, as buyers of this substitute take advantage of a lower price, which they can only do by buying more of X.

Under monopolistic competition, competing firms supply products that are close substitutes. As a result, a cross-price elasticity between two of these substitutes cannot be arbitrarily small, but must instead be greater than or equal to some minimum value, say $B > 0$. As a weighted average of these cross-price elasticities, ε^x_{Ax} must tend to a value that is no less than B as S^x_x and S_x tend to zero. Since ε_{Ax} tends to ε^x_{Ax}, the same must be true of ε_{Ax}. As a result, ε_x tends to infinity, which is therefore the only possible limiting value for ε_x. By making S^x_x and S_x small enough, ε_x can be made as large as desired.

In an environment where X has many close substitutes, the assumption that dx/x remains bounded relative to $-(dP_x/P_x)$ as S^x_x and S_x tend to zero implies that dI_{nx}/P_xx becomes unbounded above and dI_x/P_xx becomes unbounded below. A small percentage decrease in P_x, with other prices held constant, triggers an increase in I_{nx} and an offsetting decrease in I_x, both of which become unbounded relative to P_xx. That strains credulity, not only because of the relative magnitudes involved, but also because products that are complementary with X are likely to be complementary with products that are close substitutes for X. On those grounds, we would expect dI_{nx}/P_xx and dI_x/P_xx to move in the same direction. One version of downward-sloping demand at the industry level is that the sum of prices times quantity changes across the X industry caused by the fall in P_x is non-negative. That is, $(dI_x - xdP_x) \geq 0$ or $dI_x \geq xdP_x$ which implies $dI_x/P_xx \geq dP_x/P_x$. If this condition holds, ε_x must tend to infinity because dI_x remains bounded below relative to P_xx as S^x_x and S_x tend to zero.

How small does S_x have to be in order for a firm to be a *de facto* price taker? Suppose that $S_x = .05$ and $\varepsilon_{Ax} = .5$, keeping in mind that ε_{Ax} would be infinitely large in a perfectly competitive equilibrium with homogeneous products. If the difference between ε_{Ax} and ε^x_{Ax} can be ignored—since one tends to the other—ε_{Ax} is the share-weighted average cross-price elasticity over the X industry. In this context, that S_x is the share of X in I rather than in X-industry output value, I_x. If I_x is 80% of I, the share, S^x_x, of X in I_x is $1.25S_x = .0625$, suggesting an industry with 16 suppliers. In this case, $\varepsilon_x = 10.5$. If the supplier of X raised its price by 5%, it would lose more than half its market and probably be forced to close. Such a firm is a *de facto* price taker. If S_x were twice as large and I_x were again 80% of I, the share, S^x_x, of X in X industry output value would be .125, suggesting an industry with eight suppliers. Then ε_x would be 5.5, and the supplier of X is less likely to behave as a price taker, but the X industry is now an oligopoly.

It follows that Chamberlin's monopolistic competition is a type of perfect competition, although with a key exception. When products are differentiated, they and the firms that supply them have separate identities and can be distinguished from one another. It is therefore possible to advertise a specific firm's product successfully if the advertising leads potential customers to believe that it has a higher quality than they had previously perceived. For that quality, the firm is still a price taker, however.

While market failure can always result from too few competitors and entry barriers, it does not result from product differentiation with many competitors. Chamberlin's conclusion that the demand for X is downward sloping requires the share-weighted average cross-price elasticity, ε_{Ax}, to tend to zero as the share of X in I tends to zero, but either this does not happen or ε_x tends to infinity anyway.

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