PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

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ABSTRACT

This paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Despite Chamberlin's view, monopolistic competitors are price takers, even though each firm's product has no perfect substitute. There is a difference between perfect competition with product homogeneity and perfect competition with differentiated products, however. Advertising can pay off with differentiated products because products have separate identities—and price depends on quality—even though firms are price takers for given quality. Under conditions to be given, a differentiated oligopoly will resemble monopolistic competition a la Chamberlin.

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PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

This paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Under the conditions given by Chamberlin [1965] in his classic treatise on monopolistic competition, firms would be price takers and perfect competition would prevail, although there is a key exception relating to advertising. Similar results have been derived before—for example, by Fradera [1986] and Rosen [1974]—but the approach here is simpler, shorter, and freer of restrictive assumptions. It focuses on the key issue of demand elasticity.

Despite the widespread view in economics that monopolistic competitors face downward-sloping demand and produce with excess capacity and sub-optimal firm size, the existence of many imperfect substitutes for a product is enough to turn its supplier into a price taker. The best-case scenario for an industry to have the properties historically associated with monopolistic competition occurs when cross-price elasticities within the industry are not too high—so that firms do not behave strategically—and when the number of competitors is not so large that each firm is a *de facto* price taker, but not so small that firms are able to earn positive economic profit.

Monopolistic competition implies many firms in an industry—owing to free entry and exit and large market size relative to the output that minimizes average cost for any firm. Each firm supplies a single product, and as in perfect competition, has an insignificant share of industry output. Buyers of these products maximize utility, while suppliers maximize profit and reach a Nash equilibrium, in which no firm can gain by changing its price if prices of other firms remain constant. Firms supply products that are close but not perfect substitutes, the factor that distinguishes monopolistic from perfect competition.

Let X be a differentiated product in an industry called the X industry that operates under monopolistic competition. Let P_x and x be the price and quantity of X and ε_x be its own-price elasticity of demand. Suppose that P_x changes by dP_x, with all other prices in the economy remaining constant. If dP_x/P_x is numerically small, ε_x approximately equals $(dx/x)/(-dP_x/P_x)$, where dx is the change in quantity demanded of X. Let I^* be the income of the economy in which the X industry operates and E_x be the expenditure on all products that are neither substitutes for nor complements with X. If $I = I^* - E_x$, I is the sum of expenditures on X and on products that are either substitutes for or complements with X. The economy is assumed to be large enough that I^* is independent of changes in P_x and, by definition, E_x is unaffected by such changes. Thus I remains constant when P_x changes.

However, a change in P_x does cause changes in x and in each other output in I. Let products $(Y_1 \ldots Y_M)$ be all the substitutes for and complements with X, with prices $(P_1 \ldots P_M)$ and quantities $(y_1 \ldots y_M)$. Then:

$$I = P_x x + \Sigma_k P_k y_k, \qquad (1).$$

where summation is from one to M. I is the sum of prices times quantities of all the products in I. Let P_x change by one unit with $(P_1 \ldots P_M)$ held constant. Since I does not change, we have:

$$0 = x + P_x x_P + \Sigma_k P_k y_{kP}, \qquad (2).$$

where x_P and y_{kP} are the resulting changes in x and in each y_k, for $k = (1 \ldots M)$.

Let $S_x = P_x x/I$ be the share of X in I. Since $\varepsilon_x = -x(P_x x_P)$, we have $(P_x/I)(x + P_x x_P) = S_x((1 - \varepsilon_x)$. Note from (1) that $(\Sigma_k P_k y_k)/I = (1 - S_x)$, and let ε_{Ax} be the share-weighted average cross-price elasticity of demand over $(Y_1 \ldots Y_M)$. That is:

$$(1 - S_x)\varepsilon_{Ax} = \Sigma_k[(P_k y_k/I)(P_x y_{kP}/y_k)] = (P_x/I)[\Sigma_k P_k y_{kP}]. \qquad (3).$$

As a result, (2) becomes $S_x(1 - \varepsilon_x) + (1 - S_x)\varepsilon_{Ax} = 0$, when we multiply both sides by (P_x/I). Re-arranging these terms gives:

$$\varepsilon_x = 1 + [(1 - S_x)/S_x]\varepsilon_{Ax}. \qquad (4).$$

We can also divide all the products in I into those products which are in the X industry and those which are outside it. Suppose that $(Y_1 \ldots Y_m)$ are in the X industry, while $(Y_{m+1} \ldots Y_M)$ are outside it. Let I_x be total expenditure on the X industry—that is, on $(Y_1 \ldots Y_m)$—and I_{nx} be the total expenditure on substitutes for and complements with X that are not in the X industry—that is, on $(Y_{m+1} \ldots Y_M)$. Then $I = I_x + I_{nx}$. If $S^x{}_x = P_x x/I_x$ is the share of X in I_x, $S^x{}_x = S_x(I/I_x) \geq S_x$, with the strict inequality holding when $I_{nx} > 0$. Under monopolistic competition, $S^x{}_x$ is inconsequential in equilibrium, and the same must therefore be true of S_x.

In order to determine which products are in the X industry, we must define what it means for X to be a 'close' substitute for another product, Y_k. Clearly, not *all* products for which X is a substitute are in the X industry, and 'closeness' can be measured by cross-price elasticities. Thus X is a 'close' substitute for Y_k if and only if the cross-price elasticity between the two that arises when P_x changes, with other prices held constant, is greater than or equal to some positive lower bound. The choice of lower bound determines m and is arbitrary to a degree, but this is always true of the definition of an 'industry'. The basic results here are independent of the boundary chosen, but to satisfy a condition of monopolistic competition, it should be chosen low enough that in equilibrium, $S^x{}_x$ is as small as desired.

Suppose that at first only two firms are competing in the X industry and earning positive economic profits, but that these profits then attract further entry until equilibrium is reached with zero economic profits. Entry will cause m to increase for any given lower bound, and $(M - m)$ may change as well. Under monopolistic competition, $S^x{}_x$ and S_x will tend to zero, and $(1 - S_x)/S_x$

will tend to infinity. As new competitors enter the X industry, product X will face greater competition in the form of a greater variety of alternatives. As a result, ε_x will rise. At the end of the paper, I shall ask how small S_x has to be for a firm to be a *de facto* price taker. Now the task is to show that as S_x tends to zero, ε_x tends to infinity. The key is to show first that, as S_x tends to zero, either ε_{Ax} remains bounded above zero or ε_x tends to infinity. However, if ε_{Ax} remains bounded above zero as S_x tends to zero—that is, if there exists a $B > 0$ such that $\varepsilon_{Ax} \geq B$—it is clear that ε_x also tends to infinity as S_x tends to zero.

From equation (3) above, ε_{Ax} is a sum of terms, each of which equals the share of a product in I times that product's cross-price elasticity divided by $(1 - S_x)$. We can therefore write ε_{Ax} as $\varepsilon_{Ax} = \varepsilon^x{}_{Ax} + \varepsilon^{nx}{}_{Ax}$ where $\varepsilon^x{}_{Ax}$ is this sum over all products in the X industry except X, and $\varepsilon^{nx}{}_{Ax}$ is this sum over all substitutes for or complements with X that are outside the X industry.

When P_x falls by a given amount, with prices of other products held constant, let I_x change by dI_x. As entry into the X industry occurs, consider how this affects dI_x/I_x. First note that as $S^x{}_x$ and S_x tend to zero, either ε_x tends to infinity or ε_x remains finite, in which case dx/x is bounded relative to $-(dP_x/P_x)$ for all dP_x sufficiently close to zero. Suppose that the latter is true. Then dI_x/P_xx is bounded above since the fall in P_x causes the demand for substitute products to fall. That is, if $dx/x \leq -A(dP_x/P_x)$ for some finite and positive A, $dI_x/P_xx < (1 - A)(dP_x/P_x)$ holds.

In addition, dI_x/P_xx is bounded below if the fall in P_x is sufficiently small. Otherwise, as $S^x{}_x$ tends to zero, dI_x/P_xx will tend to minus infinity for any given small decrease in P_x, and dI_x/P_xx will therefore tend to minus infinity as $S^x{}_x$ and dP_x tend to zero. Suppose that dI_x/P_xx is a continuous function of $S^x{}_x$ and of dP_x everywhere in some neighborhood of $S^x{}_x = dP_x = 0$. Then if $dP_x = 0$ and $S^x{}_x$ is small enough, dI_x will be negative. With no price changes, a transfer of demand will occur from products in the X industry to products outside this industry. However, if buyers were already maximizing utility, there would be no need for such a transfer. Thus the assumptions of utility maximization and of continuity imply that $dI_x/P_x x$ remains bounded below as S_x and $S^x{}_x$ tend to zero.

Since $dI_x/I_x = S^x{}_x[dI_x/P_x x]$ and $dI_x/P_x x$ is bounded above and below, dI_x/I_x tends to zero as $S^x{}_x$ and S_x tend to zero. If I_{nx} tends to zero, note that $\varepsilon^{nx}{}_{Ax}$ also tends to zero—since in the limit there are no complements with or substitutes for X outside the X industry—and ε_{Ax} tends to $\varepsilon^x{}_{Ax}$. If I_{nx} is positive in the limit, then dI_{nx}/I_{nx} tends to zero, since in the limit:

$$0 = dI/I = (I_x/I)(dI_x/I_x) + (I_{nx}/I)(dI_{nx}/I_{nx}) = (I_{nx}/I)(dI_{nx}/I_{nx}) \quad (5).$$

when P_x falls. However, (dI_{nx}/I_{nx}) divided by (dP_x/P_x) tends to $\varepsilon^{nx}{}_{Ax}(I/I_{nx})$. Therefore, as $S^x{}_x$ and S_x tend to zero, $\varepsilon^{nx}{}_{Ax}$ also tends to zero if dx/x remains bounded relative to $-(dP_x/P_x)$. As a result, ε_{Ax} again tends to $\varepsilon^x{}_{Ax}$.

The final step is to show that ε_x tends to infinity when ε_{Ax} tends to $\varepsilon^x{}_{Ax}$. By assumption, X is a 'close' substitute for each product in the X industry. The cross-price elasticity between X and any other product in the X industry that arises when P_x changes must be no less than some positive value, say $B > 0$. As a weighted average of these cross-price elasticities, $\varepsilon^x{}_{Ax}$ must also tend to a value no less than B as $S^x{}_x$ and S_x tend to zero. Since ε_{Ax} tends to $\varepsilon^x{}_{Ax}$, ε_{Ax} also tends to a value no less than B. As a result, ε_x tends to infinity, which is therefore the only possible limiting value for ε_x. By making $S^x{}_x$ and S_x small enough, ε_x can be made as large as desired.

In the above, the values of ε_x, S_x, and ε_{Ax} are independent of B, which simply determines the boundaries of the X industry. For the selected value of B, suppose that the set of all products for which X is a 'close' substitute forms an equivalence class. That is, suppose that X is a 'close' substitute for X, which is obvious, and that when X is a 'close' substitute for any Y_k, then Y_k is also a ‘close’ substitute for X. Finally, suppose that when X is a ‘close’ substitute for Y_k and Y_k is a ‘close’ substitute for Y_j, then X is also a ‘close’ substitute for Y_j. In this case, the X industry consists of all firms whose products are ‘close’ substitutes for X, and these products are all ‘close’ substitutes for one another as well. Each firm in the X industry so defined is a price taker when the industry consists of many products, with each firm having a small share of industry output value. However, we did not need to assume that the set of all products for which X is a ‘close’ substitute forms an equivalence class to show that the supplier of X is a price taker.

How small does S_x have to be in practice for the supplier of X to be a *de facto* price taker? Suppose that $S_x = .025$ and $\varepsilon_{Ax} = .25$. If the difference between ε_{Ax} and ε^x_{Ax} can be ignored—since one tends to the other—ε_{Ax} is the share-weighted average cross-price elasticity over the X industry, which would be infinitely large if all products in this industry were perfect substitutes. With $\varepsilon_{Ax} = .25$, a 10% decrease in P_x would lower the demand faced by an average competitor by 2.5%. If I_x is 70% of I for the value of B selected, the share, S^x_x, of X in I_x is about $1.43S_x =$ .03571, implying an industry with 28 suppliers if S^x_x is an average share for this industry. In this case, $\varepsilon_x = 10.75$, and marginal revenue is more than 90% of price. If the supplier of X raised its price by 5%, it would lose more than half its demand. Such a firm is a *de facto* price taker.

If S_x were equal to .08, and I_x were again 70% of I, the share, S^x_x, of X in X industry output value would be .114, implying an industry with 9 suppliers if S^x_x is about average for this industry. Suppose that free entry and exit prevail and that ε_{Ax} again equals .25. If ε_{Ax} is low enough to prevent firms from behaving strategically, and if eight competitors under free entry/exit is enough to prevent the supplier of X from earning positive economic profit in equilibrium, it will produce with excess capacity and sub-optimal firm size, as in monopolistic competition a la Chamberlin. Here ε_x is 3.875, and marginal revenue is 74% of price. If the supplier of *X* raises its price by 5%, it will lose 19.375% of its demand and might not behave as if price and marginal revenue were the same. The best-case scenario for this occurs when cross-price elasticities are not too high and when the number of competitors under free entry and exit is not so large that each firm is a *de facto* price taker, but not so small that firms are able to earn positive economic profit.

It follows that Chamberlin's monopolistic competition with many competitors is a type of perfect competition. However, there is a key difference between perfect competition with homogeneous products and perfect competition with differentiated products. In the latter case, products and the firms that supply them have separate identities and can be distinguished from one another. It is therefore possible to advertise a specific firm's product successfully if the advertising leads potential customers to believe that it has a higher quality than they had previously perceived. For that quality, the firm is still a price taker, however. While market failure can always result from too few competitors and entry barriers, it does not result from product differentiation with many competitors.

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