Scale Preserving Smoothing of Polygons

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ABSTRACT

A smoothed version of a polygon ξ is defined as a polygon which approximates ξ according to a given criterion and which simultaneously has no more edges than ξ itself. In this paper, a scale preserving smoothing algorithm is presented. The input to the algorithm is a polygon ξ and the output is its smoothed version ξ_ε. ξ_ε, which contains all the scale information that ξ contains, is called the Linear Minimum Perimeter Polygon (LMPP) of ξ within a tolerance of ε. Using the quantity ε the degree to which ξ_ε approximates ξ can be controlled. From the LMPP a representation for a polygon approximating ξ can be procured, which is invariant to scale and translation changes. Examples of smoothing maps and characters have been presented.

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Index Terms: Smoothing, Minimum Perimeter Polygon, Scale Preserving Smoothing, Cartography, Polygonal Representation.

I. INTRODUCTION

Over the past two decades considerable research has gone into the study of the automatic recognition of shape. In this context polygons have played a major role especially since the outer boundary of an object without holes can be approximated as a polygon. The advantages of such representations can be found, for example, in [5, Chapter VII]. Consider a typical image processing environment in which the picture of an object to be recognized, silhouetted against a background is given by a two-dimensional pixel array. Using any boundary tracking algorithm a polygonal representation for the shape of the object can be obtained. It is not uncommon that such a representation involves a polygon having many hundreds of edges [3,5,8]. Since the time required for processing the polygon is dependent on the number of edges it possesses, this polygon is usually approximated using a smoothing technique, such as the split and merge technique or the linear scan technique. These techniques and their variants have been well described in Pavlidis [5, pp. 161-184].

Useful as these techniques are, none of these techniques preserves all the scale information contained in the unsmoothed boundary. To clarify this assertion, let ξ and τ be two unsmoothed polygons with τ being a scaled version of ξ, the scaling factor being $k > 0$. Let ξ^* and τ^* be the corresponding smoothed versions, the smoothing being performed using any of the algorithms known in the literature. Even though τ is a scaled version of ξ, none of the currently available techniques can guarantee that ξ^* is a scaled version of τ^*.

In this paper we propose a smoothing scheme which can indeed guarantee the preservation of scale information. The input to the scheme is a polygon ξ and its output is ξ_ε, the smoothed version of ξ, referred to as Linear Minimum Perimeter Polygon (LMPP) of ξ within the tolerance ε. The quantity ε, $0 \le \varepsilon \le 1$, is termed as the tolerance factor. The value $\varepsilon=0$ yields ξ_ε identical to ξ and as ε increases ξ_ε approximates ξ more and more crudely. Within reasonable limits of ε, ξ_ε indeed preserves the scale information in ξ and yields ε as a single control parameter by which the smoothing can be controlled.

A natural consequence of this technique is a representation for a smoothed version of a shape, which is invariant to changes in scaling and the translation of the coordinate system in which the original shape is drawn.

In the next section we shall describe the Minimum Perimeter Polygon (MPP) of a polygon. We then proceed to define the Linear Minimum Perimeter Polygon (LMPP) and demonstrate its properties. We conclude the paper with examples of the use of the LMPP in smoothing maps and characters.

II. THE MINIMUM PERIMETER POLYGON

Let ξ be any polygon specified as an ordered sequence of points in the plane as below.

$$\xi = \{P_i / i=1,..N\} \tag{1}$$

Let δ_i be a prespecified circular or polygonal constraint domain in the neighborhood of P_i. Let ξ' be any polygon specified by the sequence

$$\xi' = \{R_i / i=1,...N;\ R_i \in \delta_i\} \tag{2}$$

ξ' is any approximation of ξ in which the point P_i is perturbed to a new location R_i within the domain δ_i. The polygon ξ^* which satisfies (2) and which has the minimum perimeter is called the Minimum Perimeter Polygon (MPP) of ξ. Sklansky et al [6,7] and Montanari [3] have suggested algorithms for computing the MPP when the disjoint domains δ_i are polygonal or circular respectively. When the constraint domains are all disjoint circles with radius γ, the MPP ξ^* satisfies the following:

$$\xi^* = \text{Argument} \min_{\xi'} [f(\xi')] \tag{3}$$

where $\xi' = \{R_i / i=1,\ldots,N;\ \ ||P_i - R_i|| \leq \gamma\}$ and $f(\xi') = \sum_{i=1}^{N} ||R_i - R_{i+1}||$ with $R_{N+1} = R_1$. A vertex R_i of ξ^* is called an active vertex if it lies on the boundary of the constraint disk. In such a case

$$||P_i - R_i|| = \gamma.$$

Montanari has proved [3] that the MPP ξ^* possesses the following properties:

(i) ξ^* can be completely and uniquely specified by the sequence of active vertices.

(ii) Let R_j and R_k be the first active vertices on either side of a vertex P_i of ξ. If the line joining R_j and R_k intersects the circular constraint disk around P_i, the vertex R_i will not be an active vertex. In such a case the edges of the MPP through R_i are colinear and can therefore be merged. This is depicted in Figure 1.

(iii) If the line joining R_j and R_k lies outside the constraint disk around P_i, the vertex R_i will be active. It will lie on the boundary of the disk at the point where the bisector of the angle $R_j R_i R_k$ is normal to the boundary (See Fig. II).

We shall now extend the results of Montanari to formulate the Linear Minimum Perimeter Polygon (LMPP).

III. THE LINEAR MINIMUM PERIMETER POLYGON

The Linear Minimum Perimeter Polygon (LMPP) of a polygon ξ is its Minimum Perimeter Polygon obtained by making the radii of the constraint disks directly proportional to the perimeter of ξ. The constant of proportionality ε is called the tolerance factor, and the LMPP of ξ obtained using a tolerance factor of ε is given by ξ_ε.

Specifically, if $\xi = \{P_i/i=1,\ldots,N\}$, ξ_ε is obtained by the following minimization procedure

$$\xi_\varepsilon = \text{Argument } \underset{\xi'}{\text{Minimum}} \, [f(\xi')] \qquad (4)$$

with $\xi' = \{R_i/i=1,\ldots,N;\ ||R_i - P_i|| \leq \varepsilon L\}$

where $L = \sum_{i=1}^{N} ||P_i - P_{i+1}||$, $P_{N+1} = P_1$

and $f(\xi') = \sum_{i=1}^{N} ||R_i - R_{i+1}||$, $R_{N+1} = R_1$

Note that ξ_ε equals ξ if and only if ε is identically zero. We now demonstrate the scale preserving property of the LMPP. Let $|\mu|$ denote the perimeter of a polygon μ.

Theorem I

Let ξ be any closed boundary and ξ_ε be its LMPP within the tolerance ε. Let τ be a scaled version of ξ where the scaling factor is $k > 0$. Then τ_ε, the LMPP of τ within the same tolerance ε is the scaled version of ξ_ε, scaled by the same scale factor k, provided the constraint disks are all disjoint.

Proof: Let $\xi = \{P_i/i=1,\ldots,N\}$ have a perimeter $|\xi| = L$, and let its LMPP be $\xi_\varepsilon = \{R_i/i=1,\ldots,N\}$, with perimeter $|\xi_\varepsilon|$. Let $\tau = \{Q_i/i=1,\ldots,N\}$ be the scaled version of ξ. By hypothesis $|\tau| = kL$, and hence the constraint disks around Q_i must be of radius εkL.

Using ξ, ξ_ε and the following rules construct the polygon $\tau^* = \{S_i/i=1,\ldots,N\}$.

(a) If R_i is non-active render S_i non-active.

(b) If R_i is active, S_i is the unique point on the boundary around Q_i where the angular equality $P_{i-1}P_iR_i = Q_{i-1}Q_iS_i$, as illustrated in Fig. III.

From the construction τ^* is a scaled version of ξ_ε, and the scaling factor is exactly k. Thus $|\tau^*| = k|\xi_\varepsilon|$. We contend that τ^* is the LMPP of τ within the tolerance ε. To prove this let $\tau^+ = \{T_i/i=1,\ldots N\}$ be any polygon, distinct from τ^*, satisfying $||T_i - Q_i|| \leq \varepsilon kL$ for i=1,...N. It remains to be proved $|\tau^+| > |\tau^*|$. We prove this by contradiction.

Suppose $|\tau^+| \leq k|\xi_\varepsilon|$. Using the polygons τ and τ^+ and an analogous construction, construct a polygon ξ^+ inside ξ having the perimeter $\frac{1}{k}[|\tau^+|]$. Thus $|\xi^+| \leq |\xi_\varepsilon|$. This implies that ξ^+ is the LMPP of ξ which is impossible, since by definition, ξ_ε is the unique [3] LMPP of ξ. This in turn implies that τ^* is the LMPP of τ within the tolerance ε.

Remarks:

(i) Since the uniqueness of the MPP requires that the constraint disks are disjoint ξ_ε preserves the scale information in ξ if and only if

$$\varepsilon < \underset{j}{\text{Min.}}[||P_j - P_{j+1}||] / 2 \sum_{i=1}^{N} ||P_i - P_{i+1}|| \text{ , where } P_{N+1}=P_1. \quad (6)$$

This range of ε is sufficient for some applications.

(ii) In many applications the above range of ε is not adequate. This is because of the fact that in many image processing applications two neighboring pixels may be adjacent vertices of a polygon which represents a boundary [1,5]. In such cases, further smoothing can be achieved by preprocessing ξ to ensure that the constraint disks are disjoint. If P_k and P_{k+1} are two vertices in ξ obeying $||P_k - P_{k+1}|| \leq \varepsilon L$, a technique that we have found useful is to approximate ξ by merging P_k and P_{k+1} to a single point - for example, at their mean. The LMPP ξ_ε is computed using the approximated version of ξ. The latter approximation is pictorially shown in Fig. IV.

(iii) A consequence of Theorem I is that if ε satisfies (6), the LMPP provides a representation for a smoothed version of ξ which is invariant to scaling and translation of the original figure ξ. This representation is a sequence of angles and lengths, where the angles are those of the LMPP, and the lengths are the lengths of its edges normalized to its total perimeter. The use of this representation in pattern classification is discussed elsewhere [2].

(iv) A note comparing the technique introduced here with the techniques currently used is not out of place. Various smoothing algorithms which use error norms have been proposed in the literature. Examples of these are the linear scan and the split and merge techniques [5] and the method due to Ramer [8]. From a naive perspective it appears as if our contribution is merely that of introducing an error norm which is proportional to the perimeter of the polygon to be smoothed. Rather, we have exploited the properties of the active and nonactive vertices of the MPP and the uniqueness of the MPP. The error norm introduced here is augmented by the latter properties of the MPP to yield the scale presentation property of the LMPP claimed in Theorem I. In other words, we contend that if an error norm proportional to the perimeter is used in conjunction with other techniques such as the linear scan or the split and merge technique , the scale information of the original polygon will be destroyed primarily because of the nonuniqueness of the resulting polygon.

(v) Ramer [8] has noted that a smoothing technique should result in a polygon which possesses a minimal number of edges and simultaneously contains all the "significant features" of the original polygon. The LMPP possesses both these properties. Since all the edges which are "almost collinear" are merged the number of edges is drastically reduced. Further, since the vertices of LMPP fall on the boundaries of the constraint disks, the vertices which primarily distinguish the shape of the polygon are preserved, as in Figure II.

Example I

Let τ be the quadrilateral ABCD, given in Fig. V. The radii of the constraint disks around these vertices are given by L, where

$$L = || AB || + || BC || + || CD || + || DA ||.$$

The LMPP of τ is τ_ε given by the triangle PQR. Let the angles of the triangle be α_p, α_q, and α_r at the vertices P, Q and R respectively, and let the lengths of the edges opposite to P, Q and R be p, q and r respectively. Then, if $\Sigma=p+q+r$, the sequence of angles and lengths of the normalized LMPP will be the ordered sequence given below

$$\{(\alpha_p,\ p/\Sigma),\ (\alpha_q,q/\Sigma),\ (\alpha_r,r/\Sigma)\}$$

***

IV. APPLICATIONS OF THE LMPP

A scale preserving smoothing technique will have immense value especially in the area of cartography. Maps of islands and lakes can be drawn to great precision provided a small scale factor is used. However, if a miniature map of an original has to be obtained, a fair amount of smoothing will have to be done. A pertinent question is one of knowing whether the miniature has all the relevant scale information. The smoothing technique presented here can be used to smooth the original map and the smoothed version miniaturized with the full confidence of the map being to the appropriate scale.

To demonstrate this feature we have considered the Great Lakes of North America. The maps were obtained from the National Geographic Magazine collection [4] drawn at a scale of 32 miles/inch. They were appropriately scaled to fit an 8" by 11" frame. These pictures were then photographed using a television camera to fit a 90 x 90 pixel array. A simple boundary tracking algorithm utilizing constant thresholds was employed to extract the boundaries of the lakes.

The boundary of a map was used as the original polygon and the LMPP was constructed as its smoothed version. The effect of smoothing using various values of ε are shown for Lake Erie in Fig. VI.(a) - (e). The original is shown in Fig. VI(a) and the smoothed versions for ε values of 0.001, 0.002, 0.005 and 0.01 are shown in Fig. VI(b)-(e) respectively. It can be seen that a value of ε=0.001 gives us a very fine approximation, and the approximation increases in crudeness as ε increases. A recommended value of ε for most applications is 0.005. This value of ε usually preserves the principal vertices of ξ, and reduces the number of edges by about 60 percent.

The effect of smoothing characters can be seen from the experiments displayed in Fig. VII. The numeral "1" was written on a 8" x 11" frame and photographed into a 75 x 75 pixel array. The boundary of the figure was then smoothed as in the previous case. The original and smoothed versions are shown in Fig. VII(a)-(e). The value of ε=0.01 gives us a crude approximation of the original - but it can easily be visually recognized as the numeral "1". In such cases the smoothing algorithm can be used as a thinning technique to extract the skeletal version of a shape.

V. CONCLUSIONS

In this paper we have presented a smoothing technique which has some interesting scale preserving properties. If ξ and τ are two polygons with τ as a scaled version of ξ, their smoothed versions ξ_ε and τ_ε are exactly scaled versions of each other and the scale factor is the same provided ε obeys a simple inequality constraint. The smoothed version known as the Linear Minimum Perimeter Polygon (LMPP) is a method of approximating ξ by ξ_ε where the latter has a number of edges less than or equal to the former. It also gives us a single parameter ε to control the degree of approximation.

A consequence of this is that we can represent a polygon ξ approximately by a string of real number pairs and this string is invariant to the scale and the coordinate system of ξ.

The use of the LMPP to smooth maps and characters has been demonstrated.

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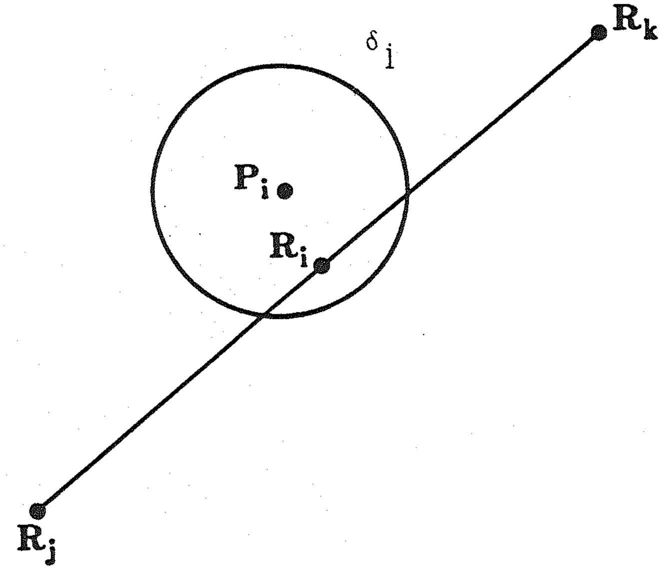

FIGURE I

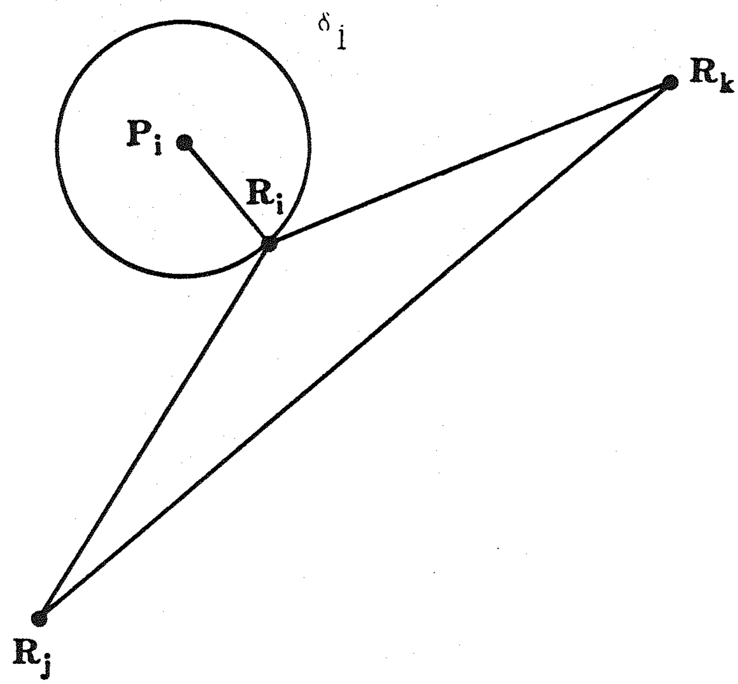

FIGURE II

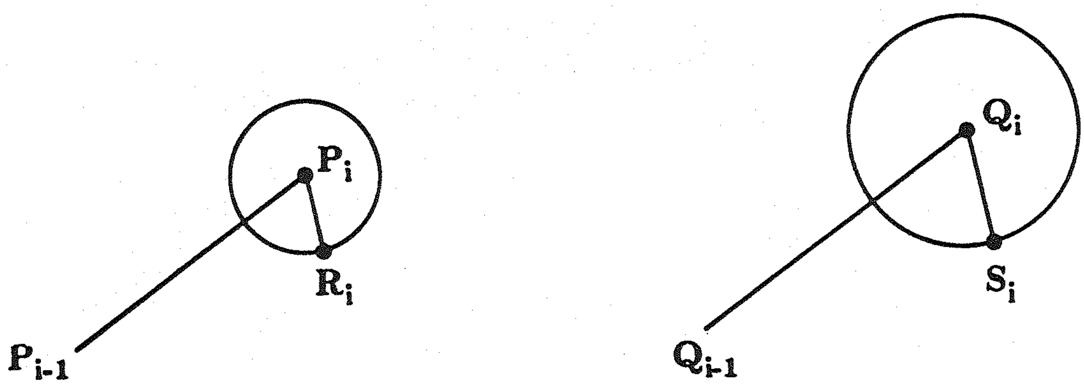

FIGURE III

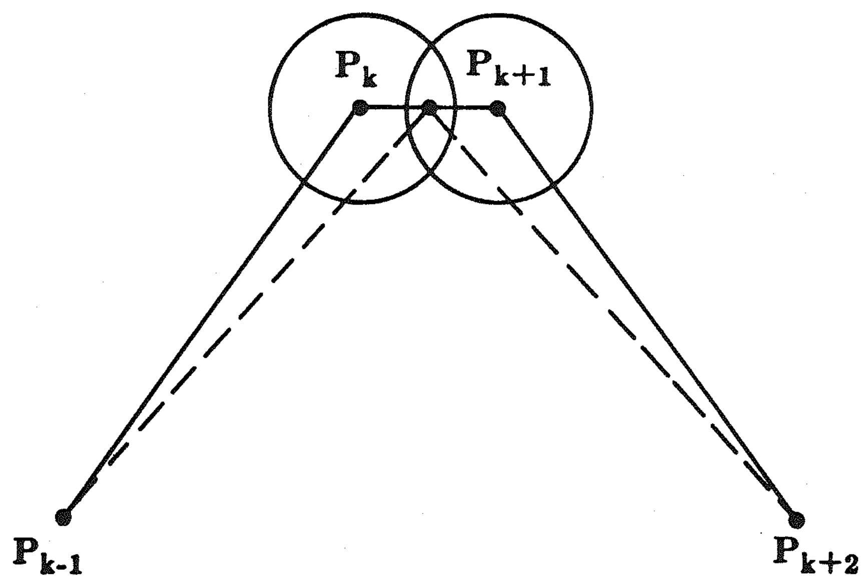

FIGURE IV

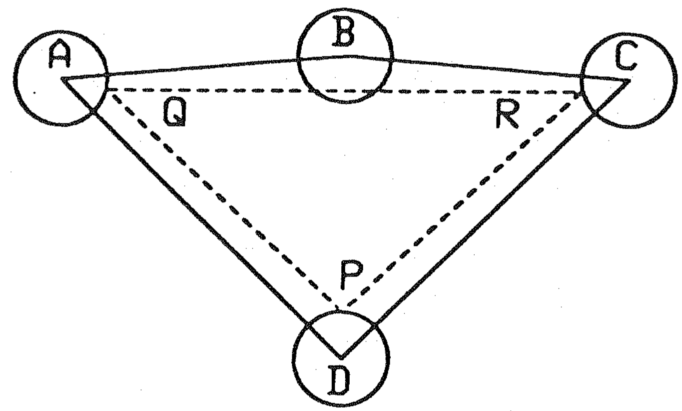

Figure V

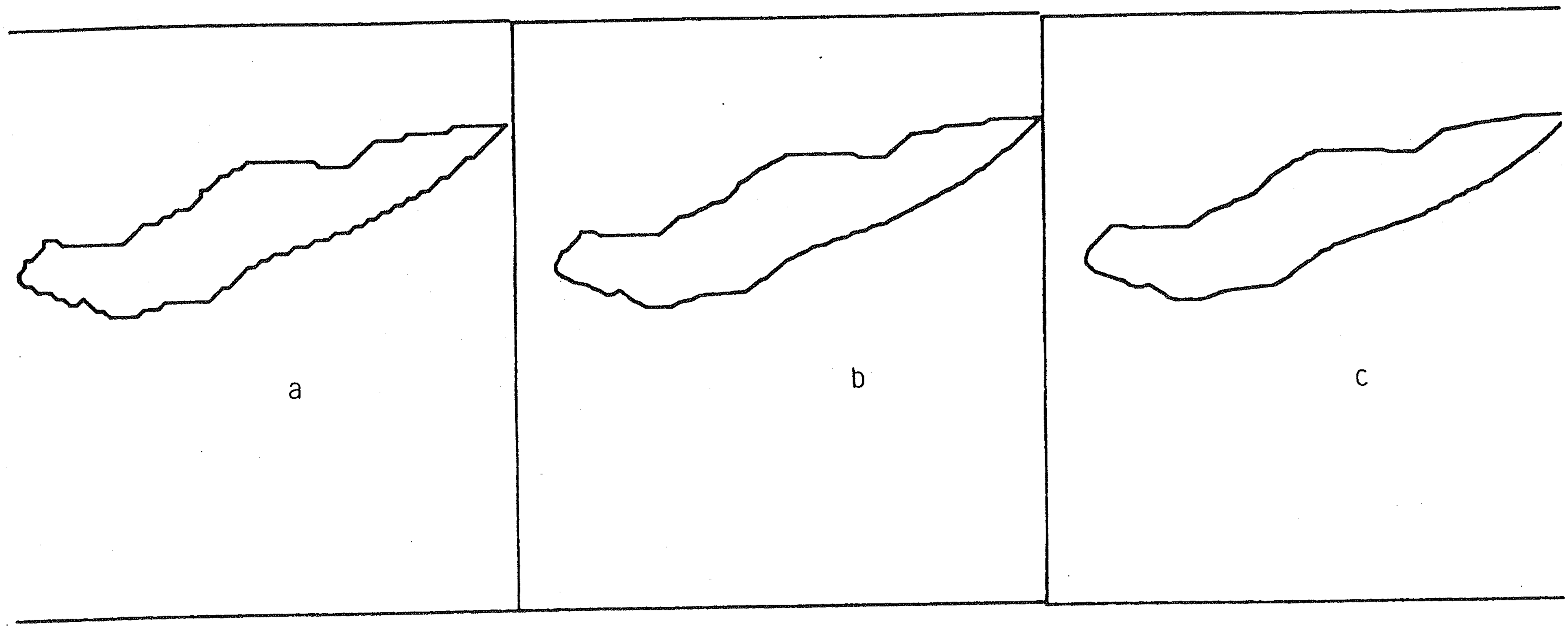

Figure VI. Examples of smoothing the boundary of Lake Erie. (a) is the original and (b)-(e) are the LMPPs drawn using ε values of 0.001, 0.002, 0.005 and 0.01 respectively.

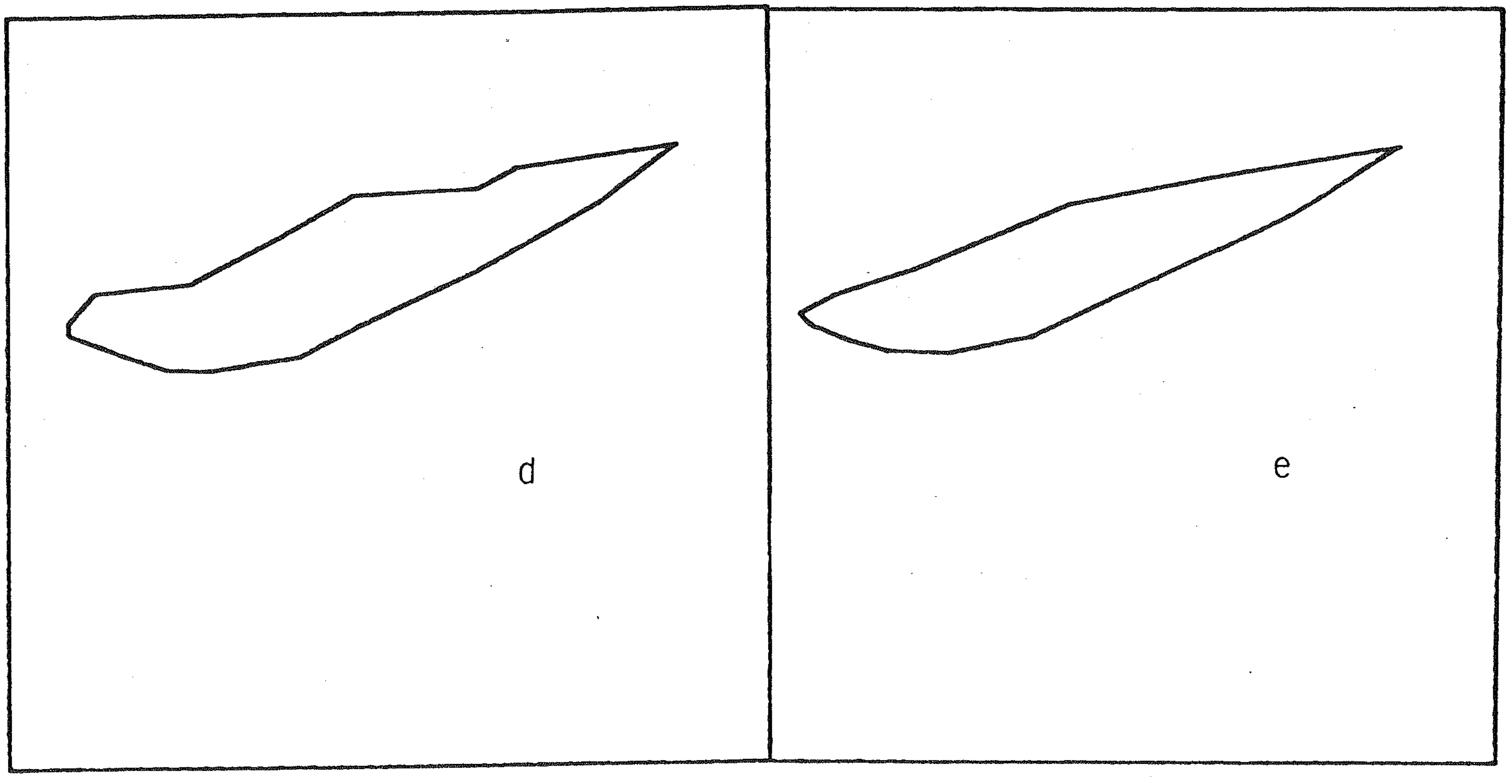

Figure VI. (continued)

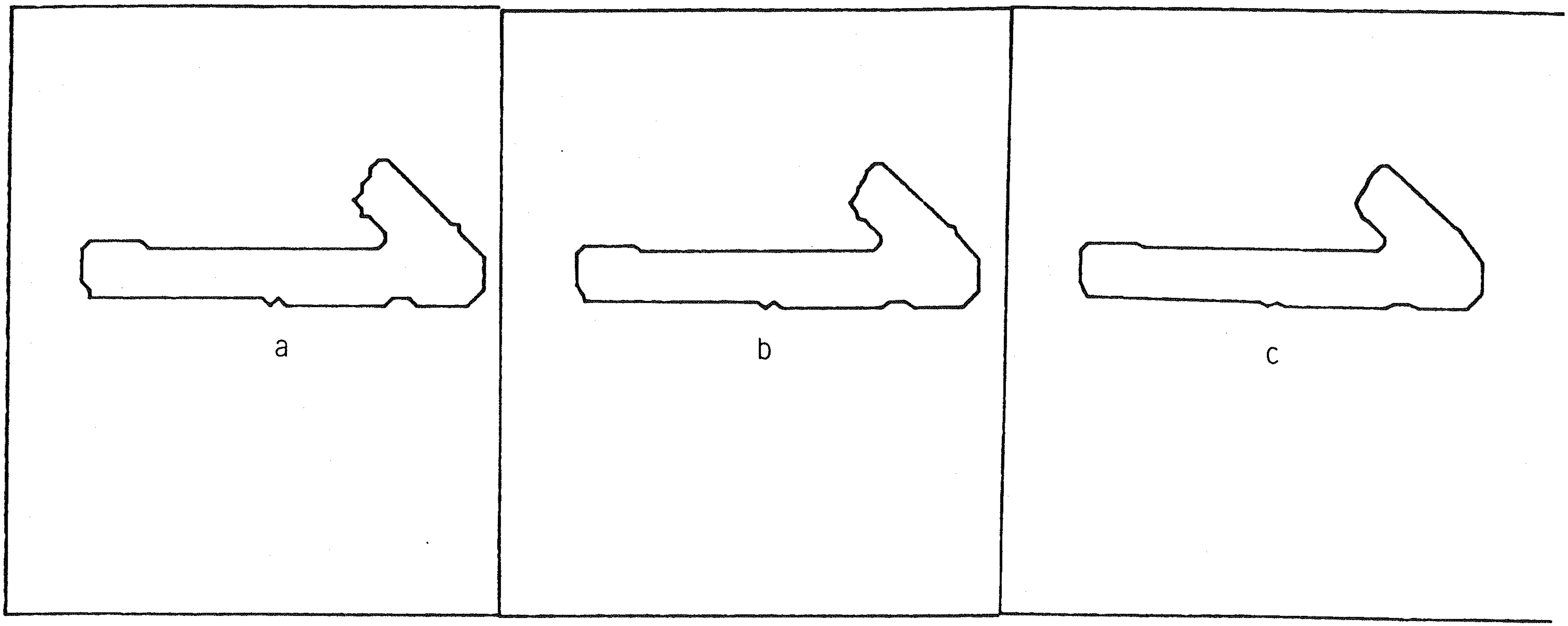

Figure VII. Examples of smoothing the numeral "1". (a) is the original and (b)-(e) are the LMPPs drawn using ε values of 0.001, 0.002, 0.005 and 0.01 respectively.

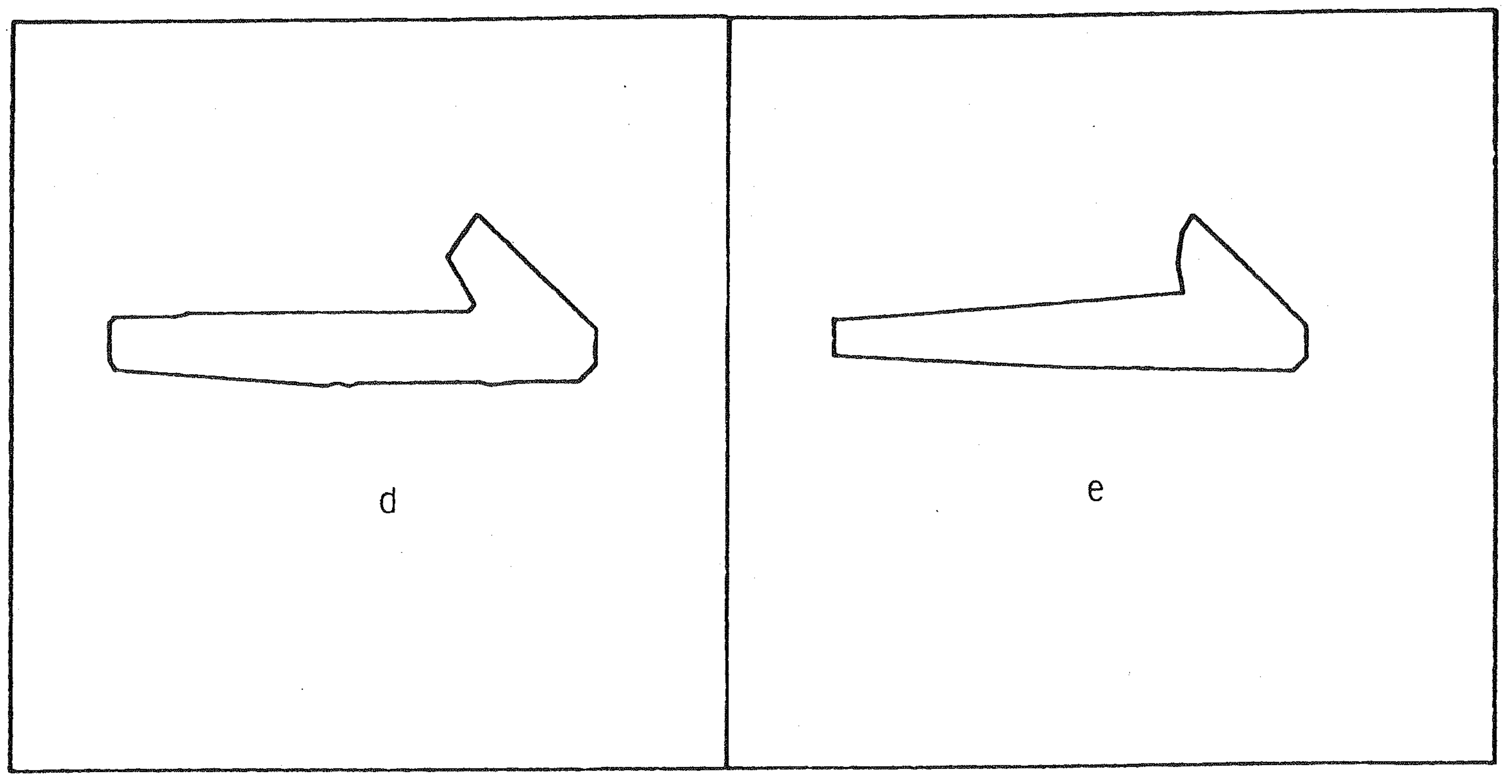

Figure VII. (continued)

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