GEOMETRIC CONTAINMENT IS NOT REDUCIBLE TO PARETO DOMINANCE

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1. INTRODUCTION

Given a family F of closed curves in the plane (e.g., polygons), the containment problem for F is the problem of determining for all A, B $\in$ F whether A can be contained in B; that is whether there exists a rotation and translation which will move A in the plane so that A and its interior lies within B.

For some families of closed curves, it is easy to find a finite dimensional real function $f: F \to \mathbb{R}^n$ satisfying the condition that, for all A,B $\in$ F, A can be contained in B if and only if $f(A) \leq f(B)$. For example, consider the family of squares, then, obviously square S_1 can be contained in square S_2 if and only if area $(S_1) \leq$ area (S_2). Obviously, the same one-dimensional function will also work for the families of equilateral triangles, circles, etc. Another example is the family of ellipses: let a_i and b_i denote the length of the minor axis and of the major axis of ellipse E_i; then E_1 can be contained in E_2 if and only if $(a_1, b_1) \leqq (a_2, b_2)$.

This implies that, for these families, establishing containment is equivalent to testing for vector (or Pareto) dominance; in other words, geometric containment for these families is reducible to Pareto dominance. Since vector dominance is easy to test for, it is natural to ask whether this reduction is always possible.

Consider the family F of all rectangles. The question becomes: Does there exist an n-dimensional real function $f(R) = (x_1(R),\ldots, x_n(R))$ such that, for all $R_1, R_2 \in F$, R_1 can be contained in R_2 if and only if $f(R_1) \leqq f(R_2)$; that is $x_i(R_1) \leqq x_i(R_2)$ for $1 \leqq i \leqq n$, where $x_i : F \to \mathbb{R}$. Obvious candidates for x_i are: length, width, area, diagonal, etc. However, for any choice of these functions, a counter-example can be found.

The main contribution of this paper is to show that these negative results are not accidental (e.g., due to poor choice of parameters), but intrinsic to the problem. In fact, it is proved that the containment problem for rectangles cannot be solved by vector dominance regardless of the choice and of the finite number of functions x_i.

2. BASIC DEFINITIONS AND RESULTS

A rectangle with width W and length $L \geq W$ is denoted by $R(W,L)$; for i an integer, the notations R_i and $R_i(W_i,L_i)$ are identical in meaning.

A realization of a rectangle $R(W,L)$ in the real plane is a subset P of points on the plane such that P forms a rectangle of dimensions W and L. The convex hull of such a P consists of a rectangle $R(W,L)$ together with its interior.

Given $R_1, R_2 \in S$, R_1 is immersible in R_2, denoted by $R_1 \to R_2$, if there exist realizations P_1 and P_2 of R_1 and R_2, respectively, such that P_1 is a subset of the convex hull of P_2; the couple (P_1, P_2) is called an immersion of R_1 in R_2. An immersion (P_1,P_2) of R_1 in R_2 is concentric if the centers of P_1 and P_2 coincide.

Given $R_1, R_2 \in R$, R_1 is tightly immersible in R_2, denoted by $R_1 \overset{T}{\to} R_2$, if $R_1 \to R_2$ and, for any $\varepsilon > 0$, $R(W_1 + \varepsilon, L_1)$ and $R(W_1, L_1 + \varepsilon)$ are not immersible in R_2. If $R_1 \overset{T}{\to} R_2$, any immersion of R_1 in R_2 is called a tight immersion.

Lemma 1 If $R_1 \to R_2$, then there exists a concentric immersion of R_1 in R_2.

Proof. Let (P_1,P_2) be an immersion of R_1 in R_2. Without loss of generality, let the vertices of P_2 be the points $(0,0)$, $(0,L_2)$, (W_2,L_2) and $(W_2,0)$; thus, the center of P_2 is the point $(W_2/2,L_2/2)$. Let (a,b) be the center of P_1; then a translation of P_1 by $-(a-W_2/2, b-L_2/2)$ will yield the claimed result. □

Lemma 2 $R_1 \to R_2$ only if $W_1 \leq W_2$.
The proof is obvious.

Remark 1: If $R_1 \overset{T}{\to} R_2$, $R_1 \neq R_2$, then any immersion (P_1,P_2) of R_1 in R_2 has the property that each edge of P_2 intersects one and only one vertex of P_1.

Remark 2: If $R_1 \overset{T}{\to} R_3$, and $R_2 \to R_3$, $R_1 \neq R_2 \neq R_3$, then R_1 is not immersible in R_2.

For $0 \leq W \leq L \leq \infty$, the function $\lambda(\cdot;W,L):[0,W] \to \mathbb{R}$ is defined by

$$\lambda(w;W,L) = \text{Max}\ \{\ell : R(w,\ell) \to R(W,L)\}$$

That is, $\lambda(w;W,L)$ is the length of the largest rectangles of width w which is immersible in $R(W,L)$. The curve in $\mathbb{R}^2$ defined by $\lambda(w;W,L)$ for $0 \leq w \leq W$ is called the containment curve of $R(W,L)$. Function λ is well-defined over the set of squares as stated by the following

Lemma 3 $\lambda(w;S,S) = \text{Max}\ \{2\sqrt{s} - w,\ S\}$ for $0 \leq w \leq S$.

The function $\lambda(w;S,S)$ is graphed in Fig. 1 over its domain $0 \leq W \leq S$; it consists of two line segments meeting at the point $((\sqrt{2} - 1)S,S)$, called the corner point of the containment curve of $R(S,S)$.

From Lemma 3, and as observable in Fig. 1., it follows that a rectangle $R(W,S)$ is immersible in $R(S,S)$ if and only if $W \leq S$ and $W \leq L \leq \text{Max}\{S, \sqrt{2}\,S - W\}$.

Consider the line segment consisting of the points in

$$L(S) = \{(W,L) : 0 \leq W < (\sqrt{2} - 1)S,\ L = \sqrt{2}\,S - W\}; \quad \text{then,}$$

by definition of λ, for any $(W,L) \in L(S)$ we have $R(W,L) \overset{T}{\rightarrow} R(SS)$. Conversely, if $(W,L) \notin L(S) \bigcup \{R(S,S)\}$ and $R(W,L) \rightarrow R(S,S)$ then $R(W,L)$ is not tightly immersible in $R(S,S)$ since at least one of W and L could be increased while retaining immersibility.

3. THE MAIN RESULT

Let $p : N \rightarrow \mathbb{R}$ be a function whose domain N is an open convex set in $\mathbb{R}^2$, and which is monotonically non-decreasing in both coordinates; and let $g(u) = Qu + b$ $(Q > 0)$ define a line which passes through N.

Lemma 4 Function p is continuous almost everywhere over the intersection of N with the line $g(u)$.

Proof. Define $h(u) = p(u, au + b)$ for $(u, au + b) \in N$. By hypothesis, $h(\cdot)$ is monotonically non-decreasing, hence continuous almost everywhere over its domain. Let u_o be a point of continuity of h, and consider the point (u_o,v_o) where $v_o = au_o+b$. Let $\{(S_i,t_i)\}$ be a sequence of points in N which converges to (w_o,v_o); and let $\varepsilon > 0$. By continuity of h, there exists values $u_1 < u_o$ and $u_2 > u_o$ such that u_1 and u_2 are in the domain of $h(\cdot)$,

$$|h(u_1)-h(u_o)| < \varepsilon \text{ and } |h(u_2)-h(u_o)| < \varepsilon; \quad \text{setting } v_i = Q\,u_i + b,$$

these last two inequalities translate into the inequalities

$$|p(u_i,v_i) - p(u_o,v_o)| < \varepsilon \quad , \qquad i = 1,2.$$

By monotonicity hypothesis, all points in the region $Q = \{(u,v) : u_1 \leq u \leq u_2, v_1 \leq v \leq v_2\}$ satisfy $|p(u,v) - p(u_o, v_o)| < \varepsilon$. Since $\lim_{i \rightarrow \infty} ((s_i,t_i)) = (u_o,v_o)$, there exists Π such that $i > \Pi$ implies $(s_i,t_i) \in Q$ which implies that p is continuous at (u_o,v_o). Since u_o was an arbitrary point of continuity of h, and since h is continuous almost everywhere, the lemma is proved. □

Theorem Geometric containment for rectangles cannot be solved by Pareto dominance in $\mathbb{R}^n$ for any n.

Proof. (By contradiction). Assume there exist n real-valued functions $x_i : A \to R$ with domain $A = \{(W,L): 0 \leq W \leq L \leq \infty\} \subseteq \mathbb{R}^2$, $1 \leq i \leq n$, such that, for all $R_1, R_2 \in F$, $R_1 \to R_2$ if and only if $f((W_1,L_1) = (x_1(W_1,L_1), \ldots, x_n(W_1,L_1)) \leq (x_1(W_2,L_2), \ldots, x_n(W_2,L_2)) = f(W_2,L_2)$. Let $g(S) = f(S,S)$ and $h(S) = f((\sqrt{2}-1)S,S)$, $S>0$. Extend f to the first quadrant by setting $x_i(W,L) = x_i(L,L)$ for $W > L$, $1 \leq i \leq n$. Each x_i is clearly monotonically non-decreasing in each of its coordinates. By Lemma 4, each coordinate of g and h, and hence the functions g and h are continuous almost everywhere; i.e., for at most countably many S, g is discontinuous; likewise for h. It follows that, except for countably many S, g and h are simultaneously continuous. Let S_o be a point of continuity for both g and h. It will now be shown by induction that $f((\sqrt{2}-1)\,S_o,S_o) = f(S_o,S_o)$. Suppose $f((\sqrt{2}-1)S_o,S_o)$ agrees with $f(S_o,S_o)$ on $k < n$ coordinates. Without loss of generality, let $x_i((\sqrt{2}-1)S_o,S_o) = x_i(S_o,S_o)$ for $i \leq k$. Consider a sequence (W_j,S_o) of points which converge to (S_o,S_o), with $(\sqrt{2}-1)S_o < W_j < S_o$ for all j (see Figure 2). By monotonicity, $x_i((\sqrt{2}-1)S_o,S_o) \leq x_i(W_j,S_o) \leq x_i(S_o,S_o) = x_i((\sqrt{2}-1)S_o,S_o)$ for $i \leq k$; thus

$$x_i(W_j,S_o) = x_i(S_o,S_o) \quad \text{for } i \leq k \quad \text{for all } j. \tag{1}$$

<u>Claim</u> If $(W,L) \in L(S)$, then $x_i(W,L) = x_i(S_o,S_o)$ for some $i > k$.

<u>Proof of Claim.</u> Since $R(W,L) \to R(S_o,S_o)$ it follows that $x_i(W,L) \leq x_i(S_o,S_o)$ for all i; by (1), it follows that

$$x_i(W,L) \leq x_i(W_j,S_o) = x_i(S_o,S_o), \qquad \text{for } i \leq k \text{ and all } j. \tag{2}$$

By contradiction, let $x_i(W,L) < x_i(S_o,S_o)$ for all $i > k$. By continuity of f at (S_o,S_o), there exists m such that

$$x_i(W_j,S_o) > x_i(W,L) \quad \text{for } i > k, \quad j > m \tag{3}$$

By (2) and (3), it follows that for $j > m$,

$$x_i(W,L) \leq x_i(W_j,S_o) \quad \text{for } 1 \leq i \leq n.$$

That is, $R(W,L) \to R(W_j,S_o) \to R(S_o,S_o)$; but since $R(W,L) \overset{T}{\to} R(S_o,S_o)$, this is impossible. (see Remark 2). Thus, the claim is proved. We now turn to the proof of the theorem.

Every point on $L(S)$ must be tied with (S_o,S_o) in at least one coordinate x_i, $i > k$. Therefore, there must exist an index $q > k$ and a sequence of points $L(S)$ converging to $((\sqrt{2}-1)S_o,S_o)$ such that every point (w,ℓ) in the sequence has $x_q(w,\ell) = x_q(S_o,S_o)$. Continuity of f at $((\sqrt{2}-1)S_o,S_o)$ implies

$x_q((\sqrt{2} - 1)S_o, S_o) = x_q(S_o, S_o)$. Thus $f((\sqrt{2} - 1)S_o, S_o)$ agrees with $f(S_o, S_o)$ in at least $k+1$ coordinates, namely $\{1, 2, \ldots, k, \ell_q\}$. Since k is an arbitrary integer in $(0, n-1)$, it follows by induction that $x_i((\sqrt{2} - 1)S_o, S_o) = x_i(S_o, S_o)$ for all i, or alternatively, $f((\sqrt{2} - 1)S_o, S_o) = f(S_o, S_o)$; that is, $R(S_o, S_o) \rightarrow R((\sqrt{2}-1)S_o, S_o)$ a contradiction. □

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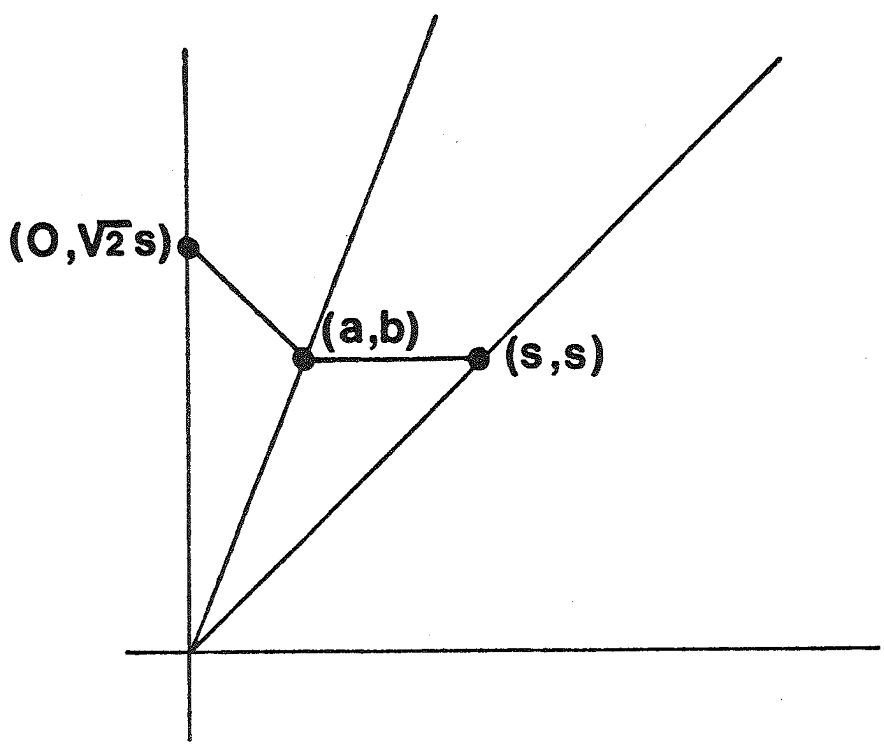

Fig.1 (a,b):((√2 -1) s, s)

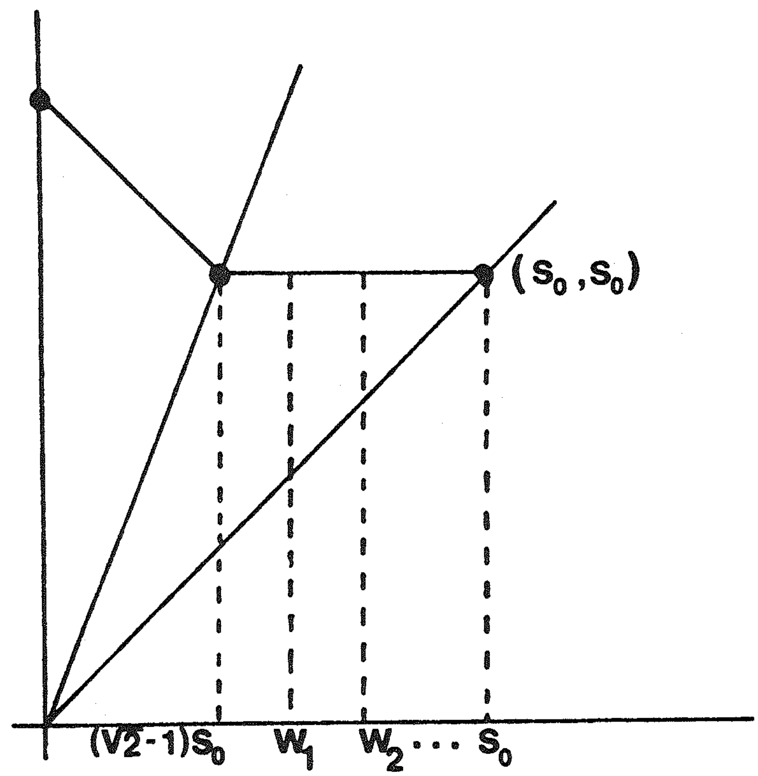

Fig. 2

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