SETS OF INTEGERS WITH DISTINCT DIFFERENCES

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1. Introduction

A distinct difference set (DDS) is a set of non-negative integers $a_0 < a_1 < \ldots < a_n$ such that the differences $a_i - a_j$ are all distinct. Such sets arise in the problem of assigning radio frequencies within a limited spectral range; the distinct difference property eliminates a certain type of interference between frequencies. In this application it is desirable that a_n be as small as possible; such a DDS is called optimal.

Some theoretical results on upper and lower bounds for a_n are presented, but the bulk of this report is a tabulation of close to optimal DDS's for $n <= 101$. This tabulation is based on the theory of perfect difference sets and on a less well known construction due to Bose [4]; we provide a brief description of each of these techniques.

We also discuss a weaker condition than the distinct difference condition:- we may allow the differences $a_i - a_j$ to represent an integer up to c times for some small constant c. For $c = 2, 3$ and $n <= 9$ we find optimal sets by using a fairly careful enumerative search.

2. Distinct difference sets and perfect difference sets.

We shall say that a set of non-negative integers $a_0, \ldots, a_n$ (in increasing order) whose non-zero differences are distinct is a distinct difference set, and that the DDS is optimal if a_n is as small as possible.

The problem of finding optimal DDS's for each integer n has been investigated by engineers [5,8] working on radio frequency assignments and coding theory, and by mathematicians [6] who have studied the problem for its own sake. Trial and error search has yielded a complete solution for all $n \le 10$. For larger n it has been observed that the theory of perfect difference sets [9] can yield DDS's which seem to be close to optimal. We have discovered that a less well known technique [4] will sometimes yield superior DDS's. For subsequent use we briefly recall the salient facts.

Suppose that v is some positive integer and that $\{a_0, a_1, \ldots, a_n\}$ is a set of $n+1$ non-negative integers such that all the differences $a_i - a_j$, $i \neq j$ when taken modulo v are distinct. We call such a set a modular distinct difference set (MDDS). Clearly a MDDS gives rise a fortiori to a DDS with $a_n < v$. Moreover by omitting elements from it we obtain DDS's of smaller size. Given a MDDS we can form another MDDS $\{b_0, b_1, \ldots, b_n\}$ by defining b_i as

$$b_i = ca_i + d, \quad (c,v) = 1$$

By a suitable choice of c and d we can often improve the initial set (in the sense of making its largest member smaller).

One way of obtaining MDDS's is via perfect difference sets. A perfect difference set (PDS) with parameters (v,k,c) is a set of k integers whose differences modulo v represent every non-zero residue from 1 to $v-1$ the same number c of times. Obviously $c(v-1) = k(k-1)$. The PDS is planar if $c=1$; in this case we put $k=n+1$ so that $v=n^2 + n + 1$. It is known that planar PDS's exist for every prime power n but, despite much research, no planar PDS has been found for other integers n. Clearly a planar PDS is also a MDDS.

There is another method [4] for obtaining MDDS's which has been used in problems concerning the additive theory of numbers. It produces, for every prime power q, a MDDS $a_0, a_1, \ldots, a_{q-1}$ with respect to the modulus $v = q^2-1$.

By using these two constructions we have found good DDS's for all $n <= 101$. However the precise value of a_n in an optimal DDS is difficult to calculate in general. Instead we give lower and upper estimates for it, and thereby deduce that a_n is asymptotic to n^2.

2.1 Bounds

An easy lower bound $a_n >= n(n+1)/2$ follows by observing that each of the $n(n+1)/2$ positive differences are distinct. An improvement on this is:

LEMMA 1. $a_n >= n(t+1)(n-t)/(t+2)$ for any t in 0..n. (For a proof of this see [1])

Note. This lower bound can be marginally improved but the bound given suffices for the following proposition.

PROPOSITION In any DDS, $a_n >= n^2 - 2n\sqrt{n}$.

Proof. In the above lemma put $t = [\sqrt{n}]$.

Remark. In view of the numerical results presented in the next subsection it would be of significant interest to improve the lower bound to $n^2 - n\sqrt{n}$.

The DDS construction shows that, when n is a prime power, the optimal a_n is bounded above by $n^2 + n$. For a general n we have the following upper bound.

LEMMA2. Let $q = n+t$, $t<q$, be a prime power greater than or equal to n. Then there exists a DDS in which $a_n <= n^2 + nt$.

Proof. (See [1])

Remarks.

1. This lemma can be slightly improved by taking more care with the inequalities; for example it can be shown that, if n is a prime power, there is a PDS with $a_n <= n^2 - n/2$.

2. The numerical evidence in the next section strongly suggests that a DDS with $a_n <= n^2$ always exists. However, to establish this the proof technique of Lemma 4 will probably need to be enhanced by exploiting that PDS's retain their property when multiplied by a residue coprime to v.

3. In [7] it is proved that, for any sufficiently large n, there exists $t <= n^{7/12}$ with $n+t$ prime. Consequently we have

THEOREM For optimal DDS's the numbers a_n are asymptotic to n^2 as $n \to \infty$.

Proof. The proposition and lemma 1 give

$$1 - 2/\sqrt{n} < a_n/n^2 < 1 + n^{19/12}/n^2.$$

2.2 Numerical results.

The two constructions of MDDS's above have been investigated for all prime powers less than 100. The initial MDDS's have been optimized using mappings x--> cx + d as explained above, and for each n < 100 we have found the best DDS $\{a_0, \ldots, a_n\}$ that can be obtained by these methods. Values of a_n are shown in table 1. (The entry in row i, column j is the value a_n for n=10i+j)

	0	1	2	3	4	5	6	7	8	9
0	0	1	3	6	11	17	28	35	45	55
1	72	85	111	127	155	179	199	216	246	283
2	333	356	372	425	480	492	553	585	623	680
3	747	784	859	938	987	1005	1099	1146	1252	1282
4	1305	1397	1507	1596	1687	1703	1804	1915	1958	2094
5	2190	2270	2347	2373	2598	2725	2773	2851	2911	3019
6	3134	3215	3391	3527	3593	3757	3819	3956	4145	4217
7	4330	4478	4513	4753	4982	5089	5204	5299	5408	5563
8	5717	5814	6020	6159	6410	6537	6708	6745	6778	6967
9	7542	7617	7726	7884	7967	8121	8357	8509	8540	8831

Table 1.

The associated DDS's are listed in full in section 3. Our belief that these sets are close to optimal is based on two observations

1. For $n <= 10$, where optimal DDS's have been found by exhaustive search, the only cases where the above a_n's are not minimal are a_7 and a_8 which can each be reduced by 1, and a_6 which can be reduced by 2.

2. The values a_n above agree within one to two percent with the expression $n^2 - n\sqrt{n}$, whereas the previous subsection demonstrated that $a_n >= n^2 - 2n\sqrt{n}$.

2.3 A related problem

A natural generalisation of the distinct difference property is to allow the differences to represent each value at most c times for some small constant c. The proof of Lemma 1 generalises easily to accommodate this hypothesis and consequently one can obtain the lower bound

$$a_n >= n^2/c \quad - \text{ lower order terms.}$$

However comparably good upper bounds are not so easy to obtain since (v,k,c) PDS's with fixed $c>1$ are not so abundant. By trial and error search we have found the following optimal sets given in Table 2.

n	c = 2	c = 3
1	0,1	0,1
2	0,1,2	0,1,2
3	0,1,2,4	0,1,2,3
4	0,1,3,5,6	0,1,2,3,5
5	0,1,4,6,8,9	0,1,2,4,5,7
6	0,1,4,6,10,11	0,1,2,4,5,8,10
7	0,1,4,6,10,15,17,18	0,1,3,4,6,8,12,13
8	0,1,4,6,11,13,19,22,23	0,1,3,5,9,10,12,15,16
9	0,1,3,8,10,14,20,25,28,29	0,1,2,5,8,11,13,15,19,20

Table 2.

2.4 Method to generate sets for table 2.

The sets in table 2 for c = 2, 3 are generalised difference sets; the differences represent each value at most c times. In such a set we may assume that $a_0=0$ in which case the n differences ("gaps") $s_i = a_{i+1} - a_i$ sum to a_n. To construct a set with a given target a_n we first find all sets $s_0, .., s_{n-1}$ whose sum is a_n. Each set is then permuted in all essentially different ways and tested for being a valid gap sequence.

An arrangement of the gap set is valid if the consecutive sums of blocks of numbers of size 1 to n - 2 in the set occur less than or equal to c times. That is for the set (1 1 2 2) n=5, the sums are 1, 1, 2, 2 for a block size one, and 2, 3, 4 for a block size two and, 4, 5 for a block size three. The sum of all the elements need not be formed since this can occur only once. If a valid permutation of gaps was found we put $a_0 = 0$ and $a_i = a_{i-1} + s_{i-1}$ for i = 1 to n.

For the previous example 1 1 2 2 if c = 2 then this arrangement would fail since the sum 2 occurs three times. The arrangement (1 2 2 1) is valid though, and produces the set (0 1 3 5 6). The sum of the elements in the gap set equals a_n. If no valid arragngements can be found for a given gap set then different values for the gaps are used (so they still sum to the previous a_n) until all sets for that a_n have been tried.
If no valid ones were found then a_n is too small and gap sets with larger sums must be tried.

2.5 Method to generate reduced perfect difference sets.

The initial difference sets for prime powers $n = p^a$, $a > 1$, were taken from a table in [3]. This table could also have been used in the prime case but it computationally more convenient to generate the PDS's by following the original construction of Singer [9].

Singer observed that from a PDS A with parameters $(n^2+n+1,n+1,1)$ could be constructed a projective plane of order n simply by taking the residues 0 .. n^2+n as the points and by taking the lines to be the translates A+i of A, i=0 .. n. This plane admits the cyclic automorphism x --> x+1. Conversly from a projective plane with a cyclic automorphism t one can construct a PDS as follows. Choose any point x and label it with the number 0; then label the point $t^m(x)$ with the number m. It then follows, by definition chasing, that each line in the projective plane has a label set which is a PDS.

The classical projective planes defined over a finite field GF(p) do admit a cyclic automorphism, and it may be formed as follows. Let V be a 3-dimensional space over GF(p). Then by taking the 1-dimensional subspaces as points and the 2-dimensional subspaces as lines we obtain a projective plane of order p. If we identify V with the field GF(p^3) the multiplicative generator of this field induces a cyclic automrphism. The minimal equation of this automorphism is a primitive cubic and once this is known the iterated images of the automorphism may be calculated.

Recasting these notions in algorithmic terms we obtain the following construction of a PDS.

Let x^3-ax^2-bx-c be a primitive cubic. Next consider the sequence of triples formed as follows:

$$(U_0,V_0,W_0) = (1,0,0)$$

subsequent triples defined as,

$$(U_i=V_{i-1},V_i=W_{i-1},U_i=cU_{i-1} + bV_{i-1} + aW_{i-1})$$

with arithmetic done modulo n.

All possible triples arise as i varies from 0 to n^3-2. Each triple has n - 1 non-zero multiples and so the triples fall into $(n^3-1)/(n-1) = n^2+n+1$ equivalence classes. In each class we pick out a normalised triple (U,V,W) such that U=1 (or, if this is impossible, with V=1; or, if this is impossible, with W=1). As the triples are generated by the rule above they are replaced by normalised equivalent triples. The normalised triples (U_i,V_i,W_i)

with $W_i=0$ define a set of values of i which comprise a PDS.

Here is an example for the equation $x^3 - x - 2$, b=1, c=2, and a=0. The mapping is $(U_i,V_i,W_i) \leftarrow (V_{i-1},W_{i-1},2U_{i-1}+V_{i-1})$. N is 3 and V is 13.

The triples are:

i	triple	normalized	i	triple	normalized
0	(1,0,0)		6	(2,1,1)	(1,2,2)
1	(0,0,2)	(0,0,1)	7	(2,2,1)	(1,1,2)
2	(0,1,0)		8	(1,2,0)	
3	(1,0,1)		9	(2,0,1)	(1,0,2)
4	(0,1,2)		10	(0,2,2)	(0,1,1)
5	(1,2,1)		11	(1,1,1)	
12	(1,1,0)				

The positions where W = 0 are (0 2 8 12) which form the difference set.

This gives the original difference set, which is then shifted cyclically by subtracting (modulo V) a suitable constant from the elements of the set. For sets whose n's are prime or prime powers the best constant is obtained by finding the place where there is the greatest difference between consecutive elements in the set (and regarding a_n, a_0 as being consecutive) This difference (say $a_i - a_j$) gives the best shift or the one that reduces a_n the most when a_i is subtracted (mod V) from the elements in the set. A set m that is not prime or prime power is generated using the latter sets and is truncated to m+1 elements. To shift these sets, the largest difference is found by taking elements with subscripts n - m + 1 apart and subtracting as above. This reduces the a_{m+1}'th element by the largest amount.

The coprimes to n from 2 to n/2 are then found. The ones from n/2 + 1 to n will produce mirror image sets to the others generated so they are ignored. The prime or prime power set n is then multiplied (mod V) by each coprime and these sets are cyclically shifted (by the above process) for n and the sets m below it. The most reduced sets are picked from these.

2.6 The Method of Bose

Let q be any prime power and let $x^2 - ax - b$ be a primitive quadratic over GF(q) with root y (a multiplicative generator for $GF(q^2)$). Every power of y can then be expressed as

$$y^k = U_k y + V_k \quad \text{with } U_k, V_k \text{ elements of } GF(q)$$

Obviously

$$U_{k+1} = aU_k + V_k$$

$$V_{k+1} = bU_k$$

So the powers of y are easily computable. Bose proved [4] that $\{k \mid U_k = 1\}$ is a MDDS of size q for the modulus q^2-1. This consruction (an easier one than the PDS construction) permits the same type of optimization with truncation as described in 2.5.

2.7 List of difference sets.

The reduced sets are put in the list as n followed by the set. All sets that are not prime or prime powers were generated using the closest prime or prime power set above them, and truncating. For the prime or prime power sets K = n + 1 is the number of elements in the set, and $V = 1 + n + n^2$ is the modulus.

3. Reduced Distinct Difference Sets.

n	set									
1	0	1								
2	0	1	3							
3	0	2	5	6						
4	0	3	4	9	11					
5	0	1	8	11	13	17				
6	0	4	11	20	25	26	28			
7	0	7	10	16	18	30	31	35		
8	0	3	9	16	20	21	35	43	45	
9	0	2	14	21	29	32	45	49	54	55
10	0	3	14	16	20	41	48	53	63	71
	72									
11	0	9	10	17	30	42	45	56	61	79
	83	85								
12	0	3	11	38	40	47	62	72	88	92
	93	105	111							
13	0	4	6	20	35	52	59	77	78	86
	89	99	122	127						
14	0	4	5	15	33	57	59	78	105	117
	125	139	142	148	155					
15	0	6	19	40	58	67	78	83	109	132
	133	162	165	169	177	179				
16	0	5	7	17	52	56	67	80	81	100
	122	138	159	165	168	191	199			
17	0	2	10	22	53	56	82	83	89	98
	130	148	153	167	188	192	205	216		

18	0	4	13	15	42	56	59	77	93	116
	126	138	146	174	214	221	240	245	246	
19	0	1	8	11	68	77	94	116	121	156
	158	179	194	208	212	228	240	253	259	283
20	0	2	24	56	77	82	83	95	129	144
	179	186	195	255	265	285	293	296	310	329
	333									
21	0	3	15	26	65	86	93	103	133	152
	177	197	228	232	234	250	286	313	342	347
	355	356								
22	0	6	22	24	43	56	95	126	137	146
	172	173	201	213	258	273	281	306	311	355
	365	369	372							
23	0	9	33	37	38	97	122	129	140	142
	152	191	205	208	252	278	286	326	332	353
	368	384	403	425						
24	0	12	29	39	72	91	146	157	160	161
	166	191	207	214	258	290	316	354	372	394
	396	431	459	467	480					
25	0	5	17	28	36	52	62	106	136	149
	174	178	234	241	243	289	292	307	329	368
	382	388	409	459	491	492				
26	0	3	15	41	66	95	97	106	142	152
	220	221	225	242	295	330	338	354	382	388
	402	415	486	504	523	546	553			
27	0	3	15	41	66	95	97	106	142	152
	220	221	225	242	295	330	338	354	382	388

	402	415	486	504	523	546	553	585		
28	0	3	28	33	41	95	156	158	193	207
	233	252	267	338	339	355	394	403	421	473
	479	502	523	570	580	592	612	616	623	
29	0	9	35	57	128	149	151	180	205	218
	281	292	336	364	370	378	394	469	473	474
	514	533	580	595	598	631	641	648	668	680
30	0	9	12	54	79	143	145	161	192	205
	226	311	349	363	382	386	439	459	466	474
	502	560	561	571	601	669	675	701	725	730
	747									
31	0	7	15	26	28	57	112	118	136	176
	177	181	211	214	258	309	318	341	389	403
	456	476	512	528	582	628	671	696	745	762
	772	784								
32	0	22	38	47	91	108	123	136	141	229
	256	263	293	319	329	358	360	400	406	505
	516	524	559	573	633	684	685	705	708	763
	767	847	859							
33	0	7	57	59	60	69	89	167	192	235
	253	259	353	398	432	468	479	507	534	551
	572	648	656	689	702	776	790	813	839	861
	903	919	934	938						
34	0	5	16	53	56	63	118	162	185	249
	262	277	350	356	382	425	454	503	522	592
	593	638	652	673	677	704	734	827	860	868
	877	951	963	985	987					
35	0	5	16	53	56	63	118	162	185	249

	262	277	350	356	382	425	454	503	522	592
	593	638	652	673	677	704	734	827	860	868
	877	951	963	985	987	1005				
36	0	14	63	87	113	169	220	286	289	328
	361	363	381	439	464	475	507	519	529	535
	566	700	717	746	798	832	839	862	877	962
	981	1002	1029	1086	1090	1091	1099			
37	0	7	57	59	60	69	89	167	192	235
	253	259	353	398	432	468	479	507	534	551
	572	648	656	689	702	776	790	813	839	861
	903	919	934	938	1050	1090	1141	1146		
38	0	36	51	94	121	125	126	147	186	257
	339	350	391	394	441	509	521	539	585	587
	593	622	699	762	802	819	918	941	960	974
	1041	1069	1079	1103	1128	1148	1236	1245	1252	
39	0	12	41	60	65	67	170	200	201	221
	313	351	374	414	417	428	450	497	592	636
	682	692	709	717	754	803	861	874	948	952
	980	1037	1076	1135	1151	1185	1203	1267	1273	1282
40	0	4	26	38	65	146	169	174	182	206
	261	308	333	411	427	446	517	523	532	599
	606	639	690	736	739	837	891	893	922	936
	966	986	1054	1111	1177	1225	1235	1246	1287	1288
	1305									
41	0	2	89	101	136	163	167	246	283	302
	307	350	383	456	500	514	536	559	641	691
	692	755	762	787	815	880	957	966	977	987

	995	1079	1085	1125	1237	1254	1306	1309	1322	1348
	1363	1397								
42	0	3	34	36	43	58	97	125	202	215
	228	302	362	400	419	437	485	588	589	593
	638	658	709	717	734	867	899	909	920	1001
	1047	1063	1115	1127	1156	1162	1234	1257	1340	1370
	1396	1480	1507							
43	0	59	71	76	98	111	162	207	275	341
	374	385	389	463	469	569	610	618	722	775
	791	801	868	898	918	941	960	988	1006	1042
	1043	1158	1214	1243	1245	1252	1277	1382	1403	1406
	1463	1524	1538	1596						
44	0	42	47	54	83	139	148	165	167	279
	319	365	426	500	501	521	531	581	599	650
	689	714	741	776	898	920	987	1030	1036	1044
	1089	1206	1279	1283	1294	1327	1457	1480	1512	1550
	1584	1608	1621	1684	1687					
45	0	2	10	23	54	115	153	237	255	295
	311	338	341	457	519	544	551	617	668	685
	705	738	780	877	927	936	1005	1050	1054	1131
	1143	1157	1179	1198	1288	1348	1456	1503	1527	1532
	1538	1566	1623	1638	1702	1703				
46	0	6	38	43	87	201	257	291	299	314
	409	421	425	549	574	651	668	722	736	772
	839	861	913	954	1018	1042	1134	1165	1185	1192
	1218	1220	1231	1351	1414	1459	1520	1541	1550	1560
	1619	1620	1692	1695	1757	1775	1804			
47	0	3	8	55	101	168	200	206	354	378

	432	489	500	515	579	615	781	790	906	919
	931	948	1023	1100	1118	1162	1166	1176	1203	1236
	1385	1424	1425	1444	1474	1505	1556	1558	1601	1629
	1664	1698	1720	1785	1792	1808	1894	1915		
48	0	17	20	86	119	140	166	227	240	255
	353	430	520	559	564	565	602	675	724	781
	817	833	905	929	961	970	980	1131	1162	1189
	1212	1319	1403	1433	1437	1451	1462	1497	1504	1589
	1601	1680	1763	1785	1825	1880	1888	1956	1958	
49	0	51	83	110	191	224	226	271	313	413
	468	514	591	619	640	683	767	790	796	865
	878	887	950	980	990	1046	1199	1211	1225	1285
	1329	1391	1525	1536	1540	1543	1577	1593	1601	1671
	1672	1710	1809	1829	1834	1973	2027	2046	2063	2094
50	0	34	44	91	95	147	207	278	332	364
	375	405	458	520	682	698	701	710	853	868
	901	946	973	1022	1080	1150	1155	1172	1240	1254
	1290	1429	1540	1546	1605	1642	1682	1684	1705	1751
	1771	1806	1835	1943	1967	2041	2151	2164	2182	2189
	2190									
51	0	34	44	91	95	147	207	278	332	364
	375	405	458	520	682	698	701	710	853	868
	901	946	973	1022	1080	1150	1155	1172	1240	1254
	1290	1429	1540	1546	1605	1642	1682	1684	1705	1751
	1771	1806	1835	1943	1967	2041	2151	2164	2182	2189
	2190	2270								
52	0	6	67	90	102	193	227	266	400	410

	495	540	543	559	640	711	728	874	878	924
	989	1098	1136	1215	1275	1293	1333	1344	1380	1385
	1387	1407	1538	1569	1639	1692	1747	1760	1775	1819
	1895	1961	1975	2017	2050	2074	2099	2136	2240	2317
	2338	2346	2347							
53	0	26	32	93	116	128	219	253	292	426
	436	521	566	569	585	666	737	754	900	904
	950	1015	1124	1162	1241	1301	1319	1359	1370	1406
	1411	1413	1433	1564	1595	1665	1718	1773	1786	1801
	1845	1921	1987	2001	2043	2076	2100	2125	2162	2266
	2343	2364	2372	2373						
54	0	2	5	18	30	67	167	199	292	326
	333	383	471	544	651	662	695	749	771	839
	956	1051	1109	1173	1211	1219	1374	1449	1458	1459
	1510	1563	1582	1606	1641	1645	1662	1756	1816	1864
	1895	1992	2037	2073	2237	2243	2314	2356	2383	2457
	2472	2486	2512	2578	2598					
55	0	12	26	100	215	220	230	231	292	317
	452	516	601	607	630	638	677	713	888	991
	995	1039	1143	1162	1171	1327	1369	1419	1422	1490
	1512	1631	1658	1671	1691	1725	1824	1841	1887	1925
	1966	1968	1998	2064	2236	2291	2409	2427	2478	2535
	2556	2591	2615	2673	2680	2725				
56	0	1	7	46	76	117	126	169	253	317
	342	345	368	492	527	751	854	856	864	883
	921	941	965	1104	1144	1203	1301	1361	1408	1423
	1496	1557	1574	1579	1610	1763	1811	1844	1865	1907
	1944	1956	2074	2087	2142	2228	2239	2334	2368	2400

	2472	2528	2542	2546	2636	2757	2773			
57	0	3	58	69	125	146	173	205	206	303
	405	454	456	673	723	768	791	832	885	959
	995	999	1183	1192	1284	1315	1323	1408	1465	1494
	1537	1703	1723	1729	1757	1799	1836	1883	1898	1944
	2096	2126	2140	2215	2231	2370	2380	2387	2392	2405
	2470	2508	2581	2693	2745	2764	2827	2851		
58	0	16	18	122	146	189	201	237	301	324
	361	369	514	611	695	721	742	748	749	811
	905	975	1014	1071	1220	1223	1303	1312	1334	1499
	1564	1598	1639	1650	1669	1711	1757	1789	1838	1855
	1931	1981	2032	2106	2227	2240	2335	2394	2438	2471
	2476	2496	2511	2693	2749	2778	2897	2907	2911	
59	0	15	123	168	171	203	208	227	284	386
	446	546	584	595	623	656	774	790	840	852
	904	910	1016	1143	1157	1241	1250	1315	1332	1378
	1407	1461	1502	1592	1732	1734	1757	1843	1851	1939
	2006	2042	2131	2141	2175	2228	2246	2372	2504	2511
	2654	2658	2684	2685	2705	2727	2806	2951	3006	3019
60	0	11	69	116	150	198	199	205	305	368
	413	432	533	546	598	658	820	855	923	931
	977	1016	1126	1169	1206	1301	1380	1385	1455	1457
	1472	1482	1508	1692	1725	1869	1883	1892	2009	2029
	2051	2147	2150	2298	2330	2359	2371	2389	2445	2502
	2523	2590	2621	2725	2763	2834	3040	3090	3106	3130
	3134									
61	0	16	21	38	63	71	74	81	200	251

	291	310	418	462	497	569	593	608	683	729
	752	766	872	967	997	1029	1095	1097	1179	1273
	1302	1351	1382	1436	1500	1506	1702	1722	1815	1819
	1867	1876	2014	2147	2175	2262	2263	2339	2366	2471
	2505	2627	2640	2729	2821	2877	2922	2963	2989	3062
	3203	3215								
62	0	22	126	143	172	187	272	278	395	429
	487	517	598	654	687	699	736	738	846	933
	976	1001	1117	1177	1244	1339	1438	1451	1470	1510
	1517	1541	1615	1619	1624	1835	2009	2020	2072	2095
	2145	2148	2308	2336	2377	2378	2432	2497	2631	2728
	2744	2764	2821	3019	3045	3083	3163	3177	3198	3225
	3373	3381	3391							
63	0	10	62	224	230	271	330	360	403	423
	452	500	527	738	794	828	851	863	945	984
	1063	1123	1209	1249	1300	1307	1403	1457	1540	1558
	1562	1846	1879	1884	1910	1991	1994	2044	2059	2105
	2129	2248	2276	2293	2371	2529	2649	2658	2660	2802
	2821	2834	2876	2901	3003	3017	3208	3209	3280	3296
	3317	3483	3519	3527						
64	0	66	76	128	290	296	337	396	426	469
	489	518	566	593	804	860	894	917	929	1011
	1050	1129	1189	1275	1315	1366	1373	1469	1523	1606
	1624	1628	1912	1945	1950	1976	2057	2060	2110	2125
	2171	2195	2314	2342	2359	2437	2595	2715	2724	2726
	2868	2887	2900	2942	2967	3069	3083	3274	3275	3346
	3362	3383	3549	3585	3593					
65	0	18	76	112	151	225	307	379	416	464

	466	678	718	823	824	966	1096	1103	1155	1167
	1274	1295	1476	1479	1517	1651	1737	1750	1889	1898
	2018	2086	2142	2184	2212	2244	2273	2295	2328	2420
	2510	2731	2755	2785	2832	2848	2852	2858	2863	3025
	3048	3082	3148	3252	3317	3331	3366	3412	3481	3534
	3577	3622	3713	3732	3740	3757				
66	0	62	80	138	174	213	287	369	441	478
	526	528	740	780	885	886	1028	1158	1165	1217
	1229	1336	1357	1538	1541	1579	1713	1799	1812	1951
	1960	2080	2148	2204	2246	2274	2306	2335	2357	2390
	2482	2572	2793	2817	2847	2894	2910	2914	2920	2925
	3087	3110	3144	3210	3314	3379	3393	3428	3474	3543
	3596	3639	3684	3775	3794	3802	3819			
67	0	14	134	207	220	417	493	527	538	580
	587	662	823	916	925	962	1137	1191	1215	1253
	1341	1353	1356	1430	1499	1611	1617	1800	1851	2025
	2060	2077	2157	2205	2207	2236	2262	2263	2358	2510
	2531	2602	2646	2727	2759	2825	2844	2992	3075	3103
	3136	3243	3259	3263	3302	3327	3392	3584	3606	3614
	3624	3715	3778	3814	3819	3886	3933	3956		
68	0	41	55	111	171	191	276	322	442	450
	495	517	697	751	763	769	853	939	996	1013
	1046	1117	1128	1155	1324	1356	1420	1462	1478	1554
	1555	1758	1872	1990	2027	2219	2244	2280	2327	2332
	2342	2371	2405	2504	2517	2552	2662	2801	2810	2950
	3015	3019	3172	3198	3301	3395	3398	3438	3539	3562
	3851	3902	3930	3932	3951	4019	4114	4138	4145	

69	0	25	27	32	42	172	240	307	446	570
	704	754	820	868	931	957	1188	1293	1301	1424
	1460	1488	1525	1570	1616	1771	1784	1787	1841	1928
	2076	2090	2125	2176	2294	2337	2533	2588	2666	2763
	2774	2889	2963	2983	2984	3067	3105	3165	3174	3348
	3407	3419	3438	3460	3499	3592	3626	3698	3728	3786
	3848	3965	4041	4074	4114	4118	4170	4193	4199	4217
70	0	13	14	31	132	212	218	316	376	526
	628	655	667	670	696	705	781	847	891	921
	953	1147	1367	1374	1421	1458	1651	1655	1745	1764
	1785	1833	1884	1920	1987	1992	2152	2233	2267	2340
	2395	2478	2543	2571	2668	2739	2876	2928	2939	3003
	3028	3061	3085	3177	3298	3300	3320	3343	3467	3650
	3729	3745	3841	3890	3900	3946	4063	4199	4269	4277
	4330									
71	0	90	217	230	360	440	448	462	472	566
	610	649	799	856	933	964	1011	1185	1190	1233
	1235	1252	1258	1286	1389	1486	1676	1752	1812	1873
	1954	1987	1990	2017	2122	2181	2294	2320	2467	2505
	2587	2633	2651	2662	2870	2905	2925	2979	2994	3050
	3066	3161	3165	3314	3477	3517	3575	3587	3596	3756
	3872	3972	4024	4065	4130	4131	4216	4253	4345	4429
	4436	4478								
72	0	9	56	94	120	130	159	256	329	586
	600	621	708	806	812	953	978	980	996	1008
	1109	1175	1254	1393	1417	1425	1477	1680	1738	1739
	1743	1853	1991	2086	2192	2272	2340	2381	2384	2426
	2459	2509	2516	2608	2745	2815	2897	2920	2966	3020

	3054	3235	3248	3297	3452	3513	3528	3630	3711	3807
	3818	3838	3855	3971	4048	4190	4209	4262	4333	4355
	4422	4473	4513							
73	0	240	249	296	334	360	370	399	496	569
	826	840	861	948	1046	1052	1193	1218	1220	1236
	1248	1349	1415	1494	1633	1657	1665	1717	1920	1978
	1979	1983	2093	2231	2326	2432	2512	2580	2621	2624
	2666	2699	2749	2756	2848	2985	3055	3137	3160	3206
	3260	3294	3475	3488	3537	3692	3753	3768	3870	3951
	4047	4058	4078	4095	4211	4288	4430	4449	4502	4573
	4595	4662	4713	4753						
74	0	7	27	52	160	274	275	454	514	570
	612	627	803	819	913	944	1020	1149	1185	1235
	1284	1313	1330	1384	1568	1592	1629	1669	1688	1781
	2084	2186	2218	2220	2342	2447	2510	2520	2720	2773
	2808	2817	2831	2882	2948	3003	3029	3033	3051	3424
	3507	3515	3528	3670	3711	3739	3857	3896	3929	3976
	4099	4247	4353	4491	4497	4559	4623	4634	4773	4811
	4816	4900	4903	4970	4982					
75	0	23	114	164	179	203	222	295	299	315
	535	644	669	817	822	859	1017	1027	1045	1071
	1102	1182	1284	1471	1488	1741	1781	1813	1891	1943
	1988	2037	2160	2194	2229	2289	2392	2551	2606	2669
	2725	2728	2776	2877	3037	3044	3143	3184	3270	3271
	3282	3403	3621	3634	3667	3705	3734	3810	3876	3955
	3964	4045	4238	4308	4432	4434	4496	4680	4707	4824
	4985	5007	5015	5021	5068	5089				

76	0	115	138	229	279	294	318	337	410	414
	430	650	759	784	932	937	974	1132	1142	1160
	1186	1217	1297	1399	1586	1603	1856	1896	1928	2006
	2058	2103	2152	2275	2309	2344	2404	2507	2666	2721
	2784	2840	2843	2891	2992	3152	3159	3258	3299	3385
	3386	3397	3518	3736	3749	3782	3820	3849	3925	3991
	4070	4079	4160	4353	4423	4547	4549	4611	4795	4822
	4939	5100	5122	5130	5136	5183	5204			
77	0	9	33	160	205	331	395	443	523	608
	817	818	875	890	937	951	979	1004	1020	1177
	1199	1211	1352	1457	1477	1568	1665	1749	1815	1915
	1918	1953	1970	2154	2204	2214	2260	2445	2515	2541
	2580	2673	2678	2825	2856	3024	3098	3246	3328	3341
	3347	3368	3565	3653	3696	3745	3747	3777	3921	4080
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	5043	5072	5080	5159	5182	5236	5295	5299		
78	0	109	118	142	269	314	440	504	552	632
	717	926	927	984	999	1046	1060	1088	1113	1129
	1286	1308	1320	1461	1566	1586	1677	1774	1858	1924
	2024	2027	2062	2079	2263	2313	2323	2369	2554	2624
	2650	2689	2782	2787	2934	2965	3133	3207	3355	3437
	3450	3456	3477	3674	3762	3805	3854	3856	3886	4030
	4189	4338	4356	4446	4567	4635	4710	4721	4975	5074
	5145	5152	5181	5189	5268	5291	5345	5404	5408	
79	0	45	82	138	293	443	445	463	596	697
	707	708	773	802	1232	1240	1317	1338	1406	1598
	1639	1702	1819	1922	1992	2139	2146	2199	2230	2413
	2477	2515	2635	2640	2652	2686	2702	2901	2981	3069

	3263	3269	3538	3580	3651	3666	3720	3724	3781	3922
	3950	4022	4057	4241	4254	4273	4370	4397	4400	4449
	4496	4588	4628	4706	4749	4793	4868	4942	4978	5114
	5138	5228	5351	5373	5406	5432	5515	5540	5554	5563
80	0	19	27	30	95	111	157	265	433	492
	561	593	732	867	907	1017	1083	1090	1114	1139
	1308	1391	1427	1512	1693	1721	1763	1907	2000	2063
	2113	2152	2165	2200	2212	2318	2332	2355	2396	2652
	2658	2748	2882	2936	3011	3015	3097	3115	3336	3462
	3523	3659	3805	3850	3879	4027	4136	4169	4174	4189
	4191	4314	4417	4568	4612	4692	4726	4797	4885	4928
	5145	5239	5346	5355	5356	5413	5528	5554	5605	5626
	5717									
81	0	24	52	66	106	142	192	235	396	428
	616	667	754	803	924	969	1054	1113	1202	1260
	1276	1329	1339	1409	1679	1740	1778	1834	1859	1898
	2012	2043	2072	2077	2089	2335	2406	2718	2719	2741
	2842	2850	2917	2985	3098	3170	3232	3258	3262	3273
	3444	3463	3541	3703	3810	3932	3967	4164	4212	4219
	4310	4414	4461	4586	4698	4782	4800	4803	4809	4959
	5003	5098	5263	5265	5380	5417	5490	5590	5698	5718
	5731	5814								
82	0	59	148	159	231	324	334	351	525	740
	885	947	952	1029	1049	1157	1186	1317	1325	1483
	1506	1541	1615	1639	1646	1725	1793	1836	1874	1920
	1989	1991	2036	2245	2549	2563	2564	2582	2603	2833
	2849	2945	2967	3066	3096	3190	3378	3412	3468	3582

	3614	3618	3705	3843	3949	3962	4012	4037	4092	4285
	4350	4398	4697	4757	4823	4849	5036	5140	5177	5218
	5262	5313	5319	5480	5533	5718	5760	5788	5895	5959
	5968	5971	6020							
83	0	15	208	278	287	461	591	603	623	631
	716	1106	1122	1155	1265	1322	1336	1434	1534	1620
	1656	1793	1858	1916	1922	1989	1994	2019	2173	2326
	2473	2606	2608	2698	2715	2774	2849	2873	2954	3082
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	3819	3850	3903	4161	4255	4281	4396	4446	4628	4641
	4682	4745	4859	4902	4905	4950	5154	5223	5371	5375
	5394	5477	5515	5664	5665	5725	5752	5791	5820	5871
	6035	6082	6117	6159						
84	0	11	69	128	135	163	220	267	317	397
	435	553	543	656	817	837	924	1039	1068	1233
	1238	1279	1562	1576	1610	1637	1682	1794	1925	2129
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	2908	3022	3101	3118	3325	3381	3396	3463	3552	3738
	3771	3893	3930	3961	3974	4219	4322	4330	4406	4431
	4432	4505	4782	4825	4916	5021	5053	5104	5107	5116
	5146	5169	5354	5406	5579	5785	5834	5911	6083	6153
	6241	6370	6388	6394	6410					
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	1238	1279	1562	1576	1610	1637	1682	1794	1925	2129
	2189	2390	2394	2535	2554	2556	2675	2730	2808	2872
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	6241	6370	6388	6394	6410	6537				
86	0	33	36	81	91	109	156	186	497	593
	639	784	821	835	852	923	945	1120	1252	1260
	1326	1367	1618	1629	1679	1746	1805	1844	1867	1923
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	4564	4590	4628	4731	4830	5013	5033	5164	5242	5322
	5326	5506	5521	5546	5758	5782	6068	6122	6131	6223
	6228	6387	6394	6481	6523	6695	6708			
87	0	3	48	58	76	123	153	464	560	606
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	2929	3051	3103	3163	3252	3301	3386	3533	3751	3772
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4564 4590 4628 4731 4830 5013 5033 5164 5242 5322
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6657 6798 6930 6982 7061 7395 7422 7431 7432 7461
7542

91 0 47 118 183 295 371 392 409 516 753
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1620 1640 1980 2148 2166 2298 2330 2456 2596 2599
2837 2924 2954 2998 3039 3098 3173 3178 3261 3303
3311 3413 3584 3591 3732 3783 4142 4197 4264 4265
4481 4494 4653 4692 4829 4991 4995 5061 5089 5180

5273 5307 5552 5566 5733 5794 5943 5989 5991 6001
6100 6195 6323 6385 6396 6640 6761 6915 6934 6940
6969 7018 7165 7261 7430 7452 7475 7512 7538 7565
7581 7617

92 0 22 60 111 229 243 274 317 330 367
407 446 603 711 802 983 1130 1258 1389 1481
1502 1729 1744 1790 1890 2056 2092 2122 2154 2446
2566 2586 2866 2968 3043 3274 3321 3404 3448 3471
3472 3687 3887 3913 3968 4141 4198 4223 4402 4408
4418 4514 4517 4739 4767 4947 4976 4980 5136 5138
5273 5281 5322 5398 5407 5425 5748 5821 5915 5986
6070 6105 6139 6191 6373 6384 6438 6543 6776 6886
6964 7022 7027 7034 7176 7347 7411 7515 7534 7587
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93 0 31 267 314 385 450 562 638 659 676
783 1020 1151 1351 1384 1440 1504 1544 1660 1806
1815 1830 1887 1907 2247 2415 2433 2565 2597 2723
2863 2866 3104 3191 3221 3265 3306 3365 3440 3445
3528 3570 3578 3680 3851 3858 3999 4050 4409 4464
4531 4532 4748 4761 4920 4959 5096 5258 5262 5328
5356 5447 5540 5574 5819 5833 6000 6061 6210 6256
6258 6268 6367 6462 6590 6652 6663 6907 7028 7182
7201 7207 7236 7285 7432 7528 7697 7719 7742 7779
7805 7832 7848 7884

94 0 70 100 148 150 225 232 335 472 509
566 645 879 1168 1200 1266 1324 1365 1396 1409
1741 1808 1927 1930 2084 2235 2321 2513 2522 2671

	2683	2725	2877	2897	2932	3113	3140	3230	3357	3396
	3584	3920	3985	4008	4046	4091	4099	4120	4267	4310
	4411	4578	4624	4769	4793	4874	5078	5092	5201	5470
	5475	5539	5543	5590	5650	5721	5814	5916	6013	6024
	6100	6330	6393	6419	6535	6552	6553	6648	6681	6901
	6920	7041	7103	7301	7341	7357	7393	7534	7638	7672
	7687	7697	7939	7945	7967					
95	0	17	106	166	237	346	430	647	863	914
	926	1006	1076	1101	1345	1503	1543	1599	1604	1643
	1658	1955	1988	2078	2106	2217	2248	2266	2487	2624
	2692	2706	2967	2991	3093	3239	3287	3308	3416	3497
	3510	3622	3642	3763	3801	3974	4036	4081	4158	4263
	4362	4435	4668	4815	4867	5001	5089	5118	5237	5301
	5303	5375	5394	5479	5546	5643	5771	5895	5925	6086
	6090	6293	6336	6601	6802	6848	6857	6889	6992	7057
	7173	7184	7210	7220	7226	7340	7548	7570	7811	7985
	8008	8035	8042	8043	8118	8121				
96	0	61	134	184	233	274	312	340	544	628
	744	747	767	974	983	1020	1036	1103	1245	1349
	1367	1499	1504	1525	1818	1852	1915	1985	2060	2100
	2102	2156	2367	2439	2443	2491	2521	2622	2711	2816
	2885	3028	3257	3263	3349	3691	3789	3802	3821	3889
	3948	3991	4036	4117	4188	4479	4486	4812	4836	4837
	4847	4998	5029	5217	5250	5416	5431	5511	5525	5674
	5721	5831	5896	6097	6133	6141	6419	6650	6910	6927
	6984	7048	7075	7087	7228	7372	7484	7577	7702	7940
	8025	8054	8076	8131	8189	8249	8357			
97	0	75	151	161	228	503	524	537	548	606

	661	721	744	883	1114	1136	1140	1142	1286	1392
	1491	1524	1616	1631	1681	1718	1860	1896	1960	2045
	2166	2523	2577	2596	2712	2737	2796	2826	2924	3149
	3292	3439	3446	3447	3509	3610	3613	3730	3746	3778
	4070	4111	4303	4321	4374	4501	4557	4569	4768	4773
	4815	5008	5028	5116	5450	5541	5572	5817	5857	5866
	5947	5976	6128	6179	6240	6257	6284	6466	6545	6642
	6768	6916	6954	7050	7085	7124	7474	7568	7769	7815
	7887	8130	8241	8307	8321	8364	8416	8509		
98	0	78	84	145	289	337	346	365	439	456
	463	666	808	879	1064	1261	1266	1490	1494	1642
	1650	1673	1685	1793	1969	2118	2133	2134	2416	2482
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	3440	3672	3710	3799	3850	3989	4207	4210	4343	4433
	4454	4557	4711	4879	4923	4948	4962	4988	5085	5254
	5361	5443	5499	5624	5658	5831	5931	6053	6147	6211
	6317	6397	6447	6449	6645	6707	6718	6864	6884	6906
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99	0	78	84	145	289	337	346	365	439	456
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8137 8250 8309 8346 8379 8425 8480 8493 8540 8831

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101 0 104 120 373 412 453 551 624 646 688
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7412 7452 7484 7503 7546 7782 7837 7912 7925 8053
8327 8358 8381 8645 8753 8798 8915 8986 9020 9144
9197 9218

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