HOP-CONGESTION TRADEOFFS FOR ATM NETWORKS

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Abstract

In ATM networks messages are transmitted through virtual paths. Packets are routed along virtual paths by maintaining a routing field whose subfields determine the intermediary destinations of the packet. In such a network it is important to construct path layouts that minimize the hop number (i.e. the number of virtual paths used to travel between any two nodes) as a function of edge-congestion (i.e. the number of virtual paths passing through a link). In this paper we construct asymptotically optimal virtual path layouts for chains and meshes.

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1 Introduction

Requirements in new and emerging information services require a new transfer mode for broadband ISDN (Integrated Services Digital Network). ATM (Asynchronous Transfer Mode) is a new multiplexing and switching technology that results in more cost effective solutions of greater flexibility than several separate individually optimized technologies [2]. Because of this, it has significant commercial as well as public service applications. This technology is thoroughly described in the literature [7, 6].

For standard networks, routing has been the topic of extensive studies in the literature, e.g., see [8, 1]. Packet routing in ATM networks is based on relatively small fixed-sized packets. The model we use in this paper is based on the Virtual Path Layout model introduced by Gerstel and Zaks [4, 5]. Messages may be transmitted through arbitrarily long virtual paths. Packets are routed along those paths by maintaining a routing field whose subfields determine intermediary destinations of the packet, i.e. end-points of virtual paths on its way to the final destination. In such a network it is important to construct path layouts that minimize the hop number (i.e. the number of virtual paths used to travel between any two nodes) as a function of edge-congestion (i.e. the number of virtual paths passing through a link).

1.1 Notation and definitions

In the sequel we use the following definitions and notations.

- $\mathcal{N}$ denotes an arbitrary connected network.
- A virtual path (VP) in $\mathcal{N}$ is a simple chain in this network (i.e. a non-repetitive sequence $(v_1, \ldots, v_k)$ of vertices such that there is a link between v_i and v_{i+1}, for all $i < k$).
- A virtual channel (VC) of length k, joining vertices u and v, is a sequence $p_1, p_2, \ldots, p_k$ of VP's such that p_1 begins at vertex u, p_k ends at vertex v and the beginning of p_{i+1} coincides with the end of p_i, for $i < k$.
- A (virtual path) layout $\mathcal{P}$ in the network $\mathcal{N}$ is a collection of virtual paths in $\mathcal{N}$, such that every pair of vertices u, v of $\mathcal{N}$ is joined by a VC composed of VP's from $\mathcal{P}$.

We assume that traversing any VP is made in a single hop and define the following parameters.

$\textsc{Hops}_{\mathcal{N}}(\mathcal{P}, u, v)$ is the minimum number of hops using VP's in $\mathcal{P}$ to go from u to v. $\textsc{Hops}_{\mathcal{N}}(\mathcal{P})$ (the hop number of $\mathcal{P}$) is the maximum (taken over all pairs of vertices u, v of the network) of $\textsc{Hops}_{\mathcal{N}}(\mathcal{P}, u, v)$.

We are interested in the hop number for layouts of bounded congestion. For a given network $\mathcal{N}$ and path layout $\mathcal{P}$ define the congestion $C_{\mathcal{N}}(\mathcal{P})$ of this layout as the maximum number of VP's from $\mathcal{P}$ passing through any link of $\mathcal{N}$. For any number $c \geq 1$ define $\textsc{Hops}_{\mathcal{N}}(c)$ as the minimum $\textsc{Hops}_{\mathcal{N}}(\mathcal{P})$, where the minimum is taken over all path layouts $\mathcal{P}$ for which $C_{\mathcal{N}}(\mathcal{P}) \leq c$.

1.2 Results of the paper

Gerstel and Zaks studied layouts for chains, rings, and meshes [5]. They imposed an additional requirement that all virtual paths are shortest paths in the network between their end-points. In this paper we consider arbitrary virtual paths and study the hop number $\textsc{Hops}_{\mathcal{N}}(c)$ as a function of the bound c on congestion. We construct asymptotically optimal layouts with given congestion, for chains and meshes. Our main results are:

- $\textsc{Hops}_{\mathcal{N}}(c) \geq \log n / \log(dc) - 1$, for any network $\mathcal{N}$ with maximum degree d, and arbitrary $c \geq 1$.
- $\sqrt{2n} - 5 < \textsc{Hops}_{L_n}(2) < \sqrt{2n} + 2$, for the chain L_n of length n.
- $\frac{1}{2} n^{1/c} \leq \textsc{Hops}_{L_n}(c) \leq cn^{1/c}$, for any $c \geq 1$.
- $\textsc{Hops}_{k \times n}(c) \in \Theta(n^{1/(kc)})$, where k, c are constants and $k \times n$ is the k by n mesh.
- $\textsc{Hops}_{n \times n}(c) \in \Theta(\log n)$, for constant c.

An immediade extension of the upper bounds for chains applies to arbitrary topologies by taking a Hamiltonian path. In general, one can use a spanning tree and a path which traverses the tree.

2 A General Lower Bound

For arbitrary networks we prove the following lower bound.

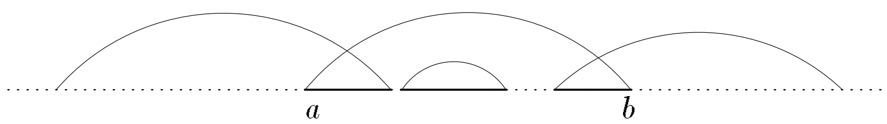

Figure 1: Induced layout $\mathcal{P}_{[a,b]}$ on the segment $[a,b]$.

THEOREM **1** *For any network $\mathcal{N}$ on n vertices, with maximal degree d, and for any $c \geq 1$ we have the following lower bound:*

$$\text{HOPS}_{\mathcal{N}}(c) \geq \frac{\log n}{\log(dc)} - 1.$$

PROOF Let $H = \text{HOPS}_{\mathcal{N}}(c)$. Choose any vertex of the network as a root, say r. Since the congestion is c and the maximum degree of a node is $\leq d$, at most dc nodes can be reached from r with one hop. More generally, with at most h hops at most $dc + (dc)^2 + \cdots + (dc)^h$ vertices can be reached from r. Put $h = H$ and it follows that $n \leq (dc)^{H+1}$, which implies that $H + 1 \geq \frac{\log n}{\log(dc)}$. This proves the theorem. ∎

3 Chain Graphs

The results in this section are stated for the chain L_n with vertices 0,1,...,n. However, they also hold for rings with only minor modifications.

3.1 Lower bounds

THEOREM **2** $\text{HOPS}_{L_n}(c) \geq \frac{1}{2}n^{1/c}$, *for any* $c \geq 1$.

PROOF We will use the following notions. Let $\mathcal{P}$ be a layout for L_n and let $I = [a,b] \subseteq L_n$ be any segment of the chain. For $J \in \mathcal{P}$ such that $J \cap I \neq \emptyset$, define $\Phi(J) = J \cap I$. Define the layout $\mathcal{P}_I$ induced by the layout $\mathcal{P}$ on the segment I as the set $\{\Phi(J) : J \in \mathcal{P}\} \setminus \{I\}$. It is clear that if $\mathcal{P}$ has congestion c then $\mathcal{P}_I$ has congestion $< c$ (see Figure 1).

It is enough to prove the following lemma.

Lemma 3 *For any layout $\mathcal{P}$ in L_n with congestion c, there is a vertex u such that*

$$\text{Hops}_{L_n}(\mathcal{P}, 1, u) \geq \frac{1}{2} n^{1/c} \text{ and } \text{Hops}_{L_n}(\mathcal{P}, u, n) \geq \frac{1}{2} n^{1/c}.$$

Proof We prove the statement by induction on c. For $c = 1$ the result is easy; take as u the mid-point of L_n. Assume the lemma is true for $c - 1$. Let $\mathcal{P}$ be an arbitrary layout of L_n with congestion c. Let $I = [a, b]$ be the largest virtual path in the layout $\mathcal{P}$. We consider two cases.

Case 1. $|I| \leq n^{(c-1)/c}$.

In this case at least $n^{1/c}$ hops are necessary to reach one end-point of the chain L_n from the other. Then take as u the mid-point of L_n and the inequality of the lemma is clearly satisfied.

Case 2. $|I| \geq n^{(c-1)/c}$.

Take u to be the mid-point of I. We only prove the first inequality. The second inequality is similar. Assume not and let $\mathcal{C} = (p_1, ..., p_k)$ be a virtual channel in $\mathcal{P}$, of length $k < \frac{1}{2} n^{1/c}$, joining 1 with u. Let $(h_0, \ldots, h_k)$ be the end-points of consecutive VP's in $\mathcal{C}$. Let h_r be the last vertex such that $h_r \leq a$ or $h_r \geq b$. Without loss of generality we may assume that $h_r \leq a$. Let q_{r+1} denote the VP joining a with h_{r+1}. Thus $\mathcal{C}' = (q_{r+1}, p_{r+2}, \ldots, p_k)$ is a virtual channel in the layout $\mathcal{P}_I$ induced by $\mathcal{P}$ on I. By the inductive hypothesis, the length of $\mathcal{C}'$ is at least

$$\frac{1}{2} |I|^{1/(c-1)} \geq \frac{1}{2} n^{1/c}.$$

Hence, $k \geq \frac{1}{2} n^{1/c}$, contradiction. This proves the lemma by induction. ∎

This completes the proof of the theorem. ∎

3.2 Asymptotically optimal path layouts

We construct two layouts for the chain L_n yielding two upper bounds on the hop number.

Theorem 4 *For any positive integer c we have that*

$$\text{Hops}_{L_n}(c) \leq c n^{1/c}.$$

Proof Let c be a positive integer. For simplicity assume that $n^{1/c}$ is an integer. The construction can be easily modified in the general case. We

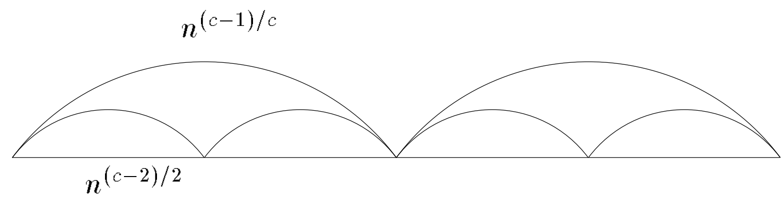

Figure 2: Layout satisfying $\textsc{Hops}_{L_n}(c) \leq cn^{1/c}$.

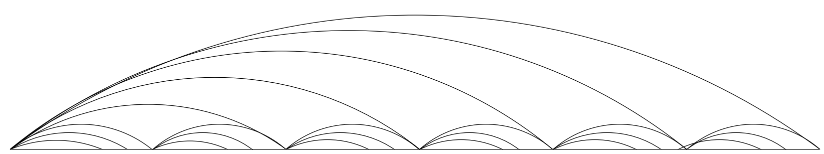

Figure 3: Layout satisfying $\textsc{Hops}_{L_n}(kn^{1/k}) \leq 2k$.

construct the layout consisting of nested virtual paths of lengths

$$n^{(c-1)/c}, n^{(c-2)/c}, \ldots, n^{1/c}, 1.$$

VP's of each length form a virtual channel joining both ends of L_n (cf. Figure 2). The layout consisting of all those VP's has congestion c and hop-number at most $cn^{1/c}$. ■

Corollary **5** *For constant c we have $\textsc{Hops}_{L_n}(c) \in \Theta(n^{1/c})$.*

The second layout is used to prove the following result.

Theorem **6** *For any integer k we have*

$$\textsc{Hops}_{L_n}(kn^{1/k}) \leq 2k.$$

Proof The layout proving this upper bound is given in Figure 3. Construct VP's with left end-point 0, of lengths

$$n^{(k-1)/k}, 2n^{(k-1)/k}, \ldots, n^{1/k}n^{(k-1)/k}.$$

From their right end-points construct VP's going right, of lengths

$$n^{(k-2)/k}, 2n^{(k-2)/k}, \ldots, n^{1/k}n^{(k-2)/k}.$$

Continue in this way with non-overlapping VP's of sizes

$$n^{(k-j)/k}, 2n^{(k-j)/k}, \ldots, n^{1/k}n^{(k-j)/k},$$

for $j = 1, ..., k$. The layout consisting of all those VP's has congestion $kn^{1/k}$ and hop-number at most $2k$. ∎

As a corollary we obtain an asymptotically tight bound for congestion $\log^2 n/\log\log n$.

COROLLARY **7** $\text{HOPS}_{L_n}(\log^2 n/\log\log n) \in \Theta(\log n/\log\log n)$.

PROOF The upper bound is obtained from theorem 6 for $k = \log n/\log\log n$. Then $n^{1/k} = \log n$ and $kn^{1/k} = \log^2 n/\log\log n$. The lower bound follows immediately from theorem 1. ∎

3.3 An almost optimal layout with congestion 2

In case of congestion 2 we have a more precise result: we give a layout for the chain which is optimal up to an additive constant.

THEOREM **8** *For the chain L_n we have*

$$\sqrt{2n} - 5 < \text{HOPS}_{L_n}(2) < \sqrt{2n} + 2.$$

PROOF We first prove the upper bound. Let $t = \lfloor\sqrt{\frac{n}{2}}\rfloor - 1$. We create two virtual channels consisting of non-overlapping paths whose lengths form arithmetic progressions. The left end of the first channel $\mathcal{C}_1$ is at vertex 0 of L_n and its right end is at vertex $2 + 4 + \cdots + 2t = t(t+1)$. The lengths of paths in this channel form an arithmetic progression $2, 4, 6, \ldots, 2i, \ldots, 2t$, left to right. The left end of the second channel $\mathcal{C}_2$ is at vertex $n - t(t+1)$ of L_n and its right end is at vertex n. The lengths of paths in this channel form an arithmetic progression $2t, 2t-2, \ldots, 6, 4, 2$, left to right. Vertices $t(t+1)$ and $n - t(t+1)$ are joined by a third virtual channel $\mathcal{C}_3$ consisting of three non-overlapping VP's of lengths differing by at most 1 (see Figure 4).

We construct the layout $\mathcal{P}$ for L_n consisting of all VP's of length 1 (links of L_n) and all VP's in chains $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$. Since all VP's in the above chains are non-overlapping, layout $\mathcal{P}$ has congestion 2.

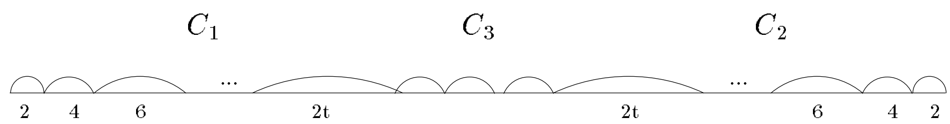

Figure 4: Layout with congestion 2 for the chain.

We shall prove that $\textsc{Hops}_{L_n}(\mathcal{P}) \leq \sqrt{2n} + 2$. The distance between end points $t(t+1)$ and $n - t(t+1)$ of channel $\mathcal{C}_3$ is

$$D = n - 2t(t+1) \leq n - 2\left(\sqrt{\frac{n}{2}} - 2\right)\left(\sqrt{\frac{n}{2}} - 1\right) \leq 6\sqrt{\frac{n}{2}} - 4.$$

Thus each VP in this channel has length at most $E = \lceil \frac{D}{3} \rceil < 2\sqrt{\frac{n}{2}}$. Consider any pair of vertices u and v in L_n.

Case 1: u is inside a VP of channel $\mathcal{C}_1$ and v is inside a VP of channel $\mathcal{C}_2$. If u is inside a VP of length $2i$ and v is inside a VP of length $2j$, we have:

$$\textsc{Hops}_{L_n}(\mathcal{P}, u, v) \leq i + (t - i) + 3 + (t - j) + j = 2t + 3 \leq \sqrt{2n} + 1.$$

Case 2: u is inside a VP of channel $\mathcal{C}_1$ and v is inside a VP of channel $\mathcal{C}_3$. If u is inside a VP of length $2i$, we have

$$\textsc{Hops}_{L_n}(\mathcal{P}, u, v) \leq i + (t - i) + 2 + \left\lceil \frac{E}{2} \right\rceil \leq \sqrt{2n} + 1.$$

All other cases when u and v are in VP's from different channels are similar. It remains to consider the situation when u and v are in VP's from the same channel. If this is channel $\mathcal{C}_1$ or $\mathcal{C}_2$, $\textsc{Hops}_{L_n}(\mathcal{P}, u, v)$ is less than in case 1. Thus the only remaining case to consider is the following:

Case 3: u and v are inside VP's of channel $\mathcal{C}_3$.

The worst case occurs when u and v are in different external VP's of $\mathcal{C}_3$. We have

$$\textsc{Hops}_{L_n}(\mathcal{P}, u, v) \leq \left\lceil \frac{E}{2} \right\rceil + 1 + \left\lceil \frac{E}{2} \right\rceil \leq E + 2 < \sqrt{2n} + 2.$$

It follows that $\textsc{Hops}_{L_n}(\mathcal{P}, u, v) < \sqrt{2n} + 2$, for any pair of nodes u, v, and consequently $\textsc{Hops}_{L_n}(\mathcal{P}) < \sqrt{2n} + 2$. This implies the upper bound.

In order to prove the lower bound, consider any layout $\mathcal{P}$ in L_n, with congestion at most 2. Let $\mathcal{Q}$ be the set of VP's in $\mathcal{P}$ of length at least 2 and let q be the size of $\mathcal{Q}$. The overlap of any VP's from $\mathcal{Q}$ can have length at most 1, hence a left-right ordering of these VP's is well defined. Let $k = \lfloor \frac{q}{2} \rfloor$. Let $A_1, A_2, ... A_k$ be the first k VP's in $\mathcal{Q}$, ordered from left to right, and let $B_k, B_{k-1}, ..., B_1$ be the last k VP's in this ordering. If q is odd, let C_0 be the remaining VP in $\mathcal{Q}$. Let a_i, b_i be the lengths of A_i, B_i, respectively, and let c_0 be the length of C_0, if q is odd, otherwise $c_0 = 0$.

We shall prove that $\text{HOPS}_{L_n}(\mathcal{P}) > X$, where $X = \sqrt{2n} - 5$. Suppose not. If u is the mid-point of C_0 and v is the end-point of this VP, we get

$$\frac{c_0}{2} \leq \text{HOPS}_{L_n}(\mathcal{P}, u, v) \leq X.$$

If u and v are mid-points of A_k and B_k we get

$$\frac{a_k + b_k}{2} - 1 \leq \text{HOPS}_{L_n}(\mathcal{P}, u, v) \leq X.$$

If u and v are mid-points of A_{k-1} and B_{k-1} we get

$$\frac{a_{k-1} + b_{k-1}}{2} - 2 + 2 \leq \text{HOPS}_{L_n}(\mathcal{P}, u, v) \leq X.$$

If u and v are mid-points of A_{k-i} and B_{k-i} we get

$$\frac{a_{k-i} + b_{k-i}}{2} - 2 + 2i \leq \text{HOPS}_{L_n}(\mathcal{P}, u, v) \leq X.$$

Finally, if u and v are mid-points of A_1 and B_1 we get

$$\frac{a_1 + b_1}{2} - 2 + 2(k-1) \leq \text{HOPS}_{L_n}(\mathcal{P}, u, v) \leq X.$$

Adding all the above inequalities we get

$$(a_1 + a_2 + \cdots + a_k) + (b_1 + b_2 + \cdots + b_k) + c_0 + 4\frac{k(k-1)}{2} \leq 2kX + 4k + 2X.$$

Since all VP's in $\mathcal{Q}$ cover L_n, this implies

$$n \leq (a_1 + a_2 + \cdots + a_k) + (b_1 + b_2 + \cdots + b_k) + c_0$$

and hence

$$n \leq 2kX - 2k^2 + 2X + 6k.$$

Let $k = \sqrt{\frac{n}{2}} - t$. Substituting $X = \sqrt{2n} - 5$ in the latter inequality we get $n \leq n + (-2t^2 + 4t - 10)$. This is a contradiction because $-2t^2 + 4t - 10$ is negative for any t. It follows that $\text{HOPS}_{L_n}(\mathcal{P}) > X$, which concludes the proof. ∎

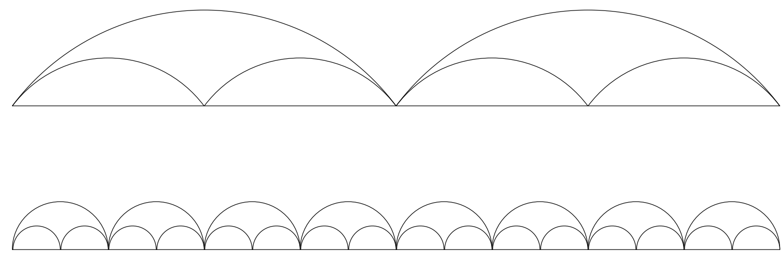

Figure 5: A nested layout with congestion 2 for the $2 \times n$ mesh (vertical links are omitted). The nested virtual paths of the layout have lengths 1, $n^{1/5}$ for the bottom chain and $n^{2/5}$, $n^{3/5}$ for the top chain.

4 The Mesh

In this section we construct asymptotically optimal layouts for rectangular meshes. A $k \times n$ mesh has k rows and n columns. Our results are based on the following lemma.

LEMMA **9** *For the $k \times n$ mesh we have*

$$\frac{1}{2}(n-1)^{1/(kc)} \leq \text{HOPS}_{k \times n}(c) \leq kc(n-1)^{1/(kc)} + 4k.$$

PROOF First we prove the lower bound. Suppose we have an optimal layout $\mathcal{P}$ with congestion c for the $k \times n$ mesh. Consider the k rows of the mesh as k copies of the chain L_{n-1} and collapse all VP's from $\mathcal{P}$ onto L_{n-1} (ignoring vertical parts of each VP). The resulting layout in L_{n-1} has congestion at most kc. Hence the lower bound follows from Theorem 2.

To prove the upper bound we construct the following layout (see Figure 5). The k rows of the $k \times n$ mesh are k copies of the chain L_{n-1}. Enumerate these k chains $R_1, ..., R_k$. For $i = 1, \ldots, k$, consider the following layout in the ith chain, starting at the leftmost vertex:

a channel of $(n-1)^{((i-1)c+1)/(kc)}$ VP's of length $(n-1)^{(kc-(i-1)c-1)/(kc)}$
a channel of $(n-1)^{((i-1)c+2)/(kc)}$ VP's of length $(n-1)^{(kc-(i-1)c-2)/(kc)}$
...
a channel of $(n-1)^{ic/(kc)}$ VP's of length $(n-1)^{(kc-ic)/(kc)}$.

Each of these layouts in respective chains (rows) has congestion c. Vertically consider all virtual paths of length 1 (edges) and only those. The layout $\mathcal{P}$ consisting of all the above VP's has congestion c.

Consider the layout $\mathcal{Q}$ in L_{n-1} obtained from $\mathcal{P}$ by collapsing all its horizontal VP's on L_{n-1}. $\mathcal{Q}$ has congestion kc. This is the layout from the proof of theorem 4, hence $\textsc{Hops}_{L_{n-1}}(\mathcal{P}) \leq kc(n-1)^{1/(kc)}$.

Consider vertex u in row i_1 and column j_1 and vertex v in row i_2 and column j_2. Let u' and v' be vertical projections of u and v on L_{n-1}. Let $\mathcal{C}$ be the shortest virtual channel in $\mathcal{Q}$ joining u' and v'. Thus the length of $\mathcal{C}$ is at most $kc(n-1)^{1/(kc)}$. The lengths of VP's in $\mathcal{C}$ are first increasing then decreasing.

We now construct a virtual channel in $\mathcal{P}$ joining u and v. First use (at most k) VP's of length 1 along column j_1 to get from u to the vertex in kth row and j_1th column. Then use consecutive VP's from $\mathcal{C}$ which are in R_k. Next use a vertical VP to get to R_{k-1} and use consecutive VP's from $\mathcal{C}$ which are in R_{k-1}. Proceed in this way until all VP's of largest length from $\mathcal{C}$ are used, travelling in the meantime vertically from R_k to R_1. Now lengths of VP's in $\mathcal{C}$ start decreasing. Use all of them, travelling in the meantime vertically from R_1 to R_k, as shorter horizontal VP's are used. Thus vertex in kth row and j_2th column is reached. Now use (at most k) VP's of length 1 along column j_2 to get to v.

The number of horizontal VP's used in this channel is equal to the length of channel $\mathcal{C}$, hence does not exceed $kc(n-1)^{1/(kc)}$. The number of vertical VP's does not exceed $4k$, which concludes the proof. ∎

The above lemma implies an asymptotically tight bound for constant k and c.

Theorem **10** *For the $k \times n$ mesh, with constant k and c, we have*

$$\textsc{Hops}_{k \times n}(c) \in \Theta(n^{1/(kc)}).$$

Our last result gives an asymptotically tight bound on the hop number for the square mesh with constant congestion.

Theorem **11** *For the $n \times n$ mesh and constant $c \geq 2$ we have*

$$\textsc{Hops}_{n \times n}(c) \in \Theta(\log n).$$

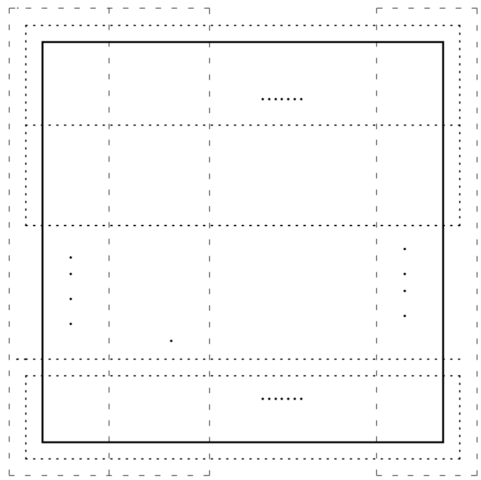

Figure 6: Partitioning the $n \times n$ mesh into stripes. Dashed lines represent the vertical partitioning, while dotted lines repreent the horizontal partitioning.

PROOF We first show a layout $\mathcal{P}$ with congestion c, such that $\mathrm{HOPS}_{n\times n}(\mathcal{P}) \in O(\log n)$. Subdivide the mesh vertically and horizontally into submeshes $(\log n) \times n$ and $n \times (\log n)$, respectively, in such a way that neighboring submeshes share one row (column). Call these submeshes horizontal (vertical) stripes (see Figure 6).

In each stripe S construct the layout $\mathcal{P}_S$ from lemma 9, for congestion $c-1$, in such a way that VP's in shared rows (columns) coincide. Lemma 9 for $k = \log n$ implies that the hop number of each of these layouts is $O(\log n)$.

Let $\mathcal{P}$ be the layout in the square mesh, consisting of all VP's from these layouts. Since all vertical VP's in horizontal stripes and horizontal VP's in vertical stripes have length 1, the amalgamation of all layouts $\mathcal{P}_S$ in $\mathcal{P}$ results in adding 1 to the congestion, hence $\mathcal{P}$ has congestion c.

Consider any vertices u and v in the square mesh. Let w be any vertex belonging to the same vertical stripe S_1 as u and the same horizontal stripe S_2 as v. Let $\mathcal{C}_1$ be the shortest VC in $\mathcal{P}_{S_1}$ joining u with w and let $\mathcal{C}_2$ be the shortest VC in $\mathcal{P}_{S_2}$ joining w with v. The concatenation $\mathcal{C}$ of these VC's is a virtual channel in $\mathcal{P}$ joining u with v, of length $O(\log n)$. This proves the upper bound. The lower bound $\Omega(\log n)$ follows immediately from Theorem 1. ■

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