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Two restrictions on possible connectives

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Introduction

If languages could lexicalize arbitrary truth tables as sentential connectives, we should be able to find a great variety of connectives in the world's languages. However, very few connectives are typologically attested, as has long been known. For example, no known language lexicalizes *if-and-only-if*, *not-and*, or McCawley (1972)'s *schmor*, a connective that returns true exactly when two or more of its arguments are true. In fact, Gazdar (1979) makes the point that the only *bona fide* non-unary connectives are $\wedge$ and $\vee$. This typological puzzle calls for an explanation, and indeed several proposals have been suggested in the literature.[1]

We examine two approaches to restricting the possible connectives. Both follow McCawley (1972), and more specifically Gazdar (1979), in assuming that connectives can only see the set of truth values of their syntactic arguments. As Gazdar notes, this eliminates sensitivity to ordering and repetitions. The first approach, growing out of Gazdar and Pullum (1976) and Gazdar (1979), takes the notion of *choice* as its starting point: if O is a connective and A its argument, then $O(A) \in A$. For example, if all the arguments are true, $A = \{1\}$, and $O(A)$ is 1. We will refer to this as the *choice-based approach*.[2] The second approach, growing out of Keenan and Faltz (1978, 1985), takes *ordering* as its starting point: the domain of truth values is assumed to be ordered, with $0 < 1$, and a connective can only choose the maximum or minimum element within its argument. We will refer to this as the *ordering-based approach*.[3]

In the classical domain, choice and ordering seem to predict the same sentential connectives. The perspectives they offer are different, though, offering the hope of divergent predictions if we go beyond the classical domain. A tempting place to look is non-classical semantics, used to implement the Frege-Strawson program for presupposition.[4] The challenge here is that there are many trivalent extensions of the classical operators, but only one

[1]A similar state of affairs holds with respect to other logical operators. See Barwise and Cooper (1981), Higginbotham and May (1981), Keenan and Stavi (1986), and van Benthem (1984) for discussion of the case of quantificational determiners.

[2]This is a generalization of Gazdar and Pullum's notion of *confessionality*, which requires that if A is $\{0\}$ then $O(A) = 0$.

[3]For a more general discussion, we should replace *maximum* with *supremum* (least upper bound) and *minimum* with *infimum* (greatest lower bound). We stay with *maximum* and *minimum* here to keep the discussion simple. We should note that Keenan and Faltz and much work inspired by it assume that the appropriate structures form Boolean algebras. This stronger assumption is incompatible with the trivalent extensions discussed below.

[4]See van Fraassen (1966) and Keenan (1972) for early proposals.

(the system of Peters (1979), modelling Karttunen (1974)) is attested in natural language.

At the time of the original proposals providing the basis for the choice-based and ordering-based approaches, the extension to trivalent semantics seemed unattractive: both approaches are committed to symmetric semantics, while the projection patterns of the connectives is inherently asymmetrical. Following Schlenker (2007, 2008), however, recent work on presupposition projection has explored the idea of a modular architecture in which a symmetric core is stated separately from an incrementalization procedure. Specifically, Fox (2008) and George (2008) provide incrementalizations of symmetric trivalent operators. This new direction allows us to return to the two explanatory accounts for binary connectives and compare their trivalent extensions. At first, as we will see, only the ordering-based approach seems to remain explanatory in the trivalent domain. We then notice that an epistemic perspective used by Fox (2008) and George (2008) allows the choice-based approach to eliminate the gains of the ordering-based approach and become explanatory once again. We conclude that the match goes on.

1 The projection problem

The projection problem for presupposition is the problem of predicting the presuppositions of a complex sentence from its constituent parts. The second conjunct in (1) carries the presupposition that John has a wetsuit, but the conjunction as a whole appears to have only the conditional presupposition that if John is a scuba diver, he has a wetsuit.[5,6] A similar state of affairs holds in (2), this time with disjunction: the disjunction as a whole inherits only the conditionalized version of the presupposition of the second disjunct. The sentences below are modeled after Karttunen (1973, 1974), whose characterization of the empirical facts of presupposition projection set the stage for much of the subsequent work on the subject.

(1) (It is possible that) John is a scuba diver, and his wetsuit is blue

(2) Either John is not a scuba diver, or his wetsuit is blue

Presupposition projection has been handled within a variety of different frameworks. Peters (1979) has observed that Karttunen's characterization of the projection facts can be captured by extending the classical 2-by-2 truth tables for the binary connectives to 3-by-3 tables, as in (3). A third option for each conjunct, marked $\circledast$, appears for each argument of the connective, as well as for the outcome, and represents presupposition failure. This failure can be thought of as undefinedness (possibly cashed out in terms of partiality of a function), a new truth value, or perhaps most usefully as uncertainty about which of the two classical truth values holds.

[5]The projection of the presupposition in the unembedded version of (1) is obscured by the fact that it is also entailed by the sentence. We have added the optional embedding *It is possible that* to highlight that this entailment is not relevant. For ease of exposition, we will mostly ignore this embedding below and refer to the unembedded version.

[6]We ignore here the stronger, unconditional presuppositions sometimes observed in sentences similar to these. Accounting for such unconditional presuppositions, called the *proviso problem* by Geurts (1996), has been a matter of lively debate. See Beaver (2001), Heim (2006), Singh (2007), and Schlenker (2011) among others.

(3)

and	0	⊛	1
0	0	0	0
⊛	⊛	⊛	⊛
1	0	⊛	1

or	0	⊛	1
0	0	⊛	1
⊛	⊛	⊛	⊛
1	1	1	1

To compute the presupposition of a conjunctive sentence of the form *p and q*, such as the unembedded version of (1), we look at the truth table for conjunction and compute the conditions under which the result is not presupposition failure (that is, not ⊛). It is easier to start by looking at when it *does* denote ⊛. This happens in one of two cases: (a) $[\![p]\!] = ⊛$, regardless of *q* (that is, failure in the first conjunct always projects), and (b) $[\![p]\!] = 1$ and $[\![q]\!] = ⊛$. In our example, *p* is not presuppositional, while *q* presupposes that John has a wetsuit.[7] Since *p* is not presuppositional, the (a) case is irrelevant, and the conjunction will denote ⊛ only in the (b) case: if $[\![p]\!] = 1$ and $[\![q]\!] = ⊛$. That is, the conjunction will denote ⊛ exactly when *p* is true and John does not have a wetsuit. The conjunction presupposes that it does *not* denote ⊛: that either $[\![p]\!] = 0$ (recall that *p* is not presuppositional, so it cannot denote ⊛) or that John has a wetsuit. This amounts to the presupposition that if John is a scuba diver, then he has a wetsuit, which matches Karttunen's characterization of the facts. The reasoning for (2) is similar.

The Peters tables in (3) are asymmetric, reflecting important observations by Karttunen (1973) and Stalnaker (1974) that suggest a left-to-right asymmetry in projection. Here is an early example:

(4) (Karttunen 1973 ex. 16)

a. Jack has children and all of Jack's children are bald

b. # All of Jack's children are bald, and Jack has children

Similarly, reversing the linear order of the two conjuncts in (1) seems to project the strong presupposition that John has a wetsuit rather than the weaker conditional presupposition of the original example:

(5) (It is possible that) John's wetsuit is blue, and he is a scuba diver

Peters (1979) used evidence of this kind to motivate the asymmetry in (3). In (4a), for example, the first conjunct satisfies the presupposition of the second conjunct. For the sentence to be ⊛, we need the first conjunct to be 1 (if it is 0, the whole conjunction is 0; it is not presuppositional and so cannot be ⊛) and the second conjunct to be ⊛. (4b), on the other hand, is odd, a judgment that has been taken to point to an inability of the second conjunct to help the first. Peters (1979)'s solution is to make presupposition failure in the first conjunct fatal, regardless of what follows.

2 The overgeneration puzzle

As pointed out by Gazdar (1979) and Heim (1983), extending the classical operators to account for presupposition projection raises an overgeneration problem:[8] we can imagine

[7] This is not quite right. In both cases, *p* presupposes at least that John exists, and *q* probably presupposes that he has a unique wetsuit. We ignore this to keep the presentation simple.

[8] The overgeneration problem was further discussed by Soames (1989) and Heim (1990) (citing a personal communication from Mats Rooth), who point out that Heim (1983)'s proposal is not explanatory in this sense.

various projection behaviors associated with the same given classical operator, and yet there is just one actual projection behavior attested across speakers and across languages for each operator. For example, we can imagine a variant of English, English′, in which a conjunction presupposes everything that at least one of its arguments presupposes. This would make (1) presuppose that John has a wetsuit. Or there could be a different variant of English, English″, in which a conjunction would presuppose only what both of its arguments presuppose. This would make (1) presuppose nothing. Such variants of English (or of other languages) are unattested. An explanatory theory should derive the actual projection behavior for a given operator in a principled way and explain why other imaginable projection options are absent.

In the framework of Peters (1979), the overgeneration problem can be stated as follows. There is a 3-by-3 truth table extending the classical table for conjunction (or disjunction, etc.) which accounts for the observed pattern of presupposition projection. But there are many other imaginable 3-valued extensions of the same classical table that would result in other, unattested projection patterns. In fact, a third truth value gives rise to a 3×3 table in which only 4 cells are already determined by the classical table, so each of the remaining 5 cells can in principle have any of the three available truth values. This means $3^5 = 243$ possible extensions for any classical binary connective such as *and* and *or*. Why is it that speakers and languages do not vary with respect to the extension that they choose?

3 Choice, ordering, and trivalence

Interest in the overgeneration puzzle for presupposition projection has been revived by Schlenker (2007, 2008), who offered a first comprehensive explanatory account. This has spurred work on explanatory theories of projection within different frameworks, leading to semantic accounts by Fox (2008) and George (2008), a pragmatic account by Chemla (2009), and an extension of Heim (1983)'s original system by Rothschild (2008), among other proposals. The overgeneration puzzle has been treated separately in the literature from the typological puzzle for connectives in the classical domain. In fact, as far as we can tell, the explanatory accounts just mentioned can extend XOR, *nand*, or any of the other unattested classical connectives just as easily as they can extend the attested ones. On the other hand, both of the approaches for restricting the classical connectives also suggest interesting restrictions on the possible trivalent extensions. Both approaches treat their input as a set, which means that both rule out any asymmetric connective. Moreover, both select the output from within the set, which limits the operators even further: if the input is the set $\{0, \circledast\}$, for example, the output cannot be 1. In total, this brings us down from the original 243 potential extensions for each connective to just four for each:

(6)

$\wedge_1$	0	$\circledast$	1
0	0	0	0
$\circledast$	0	$\circledast$	$\circledast$
1	0	$\circledast$	1

$\wedge_2$	0	$\circledast$	1
0	0	$\circledast$	0
$\circledast$	$\circledast$	$\circledast$	$\circledast$
1	0	$\circledast$	1

$\wedge_3$	0	$\circledast$	1
0	0	0	0
$\circledast$	0	$\circledast$	1
1	0	1	1

$\wedge_4$	0	$\circledast$	1
0	0	$\circledast$	0
$\circledast$	$\circledast$	$\circledast$	1
1	0	1	1

(7)

$\vee_1$	0	$\circledast$	1
0	0	$\circledast$	1
$\circledast$	$\circledast$	$\circledast$	1
1	1	1	1

$\vee_2$	0	$\circledast$	1
0	0	$\circledast$	1
$\circledast$	$\circledast$	$\circledast$	$\circledast$
1	1	$\circledast$	1

$\vee_3$	0	$\circledast$	1
0	0	0	1
$\circledast$	0	$\circledast$	1
1	1	1	1

$\vee_4$	0	$\circledast$	1
0	0	0	1
$\circledast$	0	$\circledast$	$\circledast$
1	1	$\circledast$	1

The choice-based approach stops here, leaving us with $4 \times 4 = 16$ different presuppositional systems for speakers to consider (since any choice of $\wedge$ in (6) is compatible with any choice for $\vee$ in (7)).

The ordering-based approach goes one step further. Recall that on this approach, the entries are min and max rather than arbitrary truth tables. If the three-valued domain is ordered, we will have a unique extension to each operator. The orderings to consider are the following:

(8) a. $\circledast < 0 < 1$
 b. $0 < \circledast < 1$
 c. $0 < 1 < \circledast$

If we accept any of the orderings in (8), the ordering-based approach gives us exactly two simplex trivalent connectives: (8a) licenses $\wedge_2, \vee_3$; (8b) licenses $\wedge_1, \vee_1$; and (8c) licenses $\wedge_3, \vee_2$. If we follow Beaver and Krahmer (2001) in accepting (8b), we obtain $\wedge_1$ and $\vee_1$. These two tables are those introduced in Kleene (1952) and known as *Strong Kleene*.[9]

4 Descriptive adequacy and linear asymmetry

The pattern of projection for the Strong Kleene operators predicted by the ordering-based approach (and for the additional operators predicted by the choice-based approach) can be computed using the same reasoning discussed in section 1. For example if p is non-presuppositional and q presupposes r the prediction of the Strong Kleene system is that both *p and q* (as in (1)) and *q and p* (as in (5)) presuppose $p \rightarrow r$, and that both *p or q* and *q or p* presuppose $\neg p \rightarrow r$. The other connective licensed by the choice-based approach make different predictions, but they, too, can only predict symmetric patterns of projection.

The predicted symmetry flies in the face of all the standard work on projection since Karttunen (1973), mentioned earlier, which might explain why trivalence was not used to attempt to decide between the choice-based and ordering-based approaches. Recently, however, Schlenker (2007) has argued for a more modular system, which includes both symmetric projection and an incrementalization procedure. Much of the work following Schlenker has maintained this modular view, in which a symmetric core is embedded within a bigger incremental system, and Fox (2008) describes a general procedure for incrementalizing a symmetric system. The rough idea is this: when we process a sentence from left to right, we must always be sure that the sentence does not end up denoting $\circledast$; if we are not sure, the result is presupposition failure for the sentence, even if this local uncertainty is resolved later on. In a conjunction of the form *p and q*, this requirement leads to the following difference between p and q. Suppose p suffers from presupposition failure, and suppose that q is false. As mentioned above, we can easily determine the truth value of the whole conjunction: *p and q* is false, regardless of p. However, this certainty is obtained only after we have processed q. Earlier in the sentence, when we have just processed p, it is still possible, according to our information at that point, that the whole conjunction would suffer from presupposition failure (this would happen, for example, if the

[9]The choice of $\wedge_2$ and $\vee_2$ is known as *Weak Kleene*. Following Krahmer (1998), the asymmetric Peters tables in (3) above are sometimes referred to as *Middle Kleene*.

second conjunct turned out to be true). Due to this local inability to ensure that $\circledast$ is avoided, the whole conjunction suffers from presupposition failure. In other words, presupposition failure in the first conjunct always projects. Things are different with the second conjunct, *q*. Let us reverse the scenario just described and assume that *p* is false and that *q* suffers from presupposition failure. The same global ability to resolve the uncertainty applies: one of the conjuncts is false, and so the whole conjunction is false. However, as discussed by Fox (2008), since the uncertainty appears in the second (and final) conjunct *q*, this global elimination of uncertainty is the same as local elimination of uncertainty: by the time we process *q* we already know that *p* is false, and so we are never in doubt as to whether our uncertainty about *q* will affect the larger structure. As in the symmetric case, $\circledast$ is avoided, and the presupposition failure in the second conjunct does not project. Incrementalizing the Strong Kleene connectives yields exactly the asymmetric Peters tables listed above. Experimental support for the modular view is provided by Chemla and Schlenker (2012).

If this is indeed the correct direction, we can now incrementalize the symmetric connectives derived by the choice-based and the ordering-based approaches and try to compare the two. When we do so, the ordering-based approach derives a different asymmetric system for each of the three orderings in (8), while the choice-based approach overgenerates by again deriving four different possibilities for each connective, which yields sixteen different systems in total.

5 An epistemic equalizer

At this point it might look like we have what we were looking for. The choice-based account and the ordering-based one were both explanatory accounts in the classical domain, but only the latter remained reasonably explanatory in the trivalent extension we just saw. If we could stop here, we would have an argument for the ordering-based approach.

As discussed by Fox (2008) and George (2008), however, there is an epistemic perspective, due to Kleene (1952), that can make any account of the classical operators explanatory in the trivalent domain. If we conjoin *p* and *q*, where *p* is false and the truth value of *q* is unknown (but is either true or false), we can already conclude that *p and q* is false. Similarly, the disjunction of *p* and *q*, where *p* is true and the truth value of *q* is unknown (but either true or false) is true. On the other hand, conjoining *p* and *q*, where *p* is true or unknown and where *q* is unknown does not allow us to determine whether the result is true or false, and so the result will be unknown. Similarly for disjunction of a false or unknown *p* with an unknown *q*. This natural perspective allows us to derive the complete Strong Kleene tables from the tables for the classical connectives based on considerations of knowledge.

Note that this epistemic perspective does not obviate the need for an explanatory account of the classical domain. It does, however, eliminate the hard-earned gains of the ordering-based approach in the trivalent domain, bringing us right back to our starting point.

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