On Some Missing Logical Operators in Natural Language

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1 On the Study of Natural Language

- two broad approaches to the study of natural language

1.1 Language as a Byproduct

- one view, dominant outside linguistics: there's nothing special about it
- knowledge of language will fall out from general cognitive mechanisms (eg. learning)
- two kinds of theory-data difficulties: (1) the input data is often impoverished (eg. question formation), (2) the input data is often ignored (eg. the presuppositions of the definite article)

(1) a. The man is happy
$\rightarrow$ Is the man happy?
b. The man who is tall is happy
$\rightarrow$ Is the man who is tall happy?
$\rightarrow$ *Is the man who tall is happy?

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- if the child is only presented with questions like in (a), no evidence to choose between different rules in (b) (roughly, 'move the structurally most prominent auxiliary' versus 'move the (linearly) first auxiliary')
- the second rule would seem to be simpler, yet it's not selected
- children also seem to not make errors of the kind put forth by the second rule, etc.
- thus, they must not be entertaining rules of the second kind, i.e. it's not the data that push them to the first rule, but an innate specification that bars rules of the second kind as being a part of natural language grammars[1]

(2) The king of France is bald

- the definite article carries a presupposition of existence and uniqueness
- however, a corpus study has found that most uses of the definite article are made in contexts where the presupposition is not met (Spenader (2002))
- nevertheless, the presuppositional requirement can always be detected, which means that when children learn the meaning of *the*, they ignore the apparent optionality found in their input data, and encode the presupposition as a necessary requirement
- a good data-sensitive learner would surely account for the apparent optionality (as with other optional mechanisms, eg. verbs that allow both an NP and S complement)
- but there is no optionality with the presuppositional requirement
- in fact, Spenader (2002) points out that the only cases where the requirement can be cancelled are artificially constructed cases put forth in the linguistics literature, cases which never show up in actual usage

[1] This general argument was first put forth in Chomsky (1968). The ban on verb raising from subject relative clauses is a specific case of a more general constraint banning movement from such a position.

1.2 Universal Grammar

- partly in response to the above difficulties, an entirely different approach to the study of language takes language on its own terms
- the task of the linguist is to discover what is essential to human language, i.e. to discover properties of Universal Grammar
- UG is taken to delimit the space of possible human languages
- characterizations of UG are hypotheses about what is essential to the language faculty
- our project aims to provide a characterization of one subcomponent of the language faculty, namely, its inventory of logical operators
- it begins from the objection that all previous approaches to logicality in natural language fall short of providing the desired characterization

2 Logical Semantics

- out of the lexical inventory, there is a distinguished subset composed of logical operators, eg. *some, all, or, and, not, if* (as opposed to eg. *cat, dog, rain, walk*)
- classical logic (and various enrichments): meanings in terms of contribution to truth-conditions

2.1 Classical Logic

- define a space of possible truth-functional connectives

and	0	1
0	0	0
1	0	1

or	0	1
0	0	1
1	1	1

xor	0	1
0	0	1
1	1	0

nor	0	1
0	1	0
1	0	0

nand	0	1
0	1	1
1	1	0

- this basic format allows you to derive eleven other two-place connectives
- the contribution of quantificational elements, eg. *some*, *all*, given in terms of truth of substitution instances
- *Some A B* is true iff there is an x which as A, and which is B
- *All A B* is true iff for every x which is A, x is also B
- *No A B* is true iff there is no x which is both A and B
- can define many many others
- eg. *Allnon A B* is true iff for every x which is not A, x is also B
- eg. *Nall A B* is true iff there is an x which is A, but is not also B
- etc.
- could just be an accident of English that such operators are not realized

2.2 Some Enrichments

2.2.1 Modal Operators

- qualified statements via use of qualifying operators (eg. *can, must, possible, necessary, might, believe, know*)
- eg. *might A* is true in a world w iff there is an accessible world, w', such that *A* is true in w'

2.2.2 Generalized Quantifier Theory

- some operators (eg. *most*) not classically definable – was one motivation for generalized quantifier theory (Barwise and Cooper (1981))
- eg. ***Most** A B* is true iff $|A \cap B| > |A \cap \bar{B}|$
- *Some A B* is true iff $A \cap B \neq \emptyset$
- *All A B* is true iff $A \subseteq B$
- *No A B* is true iff $A \cap B = \emptyset$
- *Allnon A B* is true iff $\bar{A} \subseteq B$
- *Nall A B* is true iff $A \cap \bar{B} \neq \emptyset$

2.2.3 Three-Valued Logic

- it has been argued (eg. Peters (1979), Beaver and Krahmer (2001), Fox (Fall 2007), George (2008), Fox (To Appear)) that a three-valued logic would be appropriate for capturing presuppositions in natural language

(3) The king of France is bald

- is the sentence true, false, or neither, given that there is no king of France?
- to capture the squeemishness one feels here, a third truth-value, $\circledast$, is added, roughly 'I don't know whether it's true or false'
- sometimes given the value 0.5

or	0	$\circledast$	1
0	0	?	0
$\circledast$	?	?	?
1	0	?	1

- in general, there are 3^9 = 19,683 definable two-place connectives, 3^5 = 243 of which are consistent with two-valued *or*
- which should be the right logic?

- different proposals provide different solutions to the so-called projection problem for presuppositions, namely, the problem of providing an algorithm for deciding what complex sentences presuppose based on the presuppositions of their parts[2]

3 Problem: Gaps in Logical Inventories

- we seem forced to extend the expressive power of classical logic to account for the existence of operators which go beyond the limits set by classical logic
- however, each such increase in expressive power brings with it problems of overgeneration
- for example, we saw that there are close to 20,000 binary connectives definable in a three-valued logic – where are all these operators?
- English only lexicalizes three: *or, and, nor*
- thus, the problem arises not only for enrichments of classical logic, but for classical logic itself (eg. it allows for 16 two-place connectives, 13 of which are never lexicalized, and never have been, as far as we have been able to discover)
- in GQT, for a domain of n individuals, there are 2^{4^n} definable quantifiers
- English lexicalizes three: *some, all, no*
- etc.
- of course, this could just be an accident of English
- but these gaps seem to be systematic, i.e. cross-linguistically, we don't find any language lexicalizing *nall*, *nand*, *xor*, *allnon*, etc.[3]

[2]See Karttunen (1973, 1974), Stalnaker (1974), Karttunen and Peters (1979), Gazdar (1979), Peters (1979), Heim (1982, 1983a,b), van der Sandt (1992) for some of the earlier classics. The problem has been revived in much recent work, eg. Geurts (1999), Beaver (2001), Beaver and Krahmer (2001), Schlenker (2007, To Appear), Chemla (2009), Fox (Fall 2007), George (2008), Fox (To Appear), Katzir and Singh (2008).

[3]See Horn (1972) for extensive discussion. We will return to Horn's account of these gaps momentarily.

- there have been serious attempts to come to grips with such overgeneration problems, but (we will argue) none are satisfactory
- Barwise and Cooper (1981) discovered a set of univeral constraints which helped narrow down the overgeneration problem
- most famously, that all quantifiers in natural language are *conservative*: QAB iff $QA(A \cap B)$, i.e. quantification is restricted to those individuals in A
- this would work to rule out operators like *Allnon*
- however, space of potential operators is still very large
- conservativity reduces the space of quantifiers from 2^{4^n} to 2^{3^n}, significant, but not enough to overcome the overgeneration problem[4]
- still allows for many operators which are unattested, eg. *nall*
- several other seemingly arbitrary choice points
- eg. does it follow from *Every man in the next room is tall* that *Some man in the next room is tall*?
- yes, only if we follow Strawson (1952) and add a constraint to the effect that QAB is defined only if $A \neq \emptyset$
- textbook logic doesn't employ such a constraint, and there has been much debate whether natural language quantifiers obey this constraint (so-called 'existential import')[5]
- what is important is that this seems like an arbitrary choice point
- for this talk, we will focus on one extended attempt to come to grips with the overgeneration problem

[4] See van Benthem (1986) for this and other results relating universal constraints to the resulting size of the sample space. For example, by also adding *permutation invariance* as a constraint on logicality, the space of operators is reduced further to $2^{(n+2)(n+1)/2}$. Again, while this is a significant reduction, the overgeneration problem remains essentially as it was. Moreover, since these stipulated constraints seem unrelated to one another, one would need to articulate why natural language chose such constraints instead of others.

[5] See Katzir and Singh (2008) for references.

3.1 Horn (1972)

- begin with Aristotle's Square of Opposition
- Horn's observation: of the four corners, one is systematically absent in the world's languages
- *some, all, no, *nall*
- *or, and, nor, *nand*
- *sometimes, always, never*(= not sometimes), **nalways*
- *may, must, *nmust*
- *either, both, neither* = (not both), **noth* (= not both)
- note that there is no cognitive restriction against the fourth corner – we often infer these meanings through scalar implicature, and can construct them analytically
- Horn: two factors responsible for this gap: (1) Scalar Implicature,[6] (2) A specific pressure against negation
- can reason from *some* to 'not all,' *or* to 'not and' (and similarly with the other squares), so that, in effect, the overall meaning conveyed by assertion of *some* is 'some and not all'[7]
- at the same time, by the same logic of implicature computation, assertion of *nall* would lead to the implicature that 'not no,' i.e. some
- thus, the overall meaning conveyed by assertion of *nall* is 'some and not all'[8]

[6] Very roughly, a scalar implicature is a pragmatic inference that strengthens the meaning of a sentence. From use of sentence S, the hearer reasons as follows: The speaker could have used a stronger sentence, T. This would have been more informative, hence 'better.' Assuming that T is relevant, and that the speaker is knowledgeable about T's truth-value, and is cooperative, they would have said T had it been true. Since they didn't, it must be false. Thus, S can be strengthened to 'S and not T.' For example, *some* is strengthened by implicature to 'some and not all,' *or* is strengthened to exclusive or, etc. See Grice (1967) and Horn (1972) for the earliest formulations.

[7] Note that this reasoning works only if we follow Strawson (1952) and adopt existential import.

[8] Eg. *Not all the boys came to the party* has an implicature that some of them did come to the party.

- a lexical inventory including *some* and *nall* would seem to be wasting resources, using up two lexical items whose contribution to communication is essentially identical

- ideally, then, natural languages would make use of either the three-membered inventory $< \exists, \forall, \neg\exists >$ (*some, all, no*) or the three-membered inventory $< \forall, \neg\forall, \neg\exists >$ (*all, nall, no*)

- while Horn doesn't provide a formal constraint to capture this, he seems to have in mind something like the following

- let $S(Op)$ be a sentence containing operator Op

Constraint on Lexicalization 1 (Scalar Implicature) *If assertion of $S(Op_1)$ conveys the same information as assertion of $S(Op_2)$ (i.e. meaning plus scalar implicatures), then don't lexicalize both of Op_1, Op_2.*

- we have two inventories that would satisfy the above constraint

- only one is ever attested

- why?

- Horn: note that the second inventory contains two instances of negation, while the first one contains only one[9]

- a specific pressure against negation determines that the first inventory 'wins' over the second

Constraint on Lexicalization 2 (Minimize Negation) *If $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_k$ are lexical inventories containing the same number of operators, the inventory $\mathcal{I}_j$ with the least instances of negation wins.*

3.2 Negation and Duality

- for Horn's result to go through, he has to ignore the duality of $\forall$ and $\exists$

- since the two are interdefinable, i.e. $\exists = \neg\forall\neg$, $\forall = \neg\exists\neg$, we only need one basic operator, and derive the rest by inner and outer negation

[9]Note that this result does not depend on whether we represent *no* as $\neg\exists$ or as $\forall\neg$.

- shape of the square: $Op, \neg Op, Op\neg, \neg Op\neg$
- once we take duality into account, the argument fails
- eg. consider the attested inventory, *some, all, no*
- if $\exists$ is basic, then we have: $\{\exists, \neg\exists\neg, \neg\exists\}$
- if $\forall$ is basic, then we have: $\{\neg\forall\neg, \forall, \forall\neg\}$
- now consider the unattested inventory, but allowed by the scalar implicature constraint: *all, no, nall*
- if $\exists$ is basic, then we have: $\{\neg\exists\neg, \neg\exists, \exists\neg\}$
- if $\forall$ is basic, then we have: $\{\forall, \forall\neg, \neg\forall\}$
- these four inventories are allowed by the SI constraint
- hence, we simply count negations to determine the optimal inventory, and now, it turns out that the unattested inventory with $\forall$ as basic wins
- the only way to avoid this, then, is to stipulate that $\exists$ is the basic operator

4 Orders and Min-Max

- Horn's constraints only make sense once we stipulate a certain space to begin with
- eg. one could imagine alternative formulations that attempt to minimize the size of the inventory entirely
- eg. in the space of two-place connectives, it is well-known that *nor* suffices to derive all the other connectives, as does *nand*[10]
- in any event, as argued above, even with the stipulation, the reasoning does not go through

[10] Let '$\uparrow$' represent *nand*, and let '$\downarrow$' represent *nor*. Then: $\neg A \equiv A \downarrow A$, $A \wedge B \equiv (A \downarrow A) \downarrow (B \downarrow B)$. Also, $\neg A \equiv A \uparrow A$, $A \vee B \equiv (A \uparrow A) \uparrow (B \uparrow B)$. From this fact, the rest of the connectives follow, since $\{\neg, \wedge\}$ and $\{\neg, \vee\}$ are sufficient to derive the rest, as is standard.

- our strategy: We will adopt Horn's Constraints, and will attempt to come up with a principled delimitation of the space of operators
- in general, we believe that the source of the overgeneration in all of the above approaches to logicality rests in their focus on *truth*
- we propose, instead, that *orderings* are fundamental, and that operators in natural language are allowed to exploit this ordering, but otherwise are not allowed any other freedom
- very roughly, we assume that operators are given ordered sets as input, and a constraint in UG dictates that simplex operators in natural language can either take the max of the set, or the min[11]

Constraint on Lexicalization 3 (Simplex Operators) *Morphologically simplex operators in natural language are limited to selecting maxima or minima of the ordered sets which they receive as input.*

- can think of this is a condtion that says: Don't Tamper – Just Choose
- eg. in the domain of truth-values, $\{0, 1\}$, with the natural order $1 > 0$, a two-place connective in natural language can take one of two forms: max, or min

min	0	1
0	0	0
1	0	1

max	0	1
0	0	1
1	1	1

- max corresponds to *or*, and min corresponds to *and*
- other operators like *xor* are just not definable
- what about *nor*?

[11]More formally, where $\mu \in \{min, max\}$, simplex operators must be of the form Op_μ, where $[[Op_\mu]] = \lambda A \subseteq D_\alpha.\lambda f_{\alpha\beta} : \beta \in Dom(\mu).\mu\{f(x) : x \in A\}$. Note that our entry does not reference specific types α, β. All that matters is that the input to μ be a non-empty, partially ordered set.

- we will extend this constraint to accommodate so-called *n-words* in just a moment
- but let's see how this works with other operators, eg. quantifiers
- eg. Op *boys walk*
- $Op\{W(b_1), W(b_2), \ldots, W(b_k)\}$, where $Op \in \{min, max\}$
- min corresponds to *all*, max corresponds to *some*

4.1 Negation and Order

- given an order, $\leq$, the reverse order is always defined: $\leq_{new} = \{< a, b >:< b, a > \in \leq\}$
- clearly, $\min_{\leq} \equiv \max_{\leq_{new}}$ (and similarly, $\max_{\leq} \equiv \min_{\leq_{new}}$)
- this suggests a role for a directionality-reversing operator, as well as a simple way to state the intuitive duality of $\min$ and $\max$
- define an operator $\circ$ (pronounced *suhrk*) that reverses ordering relations in the way just described (that is, $\circ(R) \equiv \{< a, b >:< b, a > \in R\}$), and use the following entry for one of the operators, say for $\min$

(4) $[[\min_{\leq}]] = \lambda X \in Dom(\max).\, \max_{\circ(\leq)}(X)$

- of course, could also define max in terms of min

(5) $[[\max_{\leq}]] = \lambda X \in Dom(\min).\, \min_{\circ(\leq)}(X)$

- with the *min-max* perspective, we've managed to avoid the overgeneration problem of other approaches, but we've fallen into an undergeneration problem
- while we've avoided generating *nall*, *nand*, etc. we also incorrecly predict the absence of *no*, *nor*, etc.
- so we need external negation to combine with our operators
- if min were primitive, we would predict the existence of *nall*, *nand*, etc.

- if max were primitive, we would predict the existence of *no*, *nor*, etc.
- the attested typology seems to go the second way, so it seems we need to stipulate max as primitive, and allow for our inventory to expand as follows

Constraint on Lexicalization 4 (Operators, Final version) *(a) Morphologically simplex operators in natural language are either* $min = max(\circ)$ *or* max*, (b) It is possible to lexicalize* $\circ\,\mathrm{max}$*, and the result is an n-word.*

- but notice that our result depends on a stipulation that max is primitive
- is there a way to eliminate this stipulation?

4.2 Back to the Missing Corner

- Attested System: *some, all, no*
- Unattested System: *all, nall, no*

(6) Attested System:

a. max as primitive: $\{max, max\circ, \circ max\}$
b. min as primitive: $\{min\circ, min, \circ min\circ\}$

(7) Unattested System:

a. max as primitive: $\{max\circ, \circ max\circ, \circ max\}$
b. min as primitive: $\{min, \circ min, \circ min\circ\}$

- once we move to the *min-max* approach, Horn's reasoning actually succeeds without any stipulation
- the system with the least number of negations is the one with max as primitive, as in (a)
- this leads to the attested typology, and our apparent stipulation has instead been derived by Horn's reasoning

4.3 Further Predictions

- derive the standard entry for the definite article (Link (1983)) as a max operator over the domain of individuals
- derive a strict implication analyis for conditionals
- recall the many choice points that came up in relation to other approaches
- eg. there were arbitrary stipulations concerning constraints like conservativity, permutation invariance, existential import
- in three-valued logics, previous approaches have either stipulated truth-tables (eg. Peters (1979), Beaver and Krahmer (2001)), or tried to derive them from other principles (eg. Fox (Fall 2007), George (2008), Fox (To Appear))
- however, none of these *follow* from the semantics in any way
- our *min-max* approach *derives* a solution to the projection problem, namely, we derive the so-called Strong Kleene tables (Kleene (1952))

min	0	$\circledast$	1
0	0	0	0
$\circledast$	0	$\circledast$	$\circledast$
1	0	$\circledast$	1

max	0	$\circledast$	1
0	0	$\circledast$	1
$\circledast$	$\circledast$	$\circledast$	1
1	1	1	1

- problem: this prediction goes against over thirty-five years of work on presupposition projection
- even constraints like existential import, and conservativity have been disputed in the literature, and the strict implication analysis of conditionals has been argued since Stalnaker (1968) and Lewis (1973), and much subsequent work, to be inadequate for natural language
- task: show the predictions are right, despite appearances

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