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On a New Class of Direct Adaptive Output Feedback Controllers for Nonlinear Square Systems

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Abstract— **Based on the recent development of sufficient conditions that allow nonstationary systems to become stable and strictly passive via static or dynamic output feedback, this paper presents a new class of direct adaptive output feedback controllers. The proposed control methodology uses a decentralized adaptation law mechanism based upon the Simple Adaptive Control technique. Using Lyapunov direct method and Lasalle's invariance principle for nonautonomous systems, the formal proof of stability is established. The applicability of the proposed adaptive control approach to second-order Euler-Lagrange systems, whose nonlinear dynamics is expressed in the operational space, is presented.**

I. INTRODUCTION

A well-known result in control theory that plays an important role in guaranteeing stability in adaptive control is the notion of passivity, which requires the plant be strictly passive (SP). For linear time-invariant (LTI) systems, this stability condition is equivalent to requiring the input-output transfer function be strictly positive real (SPR). However, as most real-world systems are not inherently SP, it is known that this condition can be mitigated for LTI systems for which any constant output feedback gain (unknown and not needed for implementation) could render the (fictitious) closed-loop system SP. Such systems that are only separated from strict passivity by a constant output feedback have been called almost strictly passive (ASP), and their transfer function almost strictly positive real (ASPR) [1]. Many works have attempted to clearly define what classes of systems satisfy the ASP conditions. Although some early results had been obtained for both SISO and MIMO systems [2], [3], these basic conditions have been considered difficult to satisfy and have remained unclear. It was finally shown that the ASP conditions required in order to guarantee stability with adaptive control are equivalent to requiring that an LTI system with state-space realization $\{A,B,C\}$ is ASP if it is minimum phase and the product CB is positive definite symmetric (PDS) [4], [5]. Furthermore, it was demonstrated that if a system cannot be made SP via constant output feedback, no dynamic output feedback can render it SP [6].

Over the years, a variety of direct adaptive control laws have been developed to address the problem of time-varying the gains of a controller so that the plant closed-loop characteristics match those defined by a reference model. However, most of the research in this area is based on the assumption that prior knowledge of the unknown plant to be controlled is available, and/or requires the plant to be of the same order as the reference model, and/or requires full-state feedback or observers. To mitigate these stringent requirements, the simple adaptive control (SAC) approach was developed by Sobel et al. [7], Barkana et al. [8] and Barkana and Kaufman [9]. Using the ASP results, the stability of the SAC technique for square LTI systems was rigorously established by Kaufman, Barkana and Sobel [10]. This direct adaptive output feedback method is based upon the command generator tracker methodology [11] and requires the plant to track the reference model which is an ideal representation of the plant only as far as its outputs represent the desired output behavior of the plant. For this reason, this direct model reference adaptive control (MRAC) law has been successfully applied for the control of number of large-scale systems without requiring large-order adaptive controllers. Often, for practical considerations, the SAC algorithm is robustified by adopting Ioannou and Kokotovic's idea [12], [13], where a sigma term, or *forgetting* term, is added to avoid diverge of the time-varying control gains. The stability of the adaptive algorithm with the sigma-term for LTI systems was also shown in [10].

However, although greatly reduced when compared with standard model following techniques, in some applications the SAC technique may still present a design complexity issue arising from the large number of parameters and coefficients to select. In fact, the computation of the control input involves a stabilizing output feedback gain and two feedforward control gains involving their own parameters and coefficients. To mitigate this design complexity, a modified simple adaptive control (MSAC) approach was proposed by Ulrich and de Lafontaine [14] by exploiting the idea that only the stabilizing output feedback gain could absolutely be necessary to guarantee the stability of the closed-loop system. Thus, with MSAC, the feedforward control gains are neglected. However, in [14], no proofs of stability were developed.

Recently, the ASP results for LTI systems were successfully extended to nonlinear and nonstationary systems [15], thus guaranteeing stability of nonstationary control applied to nonlinear systems. In this recent work, the stability and applicability of a reduced SAC method that uses only the integral component of the time-varying control gains was demonstrated when applied to nonlinear ASP systems. In Ulrich et al. [16], to further decrease the computational

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efforts of the MSAC approach and for ease of implementation, a decentralized MSAC (DMSAC) scheme was developed for nonlinear flexible-joint robot control, in which only the diagonal of the time-varying output feedback gain matrix is considered. Indeed, compared to centralized control approaches, the computational efficiency advantage of decentralized control techniques make them attractive for applications in complex dynamical systems, such as multilink robot manipulators. Under certain assumptions, the stability of the DMSAC was also established.

In this paper, a decentralized SAC (DSAC) output feedback control methodology for nonlinear square systems is developed and the Lyapunov proof of stability is established using the ASP theorem recently developed by Barkana [15]. The main original contributions of the present work are as follows. First, we have improved upon the work of Barkana [15] by adding proportional terms to the control gain components to increase the rate of convergence of the system, and by considering only the diagonal elements of the control gain matrices to decrease the number of required operations needed to implement the adaptive algorithm in real time. Second, compared to the DMSAC approach for nonlinear systems proposed by Ulrich et al. [16], the strategy in this paper has the advantages of injecting more knowledge of the reference model in the control structure to improve trajectory tracking performance.

II. SYSTEM AND DEFINITIONS

Consider a class of $m \times m$ nonlinear square systems described by the following formulation

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)u(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n, \qquad u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \in \mathbb{R}^m, \qquad y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \in \mathbb{R}^m$$

are the states, inputs and outputs, respectively. Note that although the nonlinear system $\{A(x,t), B(x,t), C\}$ does not need to be known to design the adaptive control law, sufficient information are required to guarantee that the ASP conditions are satisfied, so that closed-loop stability can be ensured. The following definitions and theorem applicable to the nonlinear square system (1)-(2) will be exploited in the subsequent development. Refer to [15] for more details.

Definition 1. Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1)-(2) is uniformly strictly minimum-phase if its zero dynamics is uniformly stable, or in other words, if there exist two matrices $M(x,t)$ and $N(x,t)$ satisfying the following relations

$$CM = 0 \tag{3}$$

$$NB = 0 \tag{4}$$

$$NM = I_m \tag{5}$$

such that the resulting zero dynamics given by

$$\dot{z} = (\dot{N} + NA)Mz \tag{6}$$

is uniformly asymptotically stable.

Definition 2. Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1)-(2) is strictly passive (SP) if there exists two positive definite symmetric (PDS) matrices $P(x,t)$ and $Q(x,t)$ such that the following two conditions are simultaneously satisfied

$$\dot{P} + PA + A^T P = -Q \tag{7}$$

$$PB = C^T \tag{8}$$

The Lyapunov differential equation (7) shows that an SP system is uniformly asymptotically stable, whereas the second relation (8) shows that

$$B^T PB = B^T C^T = (CB)^T = CB \tag{9}$$

which implies that the product CB is PDS.

As most real-world systems are not inherently SP, a class of almost strictly passive (ASP) systems can be defined through the following definition.

Definition 3. Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1)-(2) is ASP if there exists two PDS matrices $P(x,t)$ and $Q(x,t)$ and a constant output feedback gain $\tilde{K}_e$, such that the closed-loop system

$$\dot{x}(t) = \left[A(x,t) - B(x,t)\tilde{K}_e C\right] x(t) \tag{10}$$

$$y(t) = Cx(t) \tag{11}$$

simultaneously satisfies the following relations

$$\dot{P} + P(A - B\tilde{K}_e C) + (A - B\tilde{K}_e C)^T P = -Q \tag{12}$$

$$PB = C^T \tag{13}$$

Theorem 1. Any uniformly strictly minimum-phase nonlinear system $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1)-(2), and with the product $CB(x,t)$ being PDS, is ASP.

III. CONTROL OBJECTIVE

The control objective is to design a DSAC-based methodology which ensures that the nonlinear square system tracks the output vector $y_m(t)$ of the following (not necessarily square) reference model

$$\dot{x}_m(t) = A_m x_m(t) + B_m u(t) \tag{14}$$

$$y_m(t) = C_m x_m(t) \tag{15}$$

where

$$x_m = \begin{bmatrix} x_{m_1} \\ \vdots \\ x_{m_{n_m}} \end{bmatrix} \in \mathbb{R}^{n_m}, \qquad u_m = \begin{bmatrix} u_{m_1} \\ \vdots \\ u_{m_{pm}} \end{bmatrix} \in \mathbb{R}^{p_m},$$

$$y_m = \begin{bmatrix} y_{m_1} \\ \vdots \\ y_{m_m} \end{bmatrix} \in \mathbb{R}^{m}$$

are the reference model states, inputs and outputs, respectively. To quantify this control objective, an output tracking error, denoted by $e_y(t) \in \mathbb{R}^m$, is defined as

$$e_y \triangleq y_m - y \tag{16}$$

When the system tracks the reference model perfectly (i.e. $y_m = y^* = Cx^*$), it moves along a bounded *ideal* state trajectory, denoted by $x^*(t) \in \mathbb{R}^n$. To facilitate the subsequent analysis, a state error, denoted by $e_x(t) \in \mathbb{R}^n$, is defined as

$$e_x \triangleq x^* - x \tag{17}$$

Thus, (16) can be rewritten as

$$e_y = Cx^* - Cx = Ce_x \tag{18}$$

Assumption 1. Both the order and the number of inputs of the reference model, n_m and p_m, are multiples of m, and thus satisfy the following relationships

$$n_m = k_n m \tag{19}$$

$$p_m = k_p m \tag{20}$$

where k_n, $k_p \in \mathbb{R}$ are positive scalars.

IV. DSAC DEVELOPMENT

The standard SAC algorithm is adopted [10]

$$u = K_e(t)e_y + K_x(t)x_m + K_u(t)u_m \tag{21}$$

where $K_e(t) \in \mathbb{R}^{m\times m}$ is the time-varying stabilizing control gain matrix, and $K_x(t) \in \mathbb{R}^{m\times n_m}$ and $K_u(t) \in \mathbb{R}^{m\times m}$ are time-varying feedforward control gain matrices that contribute to maintaining the stability of the controlled system, and to bringing the output tracking error to zero. Each control gain matrix is calculated as the summation of a proportional and an integral component, as follows:

$$K_e(t) = K_{P_e}(t) + K_{I_e}(t) \tag{22}$$

$$K_x(t) = K_{P_x}(t) + K_{I_x}(t) \tag{23}$$

$$K_u(t) = K_{P_u}(t) + K_{I_u}(t) \tag{24}$$

where only the integral adaptive control terms are absolutely necessary to guarantee the stability of the direct adaptive control system. However, also including the proportional adaptive control terms increases the rate of convergence of the adaptive system toward perfect tracking.

A. Adaptation Law

Proposing a DSAC adaptation mechanism, the proportional and the integral components of the stabilizing control gain in (22), $K_{P_e}(t), K_{I_e}(t) \in \mathbb{R}^{m\times m}$, are both updated by the output tracking error, which results in the following adaptive law

$$K_{P_e}(t) = \operatorname{diag}\left\{e_y e_y^T\right\}\Gamma_{P_e} \tag{25}$$

$$\dot{K}_{I_e}(t) = \operatorname{diag}\left\{e_y e_y^T\right\}\Gamma_{I_e} \tag{26}$$

where $\operatorname{diag}\{A\}$ denotes the diagonalization operation on the square matrix $A \in \mathbb{R}^{n\times n}$ whose elements are denoted $a_{i,j}$, as follows

$$\operatorname{diag}\{A\} = \begin{bmatrix} a_{1,1} & 0 & \cdots & 0 \\ 0 & a_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n,n} \end{bmatrix} \tag{27}$$

The components of the feedforward control gain matrices $K_{P_x}(t), K_{I_x}(t) \in \mathbb{R}^{m\times n_m}$ and $K_{P_u}(t), K_{I_u}(t) \in \mathbb{R}^{m\times p_m}$ are updated as follows

$$K_{P_x}(t) = R^T \operatorname{diag}\left\{Re_y x_m^T\right\}\Gamma_{P_x} \tag{28}$$

$$\dot{K}_{I_x}(t) = R^T \operatorname{diag}\left\{Re_y x_m^T\right\}\Gamma_{I_x} \tag{29}$$

$$K_{P_u}(t) = T^T \operatorname{diag}\left\{Te_y u_m^T\right\}\Gamma_{P_u} \tag{30}$$

$$\dot{K}_{I_u}(t) = T^T \operatorname{diag}\left\{Te_y u_m^T\right\}\Gamma_{I_u} \tag{31}$$

with

$$R = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{n_m\times m}, \qquad T = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{p_m\times m} \tag{32}$$

and where $\Gamma_{P_e}, \Gamma_{I_e} \in \mathbb{R}^{m\times m}$, $\Gamma_{P_x}, \Gamma_{I_x} \in \mathbb{R}^{n_m\times n_m}$, and $\Gamma_{P_u}, \Gamma_{I_u} \in \mathbb{R}^{p_m\times p_m}$ are constant diagonal matrices that control the rate of adaptation.

The adaptive algorithm can be rewritten in the following concise form

$$u = K(t)r \tag{33}$$

where $K(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$ and $r(t) \in \mathbb{R}^{m+n_m+p_m}$ are respectively defined as:

$$K(t) \triangleq \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix} = K_P(t) + K_I(t) \tag{34}$$

$$r \triangleq \begin{bmatrix} e_y^T & x_m^T & u_m^T \end{bmatrix}^T \tag{35}$$

With this representation, the total proportional and integral adaptive control gains, denoted by $K_P(t), K_I(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$, are updated as follows:

$$K_P(t) = S^T \operatorname{diag}\left\{Se_y r^T\right\}\Gamma_P \tag{36}$$

$$\dot{K}_I(t) = S^T \operatorname{diag}\left\{Se_y r^T\right\}\Gamma_I \tag{37}$$

where Γ_P, $\Gamma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$, and the scaling matrix S is given by

$$S = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{(m+n_m+p_m)\times m} \tag{38}$$

B. Error Dynamics

The time derivative of (17) is

$$\dot{e}_x = \dot{x}^* - \dot{x} = A^*x^* + B^*u^* - Ax - Bu \tag{39}$$

Adding and subtracting Ax^* to (39), and rearranging gives

$$\dot{e}_x = Ae_x + (A^* - A)x^* + B^*u^* - Bu \tag{40}$$

Adding and subtracting Bu^* to (40), results in

$$\dot{e}_x = Ae_x + (A^* - A)x^* + B(u^* - u) + (B^* - B)u^* \tag{41}$$

Adding and subtracting $B\tilde{K}_e e_y$ to (41), and substituting e_y from (18) in the first term of the right-hand side of (41), yields

$$\begin{aligned} \dot{e}_x \;=\; & \left(A - B\tilde{K}_e C\right)e_x + (A^* - A)x^* + B(u^* - u) \\ & + (B^* - B)u^* + B\tilde{K}_e e_y \end{aligned} \tag{42}$$

Noting that the tracking error e_y along the ideal trajectory is zero, the ideal control input vector u^* is given by

$$u^* = \tilde{K}_x x_m + \tilde{K}_u u_m \tag{43}$$

Thus, substituting (33) and (43) into (42) gives

$$\begin{aligned} \dot{e}_x \;=\; & \left(A - B\tilde{K}_e C\right)e_x + (A^* - A)x^* + B\tilde{K}_x x_m + B\tilde{K}_u u_m \\ & - BK(t)r + (B^* - B)u^* + B\tilde{K}_e e_y \end{aligned} \tag{44}$$

Equation (44) can be rewritten as

$$\begin{aligned} \dot{e}_x \;=\; & \left(A - B\tilde{K}_e C\right)e_x + (A^* - A)x^* + (B^* - B)u^* \\ & - B\left(K(t) - \tilde{K}\right)r \end{aligned} \tag{45}$$

with $\tilde{K} \in \mathbb{R}^{m\times(m+n_m+p_m)}$ defined as

$$\tilde{K} \triangleq \begin{bmatrix} \tilde{K}_e & \tilde{K}_x & \tilde{K}_u \end{bmatrix} \tag{46}$$

Finally, substituting $K(t)$ from (34) yields

$$\begin{aligned} \dot{e}_x \;=\; & \left(A - B\tilde{K}_e C\right)e_x + (A^* - A)x^* + (B^* - B)u^* \\ & - BK_P(t)r - B\left(K_I(t) - \tilde{K}\right)r \end{aligned} \tag{47}$$

Note that $e_x = 0$ is not an equilibrium of (47). However, as demonstrated in the subsequent section, once the adaptive gains ultimately reach ideal values, the error may ultimately vanish.

V. STABILITY ANALYSIS

Theorem 2. The adaptive control law given by (21) with DSAC adaptation mechanism (22)-(31) ensures that all system signals are bounded under closed-loop operation, and results in asymptotic convergence of the state and output tracking errors, in the sense that

$$\left\|e_y\right\| \to 0 \quad \text{and} \quad \left\|e_x\right\| \to 0 \quad \text{as} \quad t \to \infty$$

where $\|\cdot\|$ denotes the standard Euclidean norm of a vector.

Proof. Let $V \in \mathbb{R}$ be a continuously differentiable positive-definite symmetric function given by

$$V = e_x^T P e_x + \operatorname{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right] \tag{48}$$

The time-derivative of (48) is obtained as

$$\begin{aligned} \dot{V} \;=\; & \dot{e}_x^T P e_x + e_x^T \dot{P} e_x + e_x^T P \dot{e}_x \\ & + \operatorname{tr}\left[\dot{K}_I(t)\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right] \\ & + \operatorname{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\dot{K}_I^T(t)\right] \end{aligned} \tag{49}$$

Substituting e_y from (18), $K_P(t)$ from (36), $\dot{K}_I(t)$ from (37) and $\dot{e}_x$ from (47) into (49) gives

$$\begin{aligned} \dot{V} \;=\; & e_x^T\left[\dot{P} + P\left(A - B\tilde{K}_e C\right) + \left(A - B\tilde{K}_e C\right)^T P\right]e_x \\ & - 2e_x^T PBS^T \operatorname{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & - r^T\left[K_I(t) - \tilde{K}\right]^T B^T P e_x - e_x^T PB\left[K_I(t) - \tilde{K}\right]r \\ & + \left[(A^* - A)x^* + (B^* - B)u^*\right]^T P e_x \\ & + e_x^T P\left[(A^* - A)x^* + (B^* - B)u^*\right] \\ & + \operatorname{tr}\left[S^T \operatorname{diag}\left\{SCe_x r^T\right\}\Gamma_I\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right] \\ & + \operatorname{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\Gamma_I \operatorname{diag}\left\{SCe_x r^T\right\}S\right] \end{aligned} \tag{50}$$

Using the ASP conditions (12) and (13), the expression in (50) can be simplified as

$$\begin{aligned}\dot{V} \;=\; & -e_x^T Q e_x \\ & -2e_x^T C^T S^T \mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & -r^T\left[K_I(t)-\tilde{K}\right]^T Ce_x - e_x^T C^T\left[K_I(t)-\tilde{K}\right]r \\ & +\left[(A^*-A)x^*+(B^*-B)u^*\right]^T Pe_x \\ & +e_x^T P\left[(A^*-A)x^*+(B^*-B)u^*\right] \\ & +\mathrm{tr}\left[S^T\mathrm{diag}\left\{SCe_x r^T\right\}\left(K_I(t)-\tilde{K}\right)^T\right] \\ & +\mathrm{tr}\left[\left(K_I(t)-\tilde{K}\right)\mathrm{diag}\left\{SCe_x r^T\right\}S\right] \qquad (51)\end{aligned}$$

Due to the diagonal forms of the results inside the trace functions, the following terms cancel one another

$$\mathrm{tr}\left[S^T\mathrm{diag}\left\{SCe_x r^T\right\}\left(K_I(t)-\tilde{K}\right)^T\right]-r^T\left[K_I(t)-\tilde{K}\right]^T Ce_x=0 \qquad (52)$$

and similarly

$$\mathrm{tr}\left[\left(K_I(t)-\tilde{K}\right)\mathrm{diag}\left\{SCe_x r^T\right\}S\right]-e_x^T C^T\left[K_I(t)-\tilde{K}\right]r=0 \qquad (53)$$

Thus, (51) can be simplified to

$$\begin{aligned}\dot{V} \;=\; & -e_x^T Q e_x - 2e_x^T C^T S^T\mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & +\left[(A^*-A)x^*+(B^*-B)u^*\right]^T Pe_x \\ & +e_x^T P\left[(A^*-A)x^*+(B^*-B)u^*\right] \qquad (54)\end{aligned}$$

Under the condition that the system parameters vary slowly in comparison with the control dynamics, one may assume that ultimately $x \to x^*$, such that

$$A \to A^* \quad \text{and} \quad B \to B^* \qquad (55)$$

In this particular case, (54) can be simplified to:

$$\dot{V}=-e_x^T Q e_x-2e_x^T C^T S^T\mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \qquad (56)$$

The Lyapunov derivative $\dot{V}$ in (56) is uniformly negative definite with respect to e_x, but only negative semidefinite with respect to the entire state space $[e_x, K_I(t)]$. A direct result of the Lyapunov stability theory is that all dynamic values are bounded. Also, according to LaSalle's invariance principle for nonautonomous systems [10], [17], [18], all states and adaptive gains are bounded, and all system trajectories ultimately end in the domain defined by $\dot{V}\equiv 0$. Since $\dot{V}$ is negative definite in e_x, it implies that the system ultimately ends with $e_x \equiv 0$, which proves that the adaptive control system demonstrates asymptotic convergence of the states and output errors, and boundedness of the adaptive gains.

Remark 1. Compared to the stability results obtained by Barkana [15], the additional negative term in (54) introduced by considering $K_P(t)$ in the DSAC algorithm contributes to the negativity of the Lyapunov derivative function and thus improves the rate of asymptotic convergence of the states and output tracking errors.

Remark 2. The well-known σ terms [12], [13], which adds damping to the adaptation law, can be included. These forgetting terms are introduced to ensure that the integral adaptive control gains remain bounded in cases where the tracking error would not reach zero. With this adjustment, the time-varying integral control gains are obtained as follows

$$\dot{K}_{I_e}(t)=\mathrm{diag}\left\{e_y e_y^T\right\}\Gamma_{I_e}-\sigma_e K_{I_e}(t) \qquad (57)$$

$$\dot{K}_{I_x}(t)=R^T\left(\mathrm{diag}\left\{Re_y x_m^T\right\}\Gamma_{I_x}-\mathrm{diag}\left\{\sigma_x RK_{I_x}(t)\right\}\right) \qquad (58)$$

$$\dot{K}_{I_u}(t)=T^T\left(\mathrm{diag}\left\{Te_y u_m^T\right\}\Gamma_{I_u}-\mathrm{diag}\left\{\sigma_u TK_{I_u}(t)\right\}\right) \qquad (59)$$

and similarly,

$$\dot{K}_I(t)=S^T\left(\mathrm{diag}\left\{Se_y r^T\right\}\Gamma_I-\mathrm{diag}\left\{\sigma_I SK_I\right\}\right) \qquad (60)$$

where $\sigma_e \in \mathbb{R}^{(m\times m)}$, $\sigma_x \in \mathbb{R}^{(n_m\times n_m)}$, $\sigma_u \in \mathbb{R}^{(p_m\times p_m)}$, and $\sigma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$ denote the forgetting coefficient matrices. With this modification to the DSAC algorithm, the Lyapunov derivative function becomes

$$\begin{aligned}\dot{V} \;=\; & -e_x^T Q e_x-2e_x^T C^T S^T\mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & -2\mathrm{tr}\left[S^T\mathrm{diag}\left\{\sigma_I SK_I(t)\right\}\Gamma_I^{-1}\left(K_I(t)-\tilde{K}\right)^T\right] \qquad (61)\end{aligned}$$

Thus, according to Lyapunov-Lasalle theorem, the application of the DSAC algorithm with the forgetting terms results in bounded error tracking. Note that, although it affects the proof of stability, the use of the DSAC control law with this adjustment is preferable in most applications. Indeed, without the forgetting terms the integral adaptive gains are allowed to increase for as long as there is a tracking error. When the integral gains reach certain values, they have a stabilizing effect on the system and the tracking error begins to decrease. However, if the tracking error does not reach zero for some reasons, the integral gains will continue to increase and eventually diverge. On the other hand, with the forgetting terms the integral gains increase as required (e.g. due to large tracking errors), and decrease when large gains are no longer necessary. In fact, with the forgetting terms, the integral gains are obtained as a first-order filtering of the tracking errors, and cannot diverge unless the tracking errors diverge.

Remark 3. In the general case given by (54), it can be shown that the term

$$(A^*-A)x^*+(B^*-B)u^* \qquad (62)$$

is bounded. Nevertheless, (62) affects the proof of stability, and the tracking errors converge to the final magnitude of (62). However, it is clear that the Lyapunov derivative (54) is negative semidefinite for large e_x, which guarantees that the system is stable with respect to boundedness.

□

VI. APPLICATION EXAMPLE

Consider a class of second-order Euler-Lagrange systems that are used to model various engineering systems including space robots and manipulators whose nonlinear planar dynamics is expressed in the operational space, as follows:

$$\Lambda(q)\ddot{x}_r(t) + \Pi(q,\dot{q})\dot{x}_r(t) = F(t) \tag{63}$$

where $x_r(t) \in \mathbb{R}^2$ denotes the end-effector Cartesian position, and where $\Lambda(q), \Pi(q,\dot{q}) \in \mathbb{R}^{2\times 2}$ and $F(t) \in \mathbb{R}^2$ denote the PDS pseudo-inertia matrix, the centripetal-Coriolis matrix in task space, and the control force vector, which are respectively defined as

$$\Lambda(q) = J^{-T}(q)M(q)J^{-1}(q) \tag{64}$$

$$\Pi(q,\dot{q}) = J^{-T}(q)C(q,\dot{q})J^{-1}(q) + \Lambda(q)J(q)\dot{J}^{-1}(q) \tag{65}$$

$$F(t) = J^{-T}(q)\tau(t) \tag{66}$$

where $\dot{J}^{-1}(q)$ is defined as

$$\dot{J}^{-1}(q) = \frac{\mathrm{d}}{\mathrm{d}t}\left\{J^{-1}(q)\right\} \tag{67}$$

The nonlinear system dynamics given by (63) can be expressed in a standard state-space representation with

$$A(q,\dot{q}) = \begin{bmatrix} 0 & I_2 \\ 0 & -\Lambda^{-1}(q)\Pi(q,\dot{q}) \end{bmatrix} \qquad B(q) = \begin{bmatrix} 0 \\ \Lambda^{-1}(q) \end{bmatrix} \tag{68}$$

Defining the scaled-position-plus-velocity output matrix as

$$C \triangleq \begin{bmatrix} \alpha I_2 & I_2 \end{bmatrix} \tag{69}$$

where $\alpha \in \mathbb{R}$ is a known scaling factor related to the sensors, and the state vector is given by

$$x = \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix} \tag{70}$$

It is easy to see the product CB is PDS, as follows

$$CB(q) = \begin{bmatrix} \alpha I_2 & I_2 \end{bmatrix} \begin{bmatrix} 0 \\ \Lambda^{-1}(q) \end{bmatrix} = \Lambda^{-1}(q) > 0 \tag{71}$$

Moreover, a simple selection of matrices that satisfies (3)-(5) is

$$M = \begin{bmatrix} I_2 \\ -\alpha I_2 \end{bmatrix} \qquad N = \begin{bmatrix} I_2 & 0 \end{bmatrix} \tag{72}$$

Computing

$$A_z = NA(q,\dot{q})M = -I_2 \tag{73}$$

and thus

$$\dot{z} = A_z z = -z \tag{74}$$

which shows that the zero dynamics is stable and the nonlinear dynamics is minimum-phase. This demonstrates that a two-link rigid-joint manipulator system is ASP. Thus, the application of the DSAC control methodology presented in this paper would result in a stable closed-loop system.

VII. CONCLUSIONS

Building upon the recently developed almost strictly passive conditions for nonlinear and stationary square systems, a new class of adaptive output feedback control methodology, refered to as decentralized simple adaptive control (DSAC), is developed. The decentralized control law uses proportional and integral components, and considers only the diagonal elements of the control gain matrices. Although the adaptive controller can be designed to render an asymptotically stable system under specific conditions, bounded error tracking is guaranteed in practical situations, by including forgetting terms in the adaptation law.

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