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# Decentralized simple adaptive control of nonlinear systems

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SUMMARY

Recently, the passivity results for linear time-invariant systems were successfully extended to nonlinear and nonstationary systems, thus guaranteeing stability of adaptive control of nonlinear square systems. Based on this theoretical development, this paper presents the development of a new class of direct adaptive controllers, which employ a new decentralized adaptation law mechanism that is developed from the simple adaptive control technique. The resulting direct adaptive control methodology is referred to as decentralized simple adaptive control. A simplification of this new control algorithm, referred to as decentralized modified simple adaptive control, is also presented. In addition, it is shown that both control methodologies can be modified to avoid divergence in practical situations, where the trajectory tracking errors cannot reach zero. Using Lyapunov direct method and Lasalle's invariance principle for nonautonomous systems, the formal proof of stability is established. As well, a numerical simulation study for a trajectory tracking problem by a rigid-joint manipulator is presented to illustrate the new adaptive control approaches. Copyright © 2013 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

A well-known result in control theory that plays an important role in guaranteeing stability in adaptive control is the notion of passivity, which requires the plant be strictly passive (SP). For linear time-invariant (LTI) systems, this stability condition is equivalent to requiring the input–output transfer function be strictly positive real. However, as most real-world systems are not inherently SP, it is known that this condition can be mitigated for LTI systems for which any constant output feedback gain (unknown and not needed for implementation) could render the (fictitious) closed-loop system SP. Such systems that are only separated from strict passivity by a constant output feedback have been called almost strictly passive (ASP) and their transfer function almost strictly positive real [1]. Many works have attempted to clearly define what classes of systems satisfy the ASP conditions. Although some early results and proofs had been obtained in the Russian literature for both SIMO and MIMO systems (see translations in [2, 3]), these basic conditions, despite having been recalled in [4–6], have remained unknown to the Western literature. Since then, many Western works have independently reformulated the ASP conditions [7, 8]. See introductions by Fradkov [9] and Barkana [10] for a complete historical survey on the subject. It is now well known that the ASP conditions required in order to guarantee stability with adaptive control are equivalent to requiring that a square LTI system with state-space realization $\{A, B, C\}$ be a minimum phase and the product

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$CB$ be a positive definite symmetric (PDS). A recent algebraic proof of this important statement can be found in [10].

Over the years, a variety of direct adaptive control laws have been developed to address the problem of time varying the gains of a controller so that the plant closed-loop characteristics match those defined by a reference model. However, most of the research in this area is based on the assumption that prior knowledge of the unknown plant to be controlled is available, and/or requires the plant to be of the same order as the reference model, and/or requires full-state feedback or observers. To mitigate these stringent requirements, the simple adaptive control (SAC) approach was developed by Sobel *et al.* [11], Barkana *et al.* [12] and Barkana and Kaufman [13]. Using the ASP results, the stability of the SAC technique for square LTI systems was rigorously established by Kaufman, Barkana and Sobel [14]. This direct adaptive output feedback method is based upon the command generator tracker methodology [15] and requires the plant to track the ideal model, which is an ideal representation of the plant only as far as its outputs represent the desired output behavior of the plant. For this reason, this direct adaptive control methodology has been successfully applied for the control of number of large-scale systems without requiring large-order adaptive controllers.

However, although greatly reduced when compared with standard model following techniques, in some applications the SAC technique may still present a design complexity issue arising from the large number of parameters and coefficients to select. In fact, the calculation of the control input involves a stabilizing output feedback control gain and two feedforward control gains, each calculated as the summation of a proportional and an integral control gain component. To mitigate this design complexity, the modified simple adaptive control (MSAC) idea proposed by Ulrich and de Lafontaine [16] exploited the concept that only the stabilizing output feedback gain is absolutely necessary to guarantee the stability of the closed-loop system. In other words, with MSAC, the feedforward control gains are ignored.

In spite of successful implementations of SAC/MSAC, the conditions required to guarantee the stability of these adaptive algorithms for nonlinear systems remained unclear for a long time. This is why, until recently, the SAC/MSAC adaptive algorithms were designed ad hoc and validated by simulations. Then, recently, the almost strictly positive real results for LTI systems were successfully extended to nonlinear and nonstationary systems by Barkana [17], which ensured the stability of nonstationary control applied to nonlinear square systems. In addition, this work demonstrated the stability and applicability of a reduced SAC method that used only the integral component of the time-varying control gains.

Based on this theoretical breakthrough, this paper presents the theoretical development of two novel SAC-based control schemes for nonlinear systems: (1) a decentralized simple adaptive control (DSAC) methodology and (2) a decentralized modified simple adaptive control (DMSAC) methodology. In these two control techniques, only the diagonal of the time-varying gain matrices are considered. This way, the computational requirements of the new approaches are decreased in comparison with SAC/MSAC, thus facilitating real-time implementation. Compared with centralized control approaches, the computational efficiency advantage of decentralized control techniques make them attractive for applications in complex dynamical systems, such as nonlinear multilink space robot manipulators. The stability of the developed approach for general nonlinear square ASP systems is also formally established in the sense of Lyapunov. In addition, an illustrative example to compare the tracking performance of the DSAC and DMSAC laws for robot manipulator systems is provided.

## 2. SYSTEM AND DEFINITIONS

Consider a class of $m \times m$ nonlinear square systems described by the following formulation:

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)u(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$



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where

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n, \qquad u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \in \mathbb{R}^m, \qquad y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \in \mathbb{R}^m$$

are the states, inputs and outputs, respectively. Note that although the nonlinear system $\{A(x,t), B(x,t), C\}$ does not need to be known to design the adaptive control law, sufficient informations are required to guarantee that the ASP conditions are satisfied, so that closed-loop stability can be ensured. The following definitions and theorem applicable to the nonlinear square system (1) and (2) will be exploited in the subsequent development. Refer to the study of Barkana [17] for more details.

*Definition 1*
Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1) and (2) is uniformly strictly minimum-phase if its zero dynamics is uniformly stable or in other words, if there exist two matrices $M(x,t) \in \mathbb{R}^{n\times(n-m)}$ and $N(x,t) \in \mathbb{R}^{(n-m)\times n}$ satisfying the following relations:

$$CM = 0 \tag{3}$$

$$NB = 0 \tag{4}$$

$$NM = I_{n-m} \tag{5}$$

such that the resulting zero dynamics given by

$$\dot{z} = (\dot{N} + NA)Mz \tag{6}$$

is uniformly asymptotically stable.

*Definition 2*
Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1) and (2) is SP if there exist two PDS matrices $P(x,t)$ and $Q(x,t)$ such that the following two conditions are simultaneously satisfied:

$$\dot{P} + PA + A^T P = -Q \tag{7}$$

$$PB = C^T \tag{8}$$

where $\dot{P}$ denotes the total derivative of $P(x,t)$ with respect to $t$, when $x$ depends on $t$. The Lyapunov differential equation (7) shows that an SP system is uniformly asymptotically stable, whereas the second relation (8) shows that

$$B^T PB = B^T C^T = (CB)^T = CB \tag{9}$$

which implies that the product $CB$ is PDS.

As most real-world systems are not inherently SP, a class of ASP systems can be defined through the following definition.

*Definition 3*
Any nonlinear systems $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1) and (2) is ASP if there exist two PDS matrices $P(x,t)$ and $Q(x,t)$ and a constant output feedback gain $\tilde{K}_e$, such that the closed-loop system

$$\dot{x}(t) = \left[A(x,t) - B(x,t)\tilde{K}_e C\right] x(t) \tag{10}$$



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$$y(t) = Cx(t) \tag{11}$$

simultaneously satisfies the following relations:

$$\dot{P} + P\left(A - B\tilde{K}_e C\right) + \left(A - B\tilde{K}_e C\right)^T P = -Q \tag{12}$$

$$PB = C^T. \tag{13}$$

*Theorem 1*
Any uniformly strictly minimum-phase nonlinear system $\{A(x,t), B(x,t), C\}$ with the square state-space realization (1) and (2), and with the product $CB(x,t)$ being PDS is ASP.

*Proof*
See [17].

## 3. CONTROL OBJECTIVE

The control objective is to design decentralized SAC-based control laws, which ensure that the nonlinear square system tracks the output vector $y_m(t)$ of the following (not necessarily square) ideal model:

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{14}$$

$$y_m(t) = C_m x_m(t) \tag{15}$$

where

$$x_m = \begin{bmatrix} x_{m_1} \\ \vdots \\ x_{m_{n_m}} \end{bmatrix} \in \mathbb{R}^{n_m}, \quad u_m = \begin{bmatrix} u_{m_1} \\ \vdots \\ u_{m_{p_m}} \end{bmatrix} \in \mathbb{R}^{p_m}, \quad y_m = \begin{bmatrix} y_{m_1} \\ \vdots \\ y_{m_m} \end{bmatrix} \in \mathbb{R}^{m}$$

are the ideal model states, inputs and outputs, respectively. To quantify this control objective, an output tracking error, denoted by $e_y(t) \in \mathbb{R}^m$, is defined as

$$e_y \triangleq y_m - y. \tag{16}$$

When the system tracks the ideal model perfectly (i.e., $y_m = y^* = Cx^*$), it moves along a bounded *ideal* state trajectory, denoted by $x^*(t) \in \mathbb{R}^n$. In other words, the ideal plant

$$\dot{x}^*(t) = A^* x^*(t) + B^* u^*(t) \tag{17}$$

moves along $x^*(t)$, where $A^* \equiv A(x^*, t)$ and $B^* \equiv B(x^*, t)$ and where $u^*(t)$ denotes the ideal control input (to be defined later).

To facilitate the subsequent analysis, a state error, denoted by $e_x(t) \in \mathbb{R}^n$, is defined as

$$e_x \triangleq x^* - x. \tag{18}$$

Thus, (16) can be rewritten as

$$e_y = Cx^* - Cx = Ce_x. \tag{19}$$

□

*Assumption 1*
Both the order and the number of inputs of the ideal model, $n_m$ and $p_m$, are multiples of $m$ and thus, satisfy the following relationships:

$$n_m = k_n m \tag{20}$$

$$p_m = k_p m \tag{21}$$

where $k_n, k_p \in \mathbb{R}$ are positive integer scalars.

## 4. DECENTRALIZED SIMPLE ADAPTIVE CONTROL-BASED LAWS

The standard SAC algorithm is adopted [14]:

$$u = K_e(t)e_y + K_x(t)x_m + K_u(t)u_m \tag{22}$$

where $K_e(t) \in \mathbb{R}^{m\times m}$ is the time-varying stabilizing control gain matrix, and $K_x(t) \in \mathbb{R}^{m\times n_m}$ and $K_u(t) \in \mathbb{R}^{m\times m}$ are time-varying feedforward control gain matrices that contribute to maintaining the stability of the controlled system and to bringing the output tracking error to zero. Each control gain matrix is calculated as the summation of a proportional and an integral component, as follows:

$$K_e(t) = K_{P_e}(t) + K_{I_e}(t) \tag{23}$$

$$K_x(t) = K_{P_x}(t) + K_{I_x}(t) \tag{24}$$

$$K_u(t) = K_{P_u}(t) + K_{I_u}(t) \tag{25}$$

where only the integral adaptive control terms are absolutely necessary to guarantee the stability of the direct adaptive control system. However, also including the proportional adaptive control terms increase the rate of convergence of the adaptive system toward perfect tracking, as it will be demonstrated in Section 4.2.

Proposing a DSAC adaptation mechanism, the proportional and the integral components of the stabilizing control gain in (23), $K_{P_e}(t), K_{I_e}(t) \in \mathbb{R}^{m\times m}$, are both updated by the output tracking error, which results in the following decentralized adaptation law:

$$K_{P_e}(t) = \operatorname{diag}\left\{e_y e_y^T\right\}\Gamma_{P_e} \tag{26}$$

$$\dot{K}_{I_e}(t) = \operatorname{diag}\left\{e_y e_y^T\right\}\Gamma_{I_e} \tag{27}$$

where diag$\{A\}$ denotes the diagonalization operation on the square matrix $A \in \mathbb{R}^{n\times n}$ whose elements are denoted $a_{i,j}$, as follows:

$$\operatorname{diag}\{A\} = \begin{bmatrix} a_{1,1} & 0 & \cdots & 0 \\ 0 & a_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n,n} \end{bmatrix}. \tag{28}$$

The components of the feedforward control gain matrices $K_{P_x}(t), K_{I_x}(t) \in \mathbb{R}^{m\times n_m}$ and $K_{P_u}(t), K_{I_u}(t) \in \mathbb{R}^{m\times p_m}$ are updated as follows:

$$K_{P_x}(t) = R^T \operatorname{diag}\left\{R e_y x_m^T\right\}\Gamma_{P_x} \tag{29}$$

$$\dot{K}_{I_x}(t) = R^T \operatorname{diag}\left\{R e_y x_m^T\right\}\Gamma_{I_x} \tag{30}$$

$$K_{P_u}(t) = T^T \operatorname{diag}\left\{T e_y u_m^T\right\}\Gamma_{P_u} \tag{31}$$

$$\dot{K}_{I_u}(t) = T^T \operatorname{diag}\left\{T e_y u_m^T\right\}\Gamma_{I_u} \tag{32}$$

with

$$R = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{n_m\times m}, \qquad T = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{p_m\times m} \tag{33}$$

and where $\Gamma_{P_e}, \Gamma_{I_e} \in \mathbb{R}^{m\times m}$, $\Gamma_{P_x}, \Gamma_{I_x} \in \mathbb{R}^{n_m\times n_m}$ and $\Gamma_{P_u}, \Gamma_{I_u} \in \mathbb{R}^{p_m\times p_m}$ are positive definite diagonal matrices that control the rate of adaptation.

The adaptive algorithm can be rewritten in the following concise form:

$$u = K(t)r(t) \tag{34}$$

where $K(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$ and $r(t) \in \mathbb{R}^{m+n_m+p_m}$ are respectively defined as

$$K(t) \triangleq \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix} = K_P(t) + K_I(t) \tag{35}$$

$$r \triangleq \begin{bmatrix} e_y^T & x_m^T & u_m^T \end{bmatrix}^T. \tag{36}$$

With this representation, the total proportional and integral adaptive control gains, denoted by $K_P(t), K_I(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$, are updated as follows:

$$K_P(t) = S^T \operatorname{diag}\left\{S e_y r^T\right\} \Gamma_P \tag{37}$$

$$\dot{K}_I(t) = S^T \operatorname{diag}\left\{S e_y r^T\right\} \Gamma_I \tag{38}$$

where $\Gamma_P, \Gamma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$, and the scaling matrix $S$ is given by

$$S = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{(m+n_m+p_m)\times m}. \tag{39}$$

*4.1. Error dynamics*

The time derivative of (18) is

$$\dot{e}_x = \dot{x}^* - \dot{x} = A^*x^* + B^*u^* - Ax - Bu. \tag{40}$$

By adding and subtracting $Ax^*$ to (40) and rearranging give

$$\dot{e}_x = Ae_x + (A^* - A)x^* + B^*u^* - Bu. \tag{41}$$

By adding and subtracting $Bu^*$ to (41) results in

$$\dot{e}_x = Ae_x + (A^* - A)x^* + B(u^* - u) + (B^* - B)u^*. \tag{42}$$

By adding and subtracting $B\tilde{K}_e e_y$ to (42) and substituting $e_y$ from (19) in the first term of the right-hand side of (42) yield

$$\dot{e}_x = \left(A - B\tilde{K}_e C\right) e_x + (A^* - A)x^* + B(u^* - u) + (B^* - B)u^* + B\tilde{K}_e e_y. \tag{43}$$

Noting that the tracking error $e_y$ along the ideal trajectory is zero, the underlying tracking problem assumes that there exists an ideal control input

$$u^*(t) = \tilde{K}_x x_m(t) + \tilde{K}_u u_m(t) \tag{44}$$

that can keep the plant along an ideal trajectory $x^*(t)$ that would asymptotically perform perfect tracking. Thus, by substituting (34) and (44) into (43) gives

$$\dot{e}_x = \left(A - B\tilde{K}_e C\right) e_x + (A^* - A)x^* + B\tilde{K}_x x_m + B\tilde{K}_u u_m - BK(t)r + (B^* - B)u^* + B\tilde{K}_e e_y. \tag{45}$$

Equation(45) can be rewritten as

$$\dot{e}_x = \left(A - B\tilde{K}_e C\right) e_x + (A^* - A)x^* + (B^* - B)u^* - B\left(K(t) - \tilde{K}\right) r \tag{46}$$

with $\tilde{K} \in \mathbb{R}^{m\times(m+n_m+p_m)}$ defined as

$$\tilde{K} \triangleq \begin{bmatrix} \tilde{K}_e & \tilde{K}_x & \tilde{K}_u \end{bmatrix}. \tag{47}$$

Finally, substituting $K(t)$ from (35) yields

$$\dot{e}_x = \left(A - B\tilde{K}_e C\right) e_x + (A^* - A)x^* + (B^* - B)u^* - BK_P(t)r - B\left(K_I(t) - \tilde{K}\right) r. \tag{48}$$



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*4.2. Stability analysis*

The proof of stability must considers the adaptive system defined by both (38) and (48), in order to show the asymptotic convergence of the errors and that the adaptive control gains are bounded. This is demonstrated through the following theorem.

*Theorem 2*

When applied to a square nonlinear system satisfying the ASP conditions given by (12) and (13) and under the assumptions that the parameters vary slowly in comparison with the control dynamics, such that $A^* - A$ and $B^* - B$ are close to zero, and that the ideal model satisfies Assumption 1, the adaptive control law given by (22) with DSAC adaptation mechanism (23)–(27) and (29)–(32) ensures that all adaptive control gains are bounded under closed-loop operation, and results in asymptotic convergence of the state and output tracking errors, in the sense that

$$\|e_y\| \to 0 \text{ and } \|e_x\| \to 0 \text{ as } t \to \infty$$

where $\|\cdot\|$ denotes the standard Euclidean norm of a vector.

*Proof*

Let $V \in \mathbb{R}$ be a continuously differentiable positive-definite symmetric function given by

$$V = e_x^T P e_x + \mathrm{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right]. \tag{49}$$

The time derivative of (49) is obtained as

$$\begin{aligned}\dot{V} = {} & \dot{e}_x^T P e_x + e_x^T \dot{P} e_x + e_x^T P \dot{e}_x + \mathrm{tr}\left[\dot{K}_I(t)\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right] \\ & + \mathrm{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\dot{K}_I^T(t)\right].\end{aligned} \tag{50}$$

By substituting $e_y$ from (19), $K_P(t)$ from (37), $\dot{K}_I(t)$ from (38) and $\dot{e}_x$ from (48) into (50) gives

$$\begin{aligned}\dot{V} = {} & e_x^T\left[\dot{P} + P\left(A - B\tilde{K}_e C\right) + \left(A - B\tilde{K}_e C\right)^T P\right]e_x \\ & - 2e_x^T PBS^T \mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & - r^T\left[K_I(t) - \tilde{K}\right]^T B^T P e_x - e_x^T PB\left[K_I(t) - \tilde{K}\right]r \\ & + \left[\left(A^* - A\right)x^* + \left(B^* - B\right)u^*\right]^T P e_x + e_x^T P\left[\left(A^* - A\right)x^* + \left(B^* - B\right)u^*\right] \\ & + \mathrm{tr}\left[S^T \mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_I\Gamma_I^{-1}\left(K_I(t) - \tilde{K}\right)^T\right] \\ & + \mathrm{tr}\left[\left(K_I(t) - \tilde{K}\right)\Gamma_I^{-1}\Gamma_I \mathrm{diag}\left\{SCe_x r^T\right\}S\right].\end{aligned} \tag{51}$$

Using the ASP conditions (12) and (13), the expression in (51) can be simplified as

$$\begin{aligned}\dot{V} = {} & -e_x^T Q e_x \\ & - 2e_x^T C^T S^T \mathrm{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ & - r^T\left[K_I(t) - \tilde{K}\right]^T Ce_x - e_x^T C^T\left[K_I(t) - \tilde{K}\right]r \\ & + \left[\left(A^* - A\right)x^* + \left(B^* - B\right)u^*\right]^T P e_x + e_x^T P\left[\left(A^* - A\right)x^* + \left(B^* - B\right)u^*\right] \\ & + \mathrm{tr}\left[S^T \mathrm{diag}\left\{SCe_x r^T\right\}\left(K_I(t) - \tilde{K}\right)^T\right] \\ & + \mathrm{tr}\left[\left(K_I(t) - \tilde{K}\right)\mathrm{diag}\left\{SCe_x r^T\right\}S\right].\end{aligned} \tag{52}$$

Due to the diagonal forms of the results inside the trace functions, the following terms cancel one another:

$$\mathrm{tr}\left[S^T \mathrm{diag}\left\{SCe_x r^T\right\}\left(K_I(t) - \tilde{K}\right)^T\right] - r^T\left[K_I(t) - \tilde{K}\right]^T Ce_x = 0 \tag{53}$$



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and similarly

$$\operatorname{tr}\left[\left(K_I(t)-\tilde{K}\right)\operatorname{diag}\left\{SCe_xr^T\right\}S\right]-e_x^TC^T\left[K_I(t)-\tilde{K}\right]r=0. \tag{54}$$

Thus, (52) can be simplified to

$$\begin{aligned}\dot{V}=&-e_x^TQe_x-2e_x^TC^TS^T\operatorname{diag}\left\{SCe_xr^T\right\}\Gamma_Pr\\&+[(A^*-A)x^*+(B^*-B)u^*]^TPe_x\\&+e_x^TP[(A^*-A)x^*+(B^*-B)u^*].\end{aligned} \tag{55}$$

Under the condition that the system parameters vary slowly in comparison with the control dynamics, one may assume that ultimately $A^*-A$ and $B^*-B$ are close to zero. In this particular case, (55) can be approximated by

$$\dot{V}=-e_x^TQe_x-2e_x^TC^TS^T\operatorname{diag}\left\{SCe_xr^T\right\}\Gamma_Pr. \tag{56}$$

The Lyapunov derivative $\dot{V}$ in (56) is uniformly negative definite with respect to $e_x$, but only negative semidefinite with respect to the entire state space $[e_x, K_I(t)]$. Stability of the adaptive system is therefore guanranteed from Lyapunov stability theory, and all state errors (and output errors), as well as adaptive control gains are bounded.

Furthermore, LaSalle's invariance principle for nonautonomous systems [14, 18–21] can be used to demonstrate the asymptotic stability of the tracking errors. As demonstrated in [14, p. 43], for a negative semidefinite Lyapunov derivative of the form (56), all system trajectories are contained within the domain $\Omega_0=\left\{[e_x,K_I(t)]\,|\,V([e_x,K_I(t)],t)\le V\left(\left[e_{x0},K_{I_0}(t)\right],0\right)\right\}$ (where the subscript $\{\}_0$ denotes the initial condition), and the entire state space $[e_x, K_I(t)]$ ultimately reaches the domain $\Omega_f=\Omega_0\cap\Omega$, where $\Omega$ denotes the domain defined by the Lyapunov derivative identical to zero. In other words, the state space $[e_x, K_I(t)]$ ultimately reaches the domain defined by $V([e_x,K_I(t)],t)\equiv 0$. Because $\dot{V}([e_x,K_I(t)],t)$ is negative definite in $e_x$, the system ends with $e_x\equiv 0$. Finally, because $e_x\equiv 0$ implies $e_x(t)=e_y(t)=0$, asymptotic stability of the state and output tracking errors is guaranteed.

*Remark 1*

Compared with the stability results obtained by Barkana [17], the additional negative term in (55) introduced by considering $K_P(t)$ in the DSAC algorithm contributes to the negativity of the Lyapunov derivative function and thus, improves the rate of asymptotic convergence of the states and output tracking errors.

*Remark 2*

It is well recognized that to prevent undesirable divergence of the integral time-varying control gains under nonideal conditions, the basic adaptive algorithm must be suitably modified. One possible modification is the so-called $\sigma$-modification, pioneered by the work of Narendra *et al.* [22] that studied the effects of disturbances on stability of conventional model reference adaptive control (MRAC) systems and widely popularized in the work of Ioannou and Kokotovic [23, 24], and then by Barkana and Kaufman [25, 26] and by Narendra and Annaswamy [27]. The first sigma modification for SAC (or implicit MRAC) was proposed by Fradkov [3] and Fomin *et al.* [28]. With this adjustment, the time-varying integral control gains are obtained as follows:

$$\dot{K}_{I_e}(t)=\operatorname{diag}\left\{e_ye_y^T\right\}\Gamma_{I_e}-\sigma_eK_{I_e}(t) \tag{57}$$

$$\dot{K}_{I_x}(t)=R^T\left(\operatorname{diag}\left\{Re_yx_m^T\right\}\Gamma_{I_x}-\operatorname{diag}\left\{\sigma_xRK_{I_x}(t)\right\}\right) \tag{58}$$

$$\dot{K}_{I_u}(t)=T^T\left(\operatorname{diag}\left\{Te_yu_m^T\right\}\Gamma_{I_u}-\operatorname{diag}\left\{\sigma_uTK_{I_u}(t)\right\}\right) \tag{59}$$

and similarly,

$$\dot{K}_I(t)=S^T\left(\operatorname{diag}\left\{Se_yr^T\right\}\Gamma_I-\operatorname{diag}\left\{\sigma_ISK_I\right\}\right) \tag{60}$$

where $\sigma_e \in \mathbb{R}^{(m\times m)}$, $\sigma_x \in \mathbb{R}^{(n_m\times n_m)}$, $\sigma_u \in \mathbb{R}^{(p_m\times p_m)}$ and $\sigma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$ denote the forgetting coefficient matrices. With this modification to the DSAC algorithm, the Lyapunov derivative function becomes

$$\begin{aligned}\dot{V} = &- e_x^T Q e_x - 2e_x^T C^T S^T \text{diag}\left\{SCe_x r^T\right\}\Gamma_P r \\ &- 2\text{tr}\left[S^T \text{diag}\left\{\sigma_I S K_I(t)\right\}\Gamma_I^{-1}\left(K_I(t)-\tilde{K}\right)^T\right]. \end{aligned} \tag{61}$$

Thus, according to Lyapunov–Lasalle theorem, the application of the DSAC algorithm with the forgetting terms results in bounded error tracking. Note that, although it affects the proof of stability, the use of the DSAC control law with this adjustment is preferable in most practical applications. Indeed, without the forgetting terms the integral adaptive gains are allowed to increase for as long as there is a tracking error. When the integral gains reach certain values, they have a stabilizing effect on the system, and the tracking error begins to decrease. However, if the tracking error does not reach zero for some reasons, the integral gains will continue to increase and eventually diverge. On the other hand, with the forgetting terms, the integral gains increase as required (e.g., due to large tracking errors), and decrease when large gains are no longer necessary. In fact, with the forgetting terms, the integral gains are obtained as a first-order filtering of the tracking errors and cannot diverge unless the tracking errors diverge.

*Remark 3*
In the general case given by (55), it can be shown that the term

$$(A^* - A)x^* + (B^* - B)u^* \tag{62}$$

is bounded. Nevertheless, (62) affects the proof of stability, and the tracking errors converge to the final magnitude of (62). However, it is clear that the Lyapunov derivative (55) is negative semidefinite for large $e_x$, which guarantees that the system is stable with respect to boundedness.

*Remark 4*
To further decrease the number of operations required to implement the DSAC controller, a modified version of the algorithm, referred to as the DMSAC law can be developed. The DMSAC algorithm is obtained by retaining only the error-related adaptive gains $K_{P_e}(t)$ and $K_{I_e}(t)$. In fact, as mentioned in [1], only the stabilizing control gain matrix $K_e(t)$ is absolutely required for the stability of the adaptive system. With this modification, the following DMSAC control approach is obtained:

$$u = K_e(t)e_y = \left[K_{P_e}(t) + K_{I_e}(t)\right]e_y \tag{63}$$

where $K_{P_e}(t)$ and $K_{I_e}(t)$ are adapted with the decentralized adaptation law (26) and (27).

□

## 5. APPLICATION EXAMPLE

In this section, the applicability of both adaptive control schemes for a nonlinear Euler–Lagrange system is demonstrated. To facilitate the following demonstration, we let the nonlinear rigid-joint dynamics of a planar manipulator be written in the task space as follows:

$$\Lambda(q)\ddot{x}_r(t) + \Pi(q,\dot{q})\dot{x}_r(t) = F(t) \tag{64}$$

where the actual end-effector position is denoted by $x_r \in \mathbb{R}^2$, and where $\Lambda(q), \Pi(q,\dot{q}) \in \mathbb{R}^{2\times 2}$ and $F(t) \in \mathbb{R}^2$ denote the PDS pseudo-inertia matrix, the centripetal–Coriolis matrix in task space and the control force vector, which are respectively defined as

$$\Lambda(q) = J^{-T}(q)M(q)J^{-1}(q) \tag{65}$$

$$\Pi(q,\dot{q}) = J^{-T}C(q,\dot{q})J^{-1}(q) + \Lambda(q)J(q)\dot{J}^{-1}(q) \tag{66}$$

$$F(t) = J^{-T}(q)\tau(t) \tag{67}$$

where $\dot{J}^{-1}(q)$ is defined as



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$$\dot{J}^{-1}(q) = \frac{\mathrm{d}}{\mathrm{d}t}\left\{J^{-1}(q)\right\}. \tag{68}$$

The nonlinear system dynamics given by (64) can be expressed in a standard state-space representation with

$$A(q,\dot{q}) = \begin{bmatrix} 0 & I_2 \\ 0 & -\Lambda^{-1}(q)\Pi(q,\dot{q}) \end{bmatrix} \quad B(q) = \begin{bmatrix} 0 \\ \Lambda^{-1}(q) \end{bmatrix}. \tag{69}$$

Defining the scaled-position-plus-velocity output matrix as

$$C \triangleq \begin{bmatrix} \alpha I_2 & I_2 \end{bmatrix}. \tag{70}$$

where $\alpha \in \mathbb{R}$ is a known scaling factor related to the sensors, and the state vector is given by

$$x = \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix}. \tag{71}$$

It is easy to see the product $CB(q)$ is PDS, as follows:

$$CB(q) = \begin{bmatrix} \alpha I_2 & I_2 \end{bmatrix}\begin{bmatrix} 0 \\ \Lambda^{-1}(q) \end{bmatrix} = \Lambda^{-1}(q) > 0. \tag{72}$$

Moreover, a simple selection of matrices that satisfies (3)–(5) is

$$M = \begin{bmatrix} I_2 \\ -\alpha I_2 \end{bmatrix} \quad N = \begin{bmatrix} I_2 & 0 \end{bmatrix}. \tag{73}$$

Computing

$$A_z = NA(q,\dot{q})M = -\alpha I_2 \tag{74}$$

and thus

$$\dot{z} = A_z z = -\alpha z \tag{75}$$

which shows that the zero dynamics is stable, and the nonlinear dynamics is minimum phase. This demonstrates that a two-link rigid-joint manipulator system is ASP.

The ideal model was designed to incorporate the desired input-output plant behavior, and aside from the scaling parameter which is assumed to be known, the ideal model is not based on any modeling of the plant. The matrices $A_m$ and $B_m$ are designed in terms of the ideal damping ratio $\zeta$ and undamped natural frequency $\omega_n$ as follows:

$$A_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\omega_n^2 & 0 & -2\zeta\omega_n & 0 \\ 0 & -\omega_n^2 & 0 & -2\zeta\omega_n \end{bmatrix} \quad B_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_n^2 & 0 & 0 & 0 \\ 0 & -\omega_n^2 & 0 & 0 \end{bmatrix} \tag{76}$$

and the output matrix $C_m$ is defined as

$$C_m \triangleq \begin{bmatrix} \alpha I_2 & I_2 \end{bmatrix}. \tag{77}$$

The following control gains and parameters were used:

$$\Gamma_{P_e} = 15I_2 \quad \Gamma_{I_e} = 30I_2 \quad \Gamma_{P_x} = \Gamma_{P_u} = 10I_4 \quad \Gamma_{I_x} = \Gamma_{I_u} = 15I_4 \quad \sigma_e = 0.018I_2 \quad \sigma_x = \sigma_u = 0.5I_4$$

The integral structure of the integral time-varying gains is computed online via a standard Tustin algorithm. All integral control gains were initialized to zero, and the ideal model parameters were selected as $\zeta = 0.9$, $\omega_n = 10$ rad/s and $\alpha = 2.5$.

Two simulation experiments were conducted, the first without (DMSAC) and the second with (DSAC), the time-varying control gain matrices $K_x(t)$ and $K_u(t)$. Specifically, the DMSAC control law is given by (63) where $K_{P_e}(t)$ and $K_{I_e}(t)$ are adapted with the decentralized adaptation law (26)

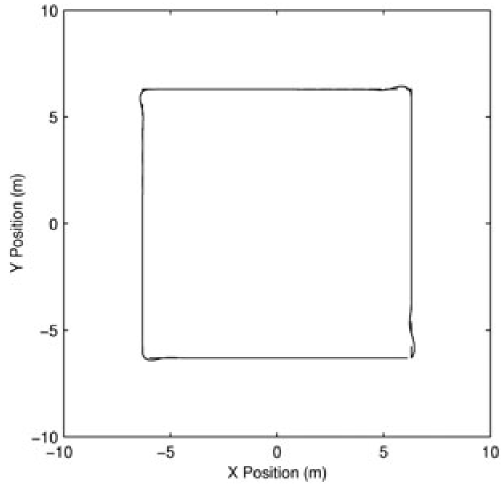


Figure 1. Trajectory tracking results obtained with the decentralized modified simple adaptive control controller. The dashed line corresponds to the desired end-effector position $x_{r_d}(t)$, and the solid line corresponds to the actual end-effector position $x_r(t)$.

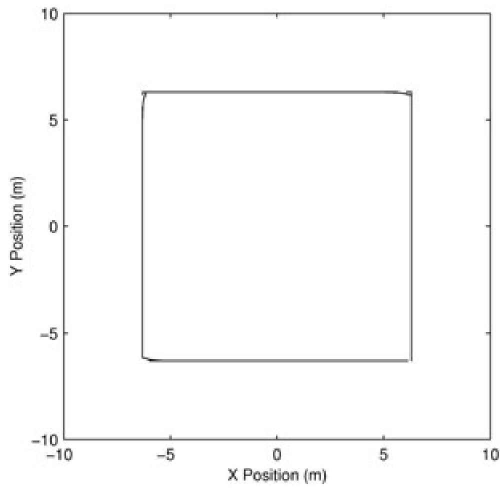


Figure 2. Trajectory tracking results obtained with the decentralized simple adaptive control controller. The dashed line corresponds to the desired end-effector position $x_{r_d}(t)$, and the solid line corresponds to the actual end-effector position $x_r(t)$.

and (27) and the DSAC control law is given by (22) with the control gain adaptation mechanisms (26), (27) and (29)–(32). Note that in this specific example, $K_x(t)$ and $K_u(t)$, can both be defined as a proportional and a derivative component, each of these multiplying its associated position and velocity signals, respectively, that is,

$$K_x(t) = \begin{bmatrix} K_{x_p}(t) & K_{x_d}(t) \end{bmatrix}$$

$$K_u(t) = \begin{bmatrix} K_{u_p}(t) & K_{u_d}(t) \end{bmatrix}.$$

The trajectory tracking results obtained with the DMSAC controller and the DSAC controller are depicted in Figures 1 and 2, respectively. The positioning overshoots achieved with the DMSAC controller are 0.142 m, 0.115 m and 0.101 m for the first, second and third direction change, respectively. Comparison with trajectory tracking results of zero overshoot for the proposed DSAC controller indicates that the DSAC yields improved tracking performance. This is also demonstrated

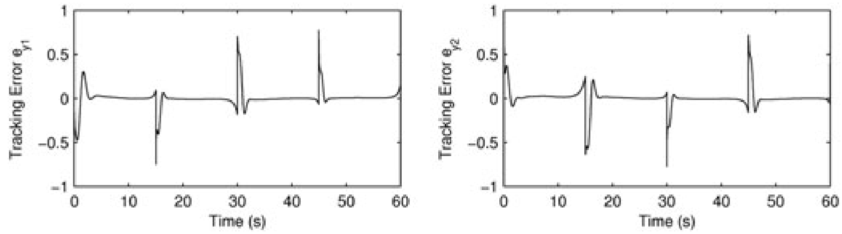


Figure 3. Trajectory tracking errors ($e_y(t) = y_m(t) - y(t)$) obtained with the decentralized modified simple adaptive control controller.

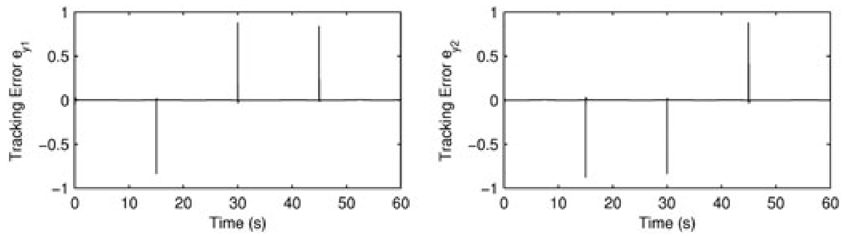


Figure 4. Trajectory tracking errors ($e_y(t) = y_m(t) - y(t)$) obtained with the decentralized simple adaptive control controller.

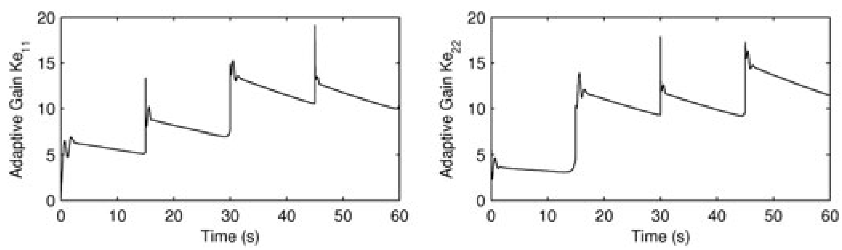


Figure 5. Adaptation history of the decentralized modified simple adaptive control controller gain $K_e(t)$.

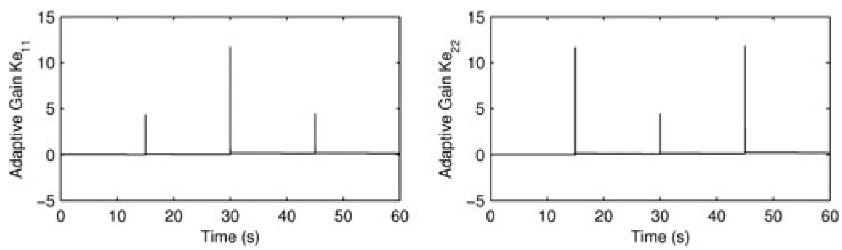


Figure 6. Adaptation history of the decentralized simple adaptive control controller gain $K_e(t)$.

in Figures 3 and 4, where an increased damping of the tracking errors with smaller settling times are obtained with the DSAC strategy. However, this increase in performance comes at the expense of greater complexity in the controller structure. The successive increase in tracking performance along each side of the trajectory for the DMSAC strategy is explained by analyzing the adaptation history of the control gains depicted in Figure 5, which shows that the gains are increasing after each direction change and thereby, providing improved tracking results. In must be noted that this particular behavior is mainly due to the specific control parameters selected herein and that a different behavior could be obtained with different parameters. On the other hand, the time-varying control gains for the DSAC algorithm shown in Figures 6–8 do not exhibit such behavior, thus providing similar tracking performance at each corner of the trajectory.



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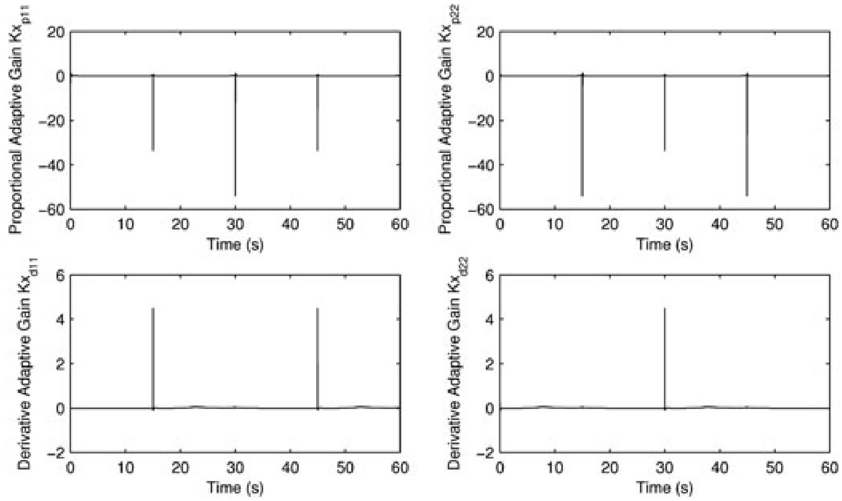


Figure 7. Adaptation history of the decentralized simple adaptive control controller gains $K_{x_p}(t)$ and $K_{x_d}(t)$.

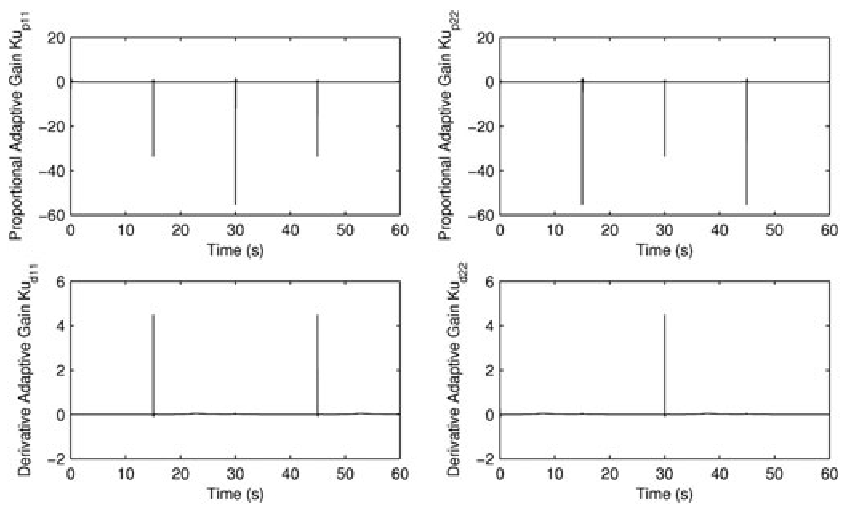


Figure 8. Adaptation history of the decentralized simple adaptive control controller gains $K_{u_p}(t)$ and $K_{u_d}(t)$.

## 6. CONCLUSION

This paper first reviewed the concept of ASP for nonlinear and nonstationary systems, based on which two new control methodologies, DSAC and DMSAC, were proposed. The DMSAC methodology represents a simplification of the DSAC method, in that the feedforward control gain matrices are ignored. Both decentralized adaptive control methodologies consider only the diagonal elements of the control gain matrices. As a result, compared with the existing SAC scheme for nonlinear systems, the number of control parameters is reduced, and the efficiency of the calculations is greatly improved. In addition, the stability analysis revealed that the rate of asymptotic convergence of the states and output tracking errors are improved, compared with the recently-developed reduced SAC control methodology for nonlinear systems. Further investigation was carried out in order to modify both methodologies to avoid divergence of the integral control gains in situations where perfect tracking cannot be achieved. As anticipated, the injection of knowledge about the ideal model in the DSAC control structure leads to improved trajectory tracking results compared with DMSAC, as demonstrated with the simulation results.



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