Nonparametric Identification of Robot Flexible Joint Space Manipulator

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***Abstract*— We consider a flexible joint two-link flexible joint space manipulator. Manipulator dynamics is derived from Euler-Lagrange formulation. The joint dynamics includes non-linear stiffness and friction components. A simplified model of the manipulator is represented by a Hammerstein model consisting of a memoryless nonlinearity followed by a dynamic, linear system. Parametric and nonparametric identification algorithms are proposed for identifying parameters of the linear and non-linear models, respectively from input-output observations of the Hammerstein system. Convergence and the rates of identification algorithm of the static nonlinearity are also discussed.**

I. Introduction

Flexible joint robotic space manipulators have been developed since 1980s. Canada and NASA developed ISS Mobile Servicing System comprising of the Space Station Remote Manipulator (SSRMS), the Special Purpose Dexterous Manipulator (SPDM), and the Mobile Remote Servicer Base System that acts as a movable platform for the SSRMS and SPDM (Salaberger [32]). The Japanese Experiment Module Remote Manipulator System, built by the Japan Aerospace Exploration Agency, is a robotic manipulator system for supporting experiments to be conducted on the Japanese Experiment Module Exposed Facility at the ISS. Another space robotic manipulator is the European Robotic Arm which was developed by the European Space Agency. It consists of an 11 m manipulator with seven degrees of freedom (DOFs) and two booms scheduled to be attached to the Russian segment of the ISS in 2012. Worldwide space robotic activities currently concentrate on the development of lightweight and autonomous robotic manipulators designed specifically for advanced and complex OSS operations. Large vibrations may occur in robotic arm when it is accelerated and stopped making positioning of the tip very difficult. For this reason accurate modeling and efficient control of a flexible joint robotic arm is important. This paper discusses modeling and identification of a two-link space robot manipulator. The rigid-joint dynamics model includes flexible joints modeled by a linear spring model and nonlinear components such as stiffness and friction effects.

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II. Flexible Robot Manipulators

Strict requirements exist for minimal vibration and precise control of long reach flexible robots deployed in spacecraft operations. Banerjee and Singhose [2] obtained excellent square tracking results by using an input shaping, inverse kinematics technique to simulate tracking square trajectories by a two-link flexible robot manipulator, for both linear and nonlinear control laws used to calculate the torque acting at the robot joints. The dynamics equations, derived from a recursive order-n algorithm in which each robot link was discretized into three rigid segments connected by torsion springs, are suitable for large bending deformations. Beres and Sąsiadek [4] formulated a Lagrange finite element dynamic model of an n-link flexible manipulator in which large rotational rigid body motion of the links and their small elastic deformations are coupled. The dynamic model, derived for a two-link flexible manipulator with two finite elements per link, may be extended to an arbitrary n-link manipulator with n-elements per link, as simulation results show but at a great computational time burden. Sąsiadek and Srinivasan [34] use a modal expansion method to model a single-link flexible manipulator with coupled dynamics, treated as a pinned-free cantilever beam with small elastic deformations together with model reference adaptive control (MRAC) that is insensitive to payload variation. Lee, Vukovich and Sąsiadek [28] applied a fuzzy logic approach to tip position control of an experimental flexible single-link manipulator, where, the fuzzy rules have position errors and rates as inputs, with motor hub speed rate as the output variable but without a mathematical model of the system. Green and Sąsiadek [11] studied tracking of a square trajectory by a two-link flexible robot manipulator with, two fuzzy controllers substituted for the nonlinear rigid link dynamics equations. In Green and Sąsiadek [12] the Linear Quadratic Gaussian scheme was generalized to include a fuzzy logic adaptive extended Kalman filter in correcting divergence when the robot was subjected to process and measurements

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noise. In this paper we discuss modeling and identification of two-link flexible robotic arm with nonlinear stiffness and friction components. Similar problem for one-link flexible manipulator was discussed in [27]. Modeling and control of two-link manipulator has been studied in Ulrich and Sąsiadek [37].

III. Robot Manipulator Dynamics

A. Robot Configuration

The two-link flexible robot has two joints: a shoulder joint revolute and an elbow joint with angles of inclination q_1 and q_2. The robot motion and vibration modes are restricted to the x-y plane. Robot parameters and physical constants used in this paper to describe the robot manipulator are taken from [2].

B. Lagrangian Formulation of Rigid Link Robot Dynamics

A conventional closed form of the nonlinear dynamics of a two-link robot manipulator with rigid links may be derived in terms of kinetic and potential energies stored in the system by the Lagrangian formulation [1]. Given an independent set of generalized coordinates, $q_i = 1,\ldots,n$, the total kinetic energy T and potential U stored in the system are given by the Lagrangian:

$$L(q_i,\dot{q}_i) = T - U, \quad i = 1,\ldots,n \tag{1}$$

and the dynamic equations of motion for the system are derived in the form given by:

$$\tau_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i}, \quad i = 1,\ldots,n \tag{2}$$

when subjected to a generalized force, τ_i, acting on a generalized coordinate q_i. For the complete robot ensemble, the kinetic energy for rigid links is given by

$$T_r = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\mathbf{q}}^T M(\mathbf{q})\dot{\mathbf{q}}, \quad i = 1,2, j = 1,2, n = 2 \tag{3}$$

where M_{ij} are elements of the robot rigid inertia matrix M and $\mathbf{q}$ is the link angle vector. Substituting equation (3) into equation (2) and applying the differential operators, we obtain the Lagrangian-Euler form of the rigid two-link robot dynamic equations as:

$$\tau = M(\mathbf{q})\ddot{\mathbf{q}} + C(\dot{\mathbf{q}},\mathbf{q}). \tag{4}$$

In the equation above the gravity terms are omitted for space operations, τ is the actuating torque vector acting on the robot joints, M is the robot inertia matrix and C is a coupling matrix of centrifugal and Coriolis force matrix given by:

$$M = \begin{bmatrix} M_{11} & M12 \\ M_{21} & M_{22} \end{bmatrix} \tag{5}$$

$$C = -m_2 l_1 l_{c2} \sin q_2 \begin{bmatrix} \dot{q}_2 & \dot{q}_1 + \dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix} \tag{6}$$

with

$$M_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2}\cos q_2) + I_1 + I_2$$
$$M_{12} = M_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) + I_2$$
$$M_{22} = m_2 l_{c2}^2 + I_2$$

where $i = 1,2$, m_i denotes the mass of link i, q_i denotes the angular displacement of the revolute joint i, l_i denotes the length of link i, l_{ci} denotes the distance from the previous joint to center of gravity of link i and I_i denotes the moment of inertia of link i about an axis perpendicular to the xy plane passing through the center of gravity of link i.

C. Lagrangian Formulation of Flexible Link Robot Dynamics

In addition to the nonlinearity of rigid-joint dynamics, achieving accurate tracking control of the two-link robot with flexible joints is compounded by the elastic deformations at the joints and the associated vibrations. To enable the design of a suitable control strategy, the robot model must capture the nonlinear flexible dynamics of the robot and the control method must adequately dampen residual joint vibrations. The introduction of the constant joint stiffness matrix to model robot dynamics captures the interaction between joint vibrations and nonlinear multibody dynamics. By further considering time-varying stiffness coefficients, the soft-windup effect and friction torques revealed by experimental studies conducted on flexible-joint mechanisms provide a complete and accurate dynamics model that describes a flexible-joint manipulator system is presented in this section.

1) Linear Joint Model: In the linear-joint dynamics model (Spong [36]) each joint is modeled as a linear torsional spring of constant stiffness and the resulting dynamics of the flexible-joint manipulators consists of two second-order differential equations. Let q_m denote the vector of angular displacements of the motor shaft angles, where the elastic joint vibrations vector is defined as $q - q_m$. The joint flexibility is taken into account by augmenting the kinetic energy of the rigid-joint space robot presented earlier with the kinetic energy of the rotors due to their own rotation and by considering the elastic potential energy of the flexible joints. The kinetic energy of the rotors, T_e, and the elastic potential energy, U_e, are given by

$$T_e = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\mathbf{q}}_m^T J_m(\mathbf{q})\dot{\mathbf{q}}_m, i,j = 1,2, n = 2$$
$$U_e = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(q_i - q_{mi})(q_j - q_{mj})$$
$$= \frac{1}{2}(\mathbf{q} - \mathbf{q}_m)^T k(\mathbf{q} - \mathbf{q}_m), i,j = 1,2, n = 2$$

where J_m denotes the positive definite motor inertia matrix and where the stiffness of the flexible joints is modeled by the k matrix, the diagonal positive definite stiffness matrix of the joints. Combining elastic terms with rigid-dynamics equations, the following dynamics equations of a flexible joint space robot with revolute joints and actuated directly by DC motors are obtained:

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = k(\mathbf{q}_m - \mathbf{q})$$
$$J_m\ddot{\mathbf{q}}_m + k(\mathbf{q}_m - \mathbf{q}) = \tau.$$

2) Nonlinear Joint Model: Since the development of the linear stiffness model by Spong [36] several researchers have conducted experiments to derive a detailed dynamics model of flexible effects in the joints of robotic manipulators. As nonlinear effects become prevalent in the system dynamics, their accurate modeling is key to the successful design of advanced flexiblejoint control laws. The most relevant nonlinear effects are related to nonlinear stiffness. According to the literature, to replicate experimental joint stiffness curves, several studies recommend approximating the stiffness torque by a nonlinear cubic function. Another important characteristic related to nonlinear stiffness was demonstrated in experiments performed at the University of Toronto by Kircanski and Goldenberg [21] who observed that the torque C torsion characteristic is deformed toward the torque axis in the region from 0 to 1 Nm. In this region, the stiffness is lower due to the soft-windup effect. Besides nonlinear-joint stiffness effects, friction torques have an important impact on the behavior of the robot manipulator system and need to be considered for accurate modeling and control. One of the most complete friction models was recently proposed by Makkar et al. [29] that included all dominant friction components, i.e., static friction, Coulomb friction, viscous friction, and the Stribeck effect. By combining the cubic nonlinear stiffness torque term, soft-windup effects, inertial coupling, and frictional torques, the following novel nonlinear-joint dynamics representation is obtained (Ulrich et al., [37]):

$$M(\mathbf{q})\ddot{\mathbf{q}} + S\ddot{\mathbf{q}}_m + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{f}(\dot{\mathbf{q}}) - k(\mathbf{q},\mathbf{q}_m)(\mathbf{q}_m - \mathbf{q}) = 0 \quad (7)$$

$$S^T\ddot{\mathbf{q}} + J_m\ddot{\mathbf{q}}_m + k(\mathbf{q},\mathbf{q}_m)(\mathbf{q}_m - \mathbf{q}) = \tau \quad (8)$$

where $\mathbf{q}$ denotes the link angle vector, $\mathbf{q}_m$ represents the motor angle vector, $M(\mathbf{q})$ denotes the symmetric positive-definite link inertia matrix, $C(\mathbf{q},\dot{\mathbf{q}})$ represents the centrifugal-Coriolis matrix, J_m denotes the positive-definite motor inertia matrix, and τ represents the control torque vector. The inertial coupling matrix is given by

$$S = \begin{bmatrix} 0 & J_{m2} \\ 0 & 0 \end{bmatrix}. \quad (9)$$

In Eqs. (7) and (8), the nonlinear stiffness torque term is specified in Kircanski and Goldenberg [21]

$$k(\mathbf{q},\mathbf{q}_m)(\mathbf{q}_m - \mathbf{q}) = \mathbf{a}_1 \begin{bmatrix} (q_{m1} - q_1)^3 \\ (q_{m2} - q_2)^3 \end{bmatrix} + \mathbf{a}_2(\mathbf{q}_m - \mathbf{q}) + K_{sw}(\mathbf{q},\mathbf{q}_m)(\mathbf{q}_m - \mathbf{q}). \quad (10)$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are positive definite diagonal matrices of stiffness coefficients and $K_{sw}(\mathbf{q},\mathbf{q}_m)$ is the soft-windup correction factor that is modeled as a saddle-shaped function

$$K_{sw}(\mathbf{q},\mathbf{q}_m) = -k_{sw} \begin{bmatrix} e^{-a_{sw}(q_{m1}-q_1)^2} & 0 \\ 0 & e^{-a_{sw}(q_{m2}-q_2)^2} \end{bmatrix} \quad (11)$$

In Eq. (7), $\mathbf{f}(\dot{\mathbf{q}})$ denotes the friction vector which is modeled by (see Makkar [29])

$$\mathbf{f}(\dot{\mathbf{q}}) = \gamma_1[\tanh(\gamma_2\dot{\mathbf{q}}) - \tanh(\gamma_3\dot{\mathbf{q}})] + \gamma_4\tanh(\gamma_5\dot{\mathbf{q}}) + \gamma_6\dot{\mathbf{q}} \quad (12)$$

where $\gamma_i, i = 1,\ldots,6$, denote positive parameters defining the different friction components.

IV. Block-oriented Models

Identification of nonlinear dynamic system is classical problem of system theory. General identification methodologies comprise two approaches: the Volterra kernels approach a nonlinear, dynamic system with input $x(t)$ and output $y(t)$ is represented by a general convolution $y(t) = \sum_{n=1}^{\infty} \int \ldots \int h_n(\tau_1,\ldots,\tau_n)x(t-\tau_1)\ldots\cdot x(t-\tau_n)d\tau_1\ldots\tau_n$, involving Volterra kernel h_n of order n and Wiener G-functionals approach where nonlinear, dynamic system is assumed to be represented by the infinite series $y(t) = \sum_{n=0}^{\infty} G_n(k_n(\tau),x(\tau);\tau \in (-\infty,t))$, where G_n are some functionals and k_n are unknown kernels, see Schetzen [35]. Both Volterra and Wiener approach impose smoothness constraint on nonlinearities thus eliminating important nonlinearities such as hard and soft limiters and quantizers. Both approaches have high computational complexity.

Owing to the problems outlined above lots of effort in the recent years has been devoted to development of simpler nonlinear dynamic models capable of representing a wide spectrum of nonlinear dynamic system behavior. The most popular models are block-oriented models which consist of simple static and dynamic blocks connected in the cascade. Such systems are identified from the input-output observations of the whole structure. Among many possible configurations one have been singled out by the researchers in the field. It is called block-oriented sandwich system, see Figure 1. In this model a nonlinear, static element is sandwiched between two linear dynamic elements. A linear combination of such systems has been shown capable of approximating with arbitrary accuracy a causal and time invariant nonlinear dynamic system, see Sandberg [33]. Such systems have found many uses, e. g. in modeling of biomedical systems [30].

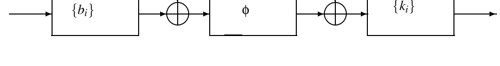

Fig. 1. Sandwich system.

Although sandwich systems are very versatile the simplified forms of such systems have become very popular. They include Wiener system comprising a linear dynamic system followed by a memoryless nonlinearity [3], see Figure 3 and Hammerstein system consisting of a memoryless nonlinearity followed by a linear dynamic system [31], [5], [6], [38] see Figure 2.

Identification of nonlinear systems has been an active field of studies over several decades, see Haber and Unbehauen [18]. Most identification techniques are parametric, however these techniques have severe limitations. They typically consider polynomial or trygonometric nonlinearities which are naturally smooth and leave out important nonlinearities such as dead-zone limiters, hard-limiters. These limitations have been relaxed by nonparametric identification techniques which do not require parametric form of the nonlinearity and are capable of identifying all measurable L_2 functions. Identification algorithms are typically derived from nonparametric regression estimators which have been studied in statistics since 1960s [7], [10], [17]. The most popular nonparametric regression estimation approaches include kernel, k nearest neighbor, Fourier and Hermite series approach and neural network approach [17]. Nonparametric techniques have been applied to identification of cascades of memoryless systems [13], [26], Hammerstein systems [15], [23], [20], [25] and Wiener systems [16]. A good survey of nonparametric identification methodology may be found in Greblicki and Pawlak [16] and comparison of parametric and nonparametric approaches in Hall [19].

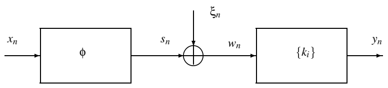

Fig. 2. Hammerstein system.

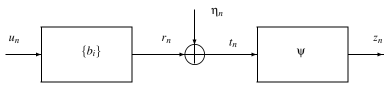

Fig. 3. Wiener system.

In the present paper we identify Hammerstein systems by kernel approach [23] using input-output measurements. We develop identification procedure in two steps. In the first step we identify static nonlinear system and establish convergence of identification algorithm. In the second step we adapt identification procedure to identification of the Hammerstein systems driven by a stationary, white noise. We estimate the linear dynamic and nonlinear memoryless subsystems from the input and output observations of the whole system. The parameters of dynamic linear subsystem are identified using correlation approach and the identification algorithm for the memoryless nonlinearity is based on kernel regression estimation. We discuss application of the Hammerstein system identification algorithm to identification of static nonlinearities and parameters of linear dynamics in the flexible two-joint space robot manipulator.

V. Identification of static nonlinearities

We start with identification of multiple input X and single output Y memoryless nonlinearity shown in Figure 4. Let X have a density f and assume $E|Y| < \infty$. Let $m(x) = E(Y|X = x)$ be a regression function and $f, g \in L_2$, where $g(x) = m(x)f(x)$. Let $(X_1,Y_1), \ldots, (X_n,Y_n)$ be independent identically distributed samples of an $R^d \times R$-valued random vector (X,Y) with $E|Y| < \infty$. We identify regression function $m(x) = E(Y|X = x)$ by the kernel regression estimate

$$m_n(x) = \frac{\sum_{i=1}^n Y_i K_{h_n}(x - X_i)}{\sum_{i=1}^n K_{h_n}(x - X_i)} \tag{13}$$

where K is an absolutely integrable kernel, h_n is the kernel bandwidth and $K_h(x) = K(\frac{x}{h})$. The kernel regression estimate was introduced by Watson [39] and was investigated by Greblicki et al. [14], Krzyzak [22], [25] and Devroye and Wagner [8].

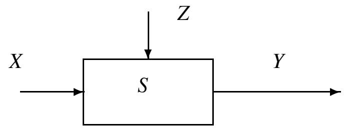

Fig. 4. Memoryless system.

The estimate depends on the smoothing parameter h, which controls the variance and the bias. Small values of h are needed for convergence, but they result in increased variance. Careful choice of h is crucial for convergence of estimate (13). First we state sufficient conditions for the pointwise convergence and the rate of estimate (13), see [23].

Theorem 1: Let K be a bounded kernel with compact support bounded away from zero and satisfying

$$c_1 H(||x||) \le K(x) \le c_2 H(||x||)$$
$$t^d H(t) \to 0 \quad t \to \infty$$

where H is bounded nonincreasing function and c_1 and c_2 are positive constants. If

$$h_n \to 0$$
$$nh_n^d \to \infty$$

as $n \to \infty$ then

$$m_n(x) - m(x) \to 0 \text{ in probability}$$

for almost all x as $n \to \infty$.

If, in addition, m satisfies Lipschitz condition of order $p, 0 < p \le 1$ then

$$|m_n(x) - m(x)| = O\left(n^{-p/(2p+d)}\right) \text{ in probability.}$$

VI. HAMMERSTEIN SYSTEM

Consider Hammerstein system shown in Figure 2.

The nonlinear memoryless subsystem (I) is given by:

$$W_n = \phi(X_n) + \xi_n, n = 0, \pm 1, \cdots \quad (14)$$

where X_n is R-valued stationary white noise with distribution μ and ξ_n is a stationary white noise with zero mean and finite variance σ_ξ^2. We assume zero correlation between ξ_n and X_n and assume without losing generality that ϕ is a scalar function. The linear subsystem dynamic (II) is given by autoregressive moving average (ARMA) model:

$$Y_n + a_1 Y_{n-1} + \cdots + a_l Y_{n-l} = b_0 W_n + b_1 W_{n-1} + \cdots + b_l W_{n-l}$$

with unknown order here l. Coefficients $a_1, \cdots, a_l$ guarantee the asymptotic stability of the system, or equivalently the roots $\mu_1, \cdots, \mu_l$ of the associated polynomial equation $z^l + a_1 z^{l-1} + \cdots + a_l = 0$ fulfill the conditions $|\mu_i| < 1, i = 1, \cdots, l$. Consequently Y_n is weakly stationary provided that W_n is weakly stationary. Subsystem II can also be represented by state equations as follows:

$$\begin{aligned} \hat{X}_{n+1} &= A\hat{X}_n + \mathbf{b}W_n \\ Y_n &= c^T \hat{X}_n + d_1 W_n. \end{aligned} \quad (15)$$

Here $\hat{X}_n$ is an l-dimensional state vector, and matrix A is assumed to be asymptotically stable, i.e. its eigenvalues of $\mathbf{A}$ all lie inside a unit circle. Hence $\hat{X}_n$ and Y_n are weakly stationary provided that W_n is weakly stationary. We can get from (15) the following formula

$$E\{Y_n|X_n\} = d_1\phi(X_n) + \alpha = m(X_n) \quad (16)$$

where $\alpha = E\phi(X)c^T(I-A)^{-1}b$.

We can also derive from (15) a weighting sequence representation (IIR) description of the linear subsystem

$$Y_n = \sum_{j=0}^{\infty} k_j W_{n-j} \quad (17)$$

where $k_0 = d_1 \neq 0, k_i = c^T A^{i-1} b, i = 1, 2, \cdots$ and condition $\sum_{i=0}^{\infty} |k_i| < \infty$ assures asymptotic stability of the linear subsystem. By taking conditional mean of the output given input (17) yields

$$E\{Y_n|X_n\} = k_0\phi(X_n) + \beta$$

where $\beta = E\phi(X)\sum_{i=1}^{\infty} k_i$.

The estimation problem is well defined if Y_n and X_n are random variables in the L_p sense, $p \geq 1$. That happens if II is asymptotically stable and

$$EW_n^p < \infty. \quad (18)$$

Relations (14), (18) together with X_n imply that W_n is weakly stationary. Condition (18) holds if $E\xi_n^p < \infty$ and either

$$\phi \in L_p(\mu) \ \text{(i.e. } E|\phi(X)|^p < \infty) \quad (19)$$

or

$$\phi(x) < P(|x|) \ \text{and} \ E|X|^{ps} < \infty \quad (20)$$

where P is a polynomial of order s. Asymptotic stability of II and (19) guarantee existance of α and β. It is clear that the class of nonlinearities satisfying (19) or (20) is so large that it cannot be finitely parameterized and admits such nonlinear functions as dead-zone limiters, hard-limiters, smooth-limiters and quantizers.

VII. IDENTIFICATION ALGORITHMS AND CONVERGENCE

We are now ready to introduce identification algorithm for the Hammerstein system. We will identify both ϕ and parameters of linear subsystem by observing random inputs and outputs of the whole system, i. e., the sequence $\{(X_i, Y_i)\}, i = 0, 1, \cdots, n-1$ of n correlated observations of the input and output. The linear subsystem identification procedure has been described in [27] and will be omitted.

Subsequently we derive identification procedure for ϕ. We start by estimating the regression function m given in (16) by the kernel regression estimate. Undetermined coefficients d_1 and α in (16) are the result of of inaccessibility of signal W_n. They can be determined if $\phi(x_1)$ and $\phi(x_2)$ were known at two points x_1 and x_2 where $m_n(x)$ converges and $\phi(x_1) \neq \phi(x_2)$. Under this condition we could estimate d_1 by

$$d_{1,n} = \frac{m_n(x_1) - m_n(x_2)}{\phi(x_1) - \phi(x_2)}$$

and α by

$$\alpha_n = m_n(x_1) - \frac{\phi(x_1)}{\phi(x_1) - \phi(x_2)}(m_n(x_1) - m_n(x_2)).$$

Relation (16) and aformentioned formulas above motivate an identification procedure for ϕ

$$\phi_n(x) = (m_n(x) - \alpha_n)/d_{n,1}.$$

Regression m can be estimated by the kernel regression estimate

$$m_n(x) = \frac{\sum_{i=1}^{n} Y_i K_{h_n}(x - X_i)}{\sum_{i=1}^{n} K_{h_n}(x - X_i)} \quad (21)$$

where $(X_1, Y_1), \ldots, (X_n, Y_n)$ are observations of an $R^d \times R$-valued random vector (X,Y) and $E|Y| < \infty$. The next result provides convergence conditions for algorithm (21). Let $g(x) = \Phi(x)f(x)$. Then we have

Theorem 2 ([23]): Let $EY^2 < \infty$, A be asymptotically stable, and $f, g \in L_2$. If N satisfies the following conditions

$$h_n \to 0 \quad (22)$$

$$nh_n^d \to \infty \quad (23)$$

then

$$m_n(x) \to m(x) \ \text{in probability}$$

as $n \to \infty$ for almost all x.

If, in addition, m satisfies Lipschitz condition of order p for some $0 < p \leq 1$ for both f, g, then

$$|m_n(x) - m(x)| = O\left(n^{-p/(2p+d)}\right) \ \text{in probability}$$

for almost all x.

VIII. Application of Hammerstein Model to Identification of Robot Manipulator

In the practical flexible link manipulator the torque is being applied to the joints using geared transmission. The friction in these joints is nonlinear and may be modeled by a static nonlinearity. The whole link can be considered as the block-oriented system with a static part modeling flexibility at the joints and dynamic part modeling flexibility of the structure. We can then apply the identification algorithm developed for the Hammerstein system to identify static nonlinearities from input-output excitations.

IX. Conclusions

In the present paper we discussed modeling and identification of two-link flexible robot manipulator by block-oriented systems. Particular attention has been given to Hammerstein systems. Identification procedures for the Hammerstein model have been introduced and their convergence and the rates discussed. Data-dependent selection of algorithms parameters is an important issue. Many techniques from machine learning field can be used for that purpose. They include cross-validation and bootstrap [9]. Machine learning techniques will be applied to parameter selection in identification algorithms in future work.

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References

[1] Asada, H. and Slotine, J. -J. E. *Robot Analysis and Control*, John Wiley and Sons Inc., 1986.

[2] Banerjee, A.K. and Singhose, W. Command Shaping in Tracking Control of a Two- Link Flexible Robot. *AIAA Journal of Guidance, Dynamics and Control*, 21(6), Engineering Note, 1012-1015, 1998.

[3] Bendat, J. *Nonlinear System Analysis and Identification from Random Data*, Wiley, New York, 1990.

[4] Beres, W. and Sąsiadek, J. Z. Finite Element Dynamic Model of Multi-link Flexible Manipulators. Applications in Mathematics and Computer Science, 5(2), 231 - 262, 1995.

[5] Chang, F. H. and Luus, R. A noniterative method for identification using Hammerstein system. *IEEE Trans. Automat. Contr*, AC-16, 464–468, 1971.

[6] Chung, H. and Sun, Y. Y. Analysis and parameter estimation of nonlinear systems with Hammerstein model using Taylor series approach. *IEEE Trans. Circuits Syst.*, CAS-35, 1533–1544, 1988.

[7] Devroye, L. and Györfi, L. *Nonparametric Density Estimation. The L1 View*. Wiley, New York, 1985.

[8] L. Devroye and T.J. Wagner, "Distribution-free consistency results in nonparametric discrimination and regression function estimation". *Ann. Statist.* vol. 8, pp. 231-239, 1980.

[9] Efron, B. and Tibshirani, R. *An Introduction to the Bootstrap*. Chapman and Hall, New York, 1993.

[10] Eubank, R. *Spline Smoothing and Nonparametric Regression*. Marcel Dekker, New York, 1988.

[11] Green, A. and Sąsiadek, J. Z. Direct, Optimal, Stochastic and Fuzzy Control of a Two-Link Flexible Manipulator. *Proceedings of the AIAA Guidance, Navigation and Control-Conference and Exhibit*, Paper No. AIAA 2000-4375, August 2000, Denver, Colorado, USA.

[12] Green, A. and Sąsiadek, J. Z. Methods of Trajectory Tracking for Flexible Manipulators. *Proceedings of the AIAA Guidance, Navigation and ControlConference and Exhibit*, Paper No. AIAA 2002-4565, August 2002, Monterey, California, USA.

[13] Greblicki, W. and Krzyżak, A. Non-parametric identification of a memoryless system with cascade structure. *Int. J. Syst. Science*, 10:1301–1310, 1979.

[14] Greblicki, W., Krzyżak, A and Pawlak, M., Distribution-free pointwise consistency of kernel regression estimate, *Annals of Statistics*, vol. 12, 1570–1575, 1984.

[15] Greblicki, W. and Pawlak, M. Hammerstein system identification by nonparametric regression estimation. *Int. J. Contr.*, 45:343–354, 1987.

[16] Greblicki, W. and Pawlak, M. *Nonparametric System Identification*, Cambridge University Press, Cambridge, 2008.

[17] Györfi, L., Kohler, M., Krzyżak, A. and Walk, H. *A Distribution-free Theory of Nonparametric Regression*. Springer-Verlag, New York, 2002.

[18] Haber, R. and Unbehauen, H. Structure identification of nonlinear dynamic systems - a survey on input/output approaches. *Automatica*, 26:651–677, 1990.

[19] Hall, P. On convergence rates in nonparametric problems. *Int. Statist. Review*, 57:45–58, 1989.

[20] Kegl, B., Krzyżak, A. and Sąsiadek, J.Z. Non-parametric identification of dynamic non-linear systems by a Hermite Series Approach, *Int. J. Syst. Science*, 32:1261-1285, 2001.

[21] Kircanski N. M., Goldenberg A. A. (1997) An experimental study of nonlinear stiffness, hysteresis, and friction effects in robot joints with harmonic drives and torque sensors. *The Int. J. of Robot. Res.* 16: 214-239

[22] A. Krzyżak, "The rates of convergence of kernel regression estimates and classification rules". *IEEE Trans. Information Theory*, vol. IT-32, pp. 668-679, 1986.

[23] Krzyżak, A. On estimation of a class of nonlinear systems by the kernel regression estimate. *IEEE Trans. Inform. Theory*, IT-36:141–152, 1990.

[24] Krzyżak, A. Global convergence of the recursive kernel regression estimates with applications in classification and nonlinear system estimation. *IEEE Trans. Inform. Theory*, IT-38:1323–1338, 1992.

[25] Krzyżak, A. On nonparametric estimation of nonlinear dynamic systems by the Fourier series estimate. *Signal Processing*, 52:299–321, 1996.

[26] Krzyżak, A. and Partyka, M. Identification of block oriented systems by nonparametric techniques. *International Journal of Systems Science*, 24:1049–1066, 1993.

[27] A. Krzyżak, J.Z. Sąsiadek and A. Green, Identification of nonlinear systems by nonparametric approach with application to control of flexible robot manipulator, *Proceedings of the 14th IEEE International Conference on Methods and Models in Automation and Robotics*, Miedzyzdroje, Poland, August 19-21, 2009.

[28] Lee, J.X., Vukovich, G. and J.Z. Sąsiadek. Fuzzy Control of a Flexible Link Manipulator. Paper WM7-1: 50, *Proceedings of the American Control Conference*, June 1994, Baltimore, Maryland, USA.

[29] Makkar C, Dixon WE, Sawyer WG et al, A new continuously differentiable friction model for control systems design. In: *IEEE/ASME Int. Conf. on Adv. Intel. Mecha.*, IEEE Press, Piscataway, New Jersey, 600-605, 2005.

[30] Marmarelis, P. and Marmarelis, V. *Analysis of Physiological Systems: the White Noise Approach*. Plenum Press, New York, 1978.

[31] Narendra, K. and Gallman, P. (1966). An iterative method for the identification of nonlinear systems using the Hammerstein model. *IEEE Trans. Automat. Contr.*, AC-11:546–550, 1966.

[32] Salaberger, C. Canadian space robotics activities *Acta Astronautica*, 239-246, 1998.

[33] Sandberg, I. Approximations of nonlinear functionals. *IEEE Trans. Circuits and Systems*, CAS-39:65–67, 1992.

[34] Sąsiadek, J. Z. and Srinivasan, R. Dynamic modeling and adaptive control of a single-link flexible manipulator. *AIAA Journal of Guidance, Dynamics and Control*, 12(6), 838-844, 1989.

[35] Schetzen, M. *The Volterra and Wiener Theories of Nonlinear Systems*. Wiley-Interscience, New York, 1980.

[36] Spong MW (1987) Modeling and Control of Elastic Joint Robots. J. Dyn. Syst., Meas., Contr. 109:310-319

[37] Ulrich, S. and Sąsiadek, J. Z., Trajectory tracking control of flexible-joint space manipulators, *Canadian Aeronautical Space Journal*, vol. 58, 1–13, 2012.

[38] Verhaegen, M. and Westwick, D. Identifying MIMO Hammerstein systems in the context of subspace identification methods, *International Journal of Control*, 63: 331–349, 1996.

[39] Watson, G. S. Smooth regression analysis, *Sankhva*, Ser. A, vol. 26, 355-372. 1964. R. L. Wheeden and A. Zygmund, Measure a