Performance Enhancement for GPS/INS Fusion by Using a Fuzzy Adaptive Unscented Kalman Filter

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***Abstract*—Kalman filter requires that the process noises to be zero mean white noise; otherwise, the divergence will occur. Adaptive tuning of a Kalman filter via fuzzy logic has been one of the promising strategies to cope with divergence when dealing with non-white noise. The fuzzy logic adaptive controller (FLAC) will continually adjust the noise strengths in the filter's internal model and tune the filter. This paper presents a new INS/GPS sensor fusion scheme based on Fuzzy Adaptive Unscented Kalman Filter (FAUKF). The FAUKF is based on the combination of the unscented Kalman filter and the fuzzy logic controller which performs adaptation task for dynamic characteristics. Results obtained by FAUKF were compared to the Extended Kalman filter (EKF), Unscented Kalman Filter (UKF) and Fuzzy Adaptive Extended Kalman Filter (FAEKF). This comparative study has demonstrated that the FAUKF leads to very promising results as compared the other three Kalman filters.**

I. INTRODUCTION

Inertial Navigation System (INS) and Global positioning system (GPS) are extensively used for positioning and attitude determination for aircraft and vehicle navigation or tracking, marine applications, etc. It is well known that GPS and GPS-like signals are susceptible to jamming or being lost due to the electromagnetic waves limitation. In addition, all inertial navigation systems suffer from integration drift; small errors in the measurement of angular velocity and acceleration are integrated into larger errors in velocity and position. Due to these limitations, the GPS/INS integration provides more accurate performance in comparison with GPS or INS stand-alone systems. An Extended Kalman Filter (EKF) is one of the most widely used methods for navigation sensor fusion. The Extended Kalman based algorithm for sensor fusion was initially proposed by D. Willer et al. [1]. The EKF highly depends on pre-defined dynamics model. In addition, due to the linearization process, the EKF might suffer from divergence and performance degradation. Better alternative for GPS/INS integration is an Unscented Kalman Filter (UKF). The UKF employs set of sigma points by deterministic sampling. Due to the fact that the UKF is able to deal with the nonlinear model, unlike the conventional EKF, linearization process is not required for UKF; hence, linearization error can be avoided. Another remarkable advantage is that the overall computational complexity of the UKF is the same as that of the EKF. The EKF and UKF performances were compared by several authors [2-3]. The major difficulties in designing EKF or UKF for sensor fusion are incomplete prior information on covariance matrices of $\mathbf{Q}$ and $\mathbf{R}$, as well as, the issue with the noise process. The Kalman filter requires that the process and measurement are corrupted by zero-mean white Gaussian noise and all the plant dynamics and noise processes are exactly known. If the theoretical assumptions of the filter and it's actual conditions do not agree, then divergence will happen. The Adaptive Kalman Filter (AKF) approach has been one of the most promising strategies developed in last twenty years [4-6]. The application of fuzzy logic has been very popular especially in the field of the adaptive control for dealing with nonlinear systems with dynamical uncertainties. Adaptive fuzzy filter represents a filter which is especially powerful in coping with nonlinearities and modeling errors as they do not require any mathematical model of the system. Many efforts have been made to improve the estimation of the covariance matrices based on the innovation estimation approach. An adaptive filter called Fuzzy Adaptive Extended Kalman Filter (FAEKF) for adapting the process and measurement noise covariance matrices was proposed by J. Z .Sasisadek et al. [7-10]. The method was based on exponential data weighting to prevent the EKF from divergence. In this work the EKF has been modified using the fuzzy logic controller. Mean value and covariance of innovation were used as two inputs of the Fuzzy Logic Adaptive Controller (FLAC). The output is then used to weight process noise and measurement noise covariance in EKF. D. Jwo et al. [11] proposed a Fuzzy Adaptive Strong Tracking Unscented Kalman Filter (FASTUKF) for ultratight GPS/INS integration. In their work the performance of the STUKF was improved by adaptively adjusting the suboptimal fading factor by implementing the fuzzy logic. The main problem associate with this method is that the calculation of the suboptimal fading factor needs the cumbersome evaluation of Jacobian matrix of system models. In addition, the fading factor is incorporated in the entire filtering process [12-13]. Fuzzy Interacting Multiple Model Unscented Kalman filter (FUZZY-IMMUKF) for maneuvering vehicle was presented by D.Jwo and C.Tseng [14]. In their algorithm an appropriate value for the process noise covariance was determined by

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changing filters. As a disadvantage, this method requires more than one filter running simultaneously.

In this paper, the new Fuzzy Adaptive Unscented Kalman Filter (FAUKF) is proposed. The proposed algorithm is based on the correction of both the process noise covariance matrix **Q**, and the measurement error covariance **R**. The exponential data weighting is used to prevent the filter from divergence by keeping it from discounting measurement for the large K. The Fuzzy Logic Controller (FLC) is used to adjust the exponential weighting of weighted UKF. The rest of the paper is organized as follows. Section II describes the weighted EKF and weighted UKF. Section III introduces the proposed Fuzzy Adaptive Unscented Kalman Filter (FAUKF) where fuzzy logic and Kalman filter are integrated. In order to show the effectiveness of the proposed filter, simulation results are discussed in section IV and finally, in section V, the conclusions of this work are given.

II. WEIGHTED KALMAN FILTER

GPS and INS measurement are both nonlinear, stochastic dynamic and measurement model can be written as:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \tag{2}$$

where

$$\mathbf{w}_k \sim N(0, \mathbf{Q}) \tag{3}$$

$$\mathbf{v}_k \sim N(0, \mathbf{R}) \tag{4}$$

The vectors $\mathbf{x}_k \in \Re^8$ and $\mathbf{z}_k \in \Re^4$ represent the state of the system and measurement at time instant k, respectively. $\mathbf{w}_k$, $\mathbf{v}_k$ are the process and measurement noises. To fuse the measurement from the GPS and INS, the UKF and EKF are used. In order to prevent divergence, and keeping the filter from discounting measurement for large K, the exponential data weighting is used [15].

A. Weighted Extended Kalman Filter

To prevent the Kalman filter from divergence the exponential weighting has been used.

The covariance matrices for the weighted EKF are defined as:

$$\mathbf{R}_k = \mathbf{R}\alpha^{-2(k+1)} \tag{5}$$

$$\mathbf{Q}_k = \mathbf{Q}\alpha^{-2(k+1)} \tag{6}$$

The weighted covariance defined as:

$$\mathbf{P}_k^{a-} = \mathbf{P}_k^{-}\alpha^{2(k)} \tag{7}$$

Then the Kalman Gain can be computed as:

$$\mathbf{K}_k = \mathbf{P}_k^{-}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^{-}\mathbf{H}_k^T + \mathbf{R}\alpha^{-2(k+1)})^{-1} \tag{8}$$

Substituting equation (7) in to the equation (11), the Kalman Gain can be rewritten as:

$$\mathbf{K}_k = \mathbf{P}_k^{a-}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^{a-}\mathbf{H}_k^T + \mathbf{R}/\alpha^2)^{-1} \tag{9}$$

where

$$\mathbf{H}_k \approx \frac{\partial h(x,k)}{\partial x} \tag{10}$$

A priori covariance matrix can be computed as:

$$\mathbf{P}_{k+1}^{-} = \boldsymbol{\psi}_k\mathbf{P}_k\boldsymbol{\psi}_k^T + \mathbf{Q}\alpha^{-2(k+1)} \tag{11}$$

Substituting equation (6) in to the equation (10), the priori covariance matrix can be rewritten as:

$$\mathbf{P}_{k+1}^{a-} = \alpha^2\boldsymbol{\psi}_k\mathbf{P}_k^a\boldsymbol{\psi}_k^T + \mathbf{Q} \tag{12}$$

where $\boldsymbol{\psi}$, the linear approximation equation can be present in form of:

$$\boldsymbol{\psi}_k \approx \frac{\partial f(x,k)}{\partial x} \tag{13}$$

And the posterior covariance matrix can be defined as:

$$\mathbf{P}_k^a = (I - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^{a-} \tag{14}$$

where

$$\mathbf{P}_0^{a-} = \mathbf{P}_0 \tag{15}$$

More detailed description of Weighted EKF approach is given in J. Z. Sasisadek et al. [7-8].

B. Weighted Unscented Kalman Filter

The first step in implementing UKF is to generate the sigma point through the scaled Unscented Transformation (UT). The scaling of UT can be fully represented by three scaling parameters [16]. The scaling parameter σ, determines the spread of sigma points. β, is used to include prior distribution information, and the scaling parameter κ influences accuracy of the approximation. It should be noted that the scaling parameter is typically assumed constant throughout UKF. Implementing these three scaling parameters, the mean weight vector $\eta_i^{(m)}$, and the covariance weight vector $\eta_i^{(c)}$, associated with the i_{th} point are defined

$$\begin{gathered} \lambda = \sigma^2(L+\kappa) - L \\ \eta_0^{(m)} = \lambda/(L+\lambda) \\ \eta_0^{(c)} = \lambda/(L+\lambda) + 1 - \sigma^2 + \beta \\ \eta_i^{(c)} = \eta_i^{(m)} = 1/(2(L+\lambda)) \quad i = 1, ..., 2L \end{gathered} \tag{16}$$

where L is the length of the state vector. The sigma vector χ generated by unscented transformation given by

$$\boldsymbol{\chi}_{k-1} = [\hat{\mathbf{x}}_{k-1} \quad \hat{\mathbf{x}}_{k-1} + \sqrt{L+\lambda}\sqrt{\mathbf{P}_\mathbf{x}} \quad \hat{\mathbf{x}}_{k-1} - \sqrt{L+\lambda}\sqrt{\mathbf{P}_x}] \tag{17}$$

where $\boldsymbol{\chi}$ is a $L(2L+1)$ matrix and each column of $\boldsymbol{\chi}$ matrix represents a sigma point. Note that the covariance P_x is calculated by using Cholesky decomposition method [17].

Once the sigma points have been generated, each point is propagated through the nonlinear function as

$$\varphi^{(i)}_{k|k-1} = f(\chi_{k-1}, u_{k-1}) \quad i = 0, ...2L \tag{18}$$

The mean and covariance are approximated by a weighted average mean and covariance of the transformed sigma points as follows:

$$\hat{\mathbf{x}}^{-}_{k|k-1} = \sum_{i=1}^{2L} \eta^{(m)}_i \varphi^{(i)}_{k|k+1} \tag{19}$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q} + \sum_{i=0}^{2L} \eta^{(c)}_i (\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}^{-}_{k|k-1})(\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}^{-}_{k|k-1})^T \tag{20}$$

The process noise covariance matrix $\mathbf{Q}$, is added to the covariance matrix due to the additive noise assumption. As for prediction, the measurement defined as:

$$\gamma^{(i)}_{k|k-1} = h(\chi_{k-1}, u_{k-1}) \tag{21}$$

The sigma points matrix χ_{k-1} then used to calculate the predicted covariance matrix.

$$\hat{\mathbf{y}}_{k|k-1} = \sum_{i=1}^{2L} \eta^{(m)}_i \gamma^{(i)}_{k|k-1} \tag{22}$$

$$\mathbf{P}^{yy}_k = \mathbf{R} + \sum_{i=0}^{2L} \eta^{(c)}_i (\gamma^{(i)}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1})(\gamma^{(i)}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1})^T \tag{23}$$

Cross covariance between the state and output defined as:

$$\mathbf{P}^{xy}_k = \sum_{i=0}^{2L} \eta^{(c)}_i (\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}^{-}_{k|k-1})(\gamma^{(i)}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1})^T \tag{24}$$

Using these covariance matrices, the Kalman gain defined as:

$$\mathbf{K}_k = \mathbf{P}^{xy}_k (\mathbf{P}^{yy}_k)^{-1} \tag{25}$$

This Kalman Gain is then used to update both the state and covariance estimates as:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}^{-}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \hat{\mathbf{y}}_{k|k-1}) \tag{26}$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}^{yy}_k \mathbf{K}^T_k \tag{27}$$

where $\hat{\mathbf{x}}_k$ and $\mathbf{P}_k$ are the posterior state and covariance estimates. The term $\mathbf{z}_k - \hat{\mathbf{y}}_{k|k-1}$ is called residual or innovation which is the differences of GPS measured pseudo ranges and INS estimated ranges, it reflects the degree to which the model fits the data.

C. The Adaptive Strategy

The models and implementation equations for the covariance matrices of weighted unscented Kalman filter are set as:

$$\mathbf{P}_{k|k-1} = \mathbf{Q}\alpha^{-2(k+1)} + \sum_{i=0}^{2L} \eta^{(c)}_i (\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}^{-}_{k|k-1})(\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}^{-}_{k|k-1})^T \tag{28}$$

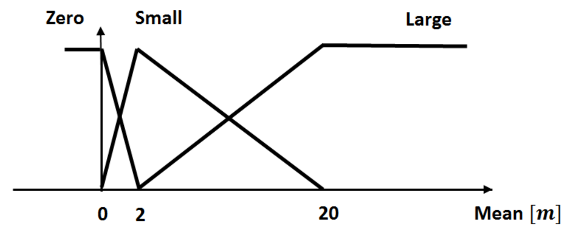

Fig. 1: Mean value membership functions.

$$\mathbf{P}^{yy}_k = \mathbf{R}\alpha^{-2} + \sum_{i=0}^{2L} \eta^{(c)}_i (\gamma^{(i)}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1})(\gamma^{(i)}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1})^T \tag{29}$$

$$\mathbf{P}^{xy}_k = \alpha^{-2(k+1)} (\sum_{i=0}^{2L} \eta^{(c)}_i (\varphi^{(i)}_{k|k-1} - \hat{\mathbf{x}}_{k|k-1})(\gamma^{(i)}_{k|k+1} - \hat{\mathbf{y}}_{k|k-1})^T) \tag{30}$$

where $\alpha \geq 1$, $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices. For $\alpha > 1$, as time increases the $\mathbf{R}$ and $\mathbf{Q}$ decrease. We are giving more credibility to the recent data by decreasing the noise covariance exponentially with K, and if $\alpha = 1$ the filter becomes a regular UKF.

III. Fuzzy logic adaptive system for non-white process noise

If the Kalman filter is based on complete and perfect tuned model, the residuals should be zero mean white noise process. If the residuals are not white noise and correlated with itself, the Kalman filter will diverge or coverage to a large bound. In practice, sometime the w_k may be not zero mean. One alternative to this problem is adding a shaping filter to manufacture a color noises with a given spectral density from white noise; however, increasing the state variables is the main drawback of this methods. The better alternative is implementing the fuzzy logic controller to adapt the filter online. The Fuzzy Logic Controller (FLC) has been one of the methods to adapt the filter while does not increase the state variables; hence, require less computing time. The purpose of the fuzzy logic adaptive controller is to detect the divergences and adapt the gain to prevent the Kalman filter from divergence. In this paper, the residual used to adapt the filter. The covariance and mean values of the residuals are used as inputs to the FLC to decide the degrees of divergence and the exponential weighting alpha is the output. The FLC will adapt the Kalman filter optimally by selecting appropriate α and trying to keep the innovation sequence acting as zero mean white noise. Generally, when the covariance of the residual is becoming large then the mean value is moving away from zero and subsequently the Kalman filter is becoming unstable. In this case, the FLC applies a large α. The characteristics of fuzzy system highly depend on the fuzzy rules. The Proposed fuzzy logic controller, FAUKF, used 9 rules such as:

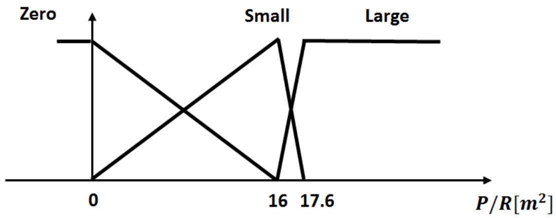

Fig. 2: Covariance membership functions.

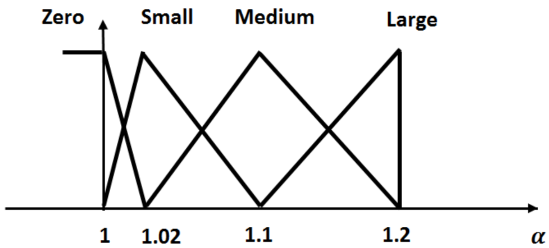

Fig. 3: α membership functions.

- **If** the covariance of residuals is zero **and** the mean value is zero **then** α is small.
- **If** the covariance of residuals is large **and** the mean value is zero **then** α is large.

The membership function of two fuzzy controller inputs (mean and covariance of residual) and output variable of the fuzzy logic controller, (α), are illustrated in Figure 1, Figure 2, and Figure 3 respectively. The rule table for FLC is shown in table I.

IV. SIMULATION RESULTS

Simulation experiments have been done to evaluate and compare the performance of the EKF, UKF, FAEKF, and FAUKF for INS/GPS integration. The state vectors used in simulation included eight states. Three states for position errors, three for velocity errors and two states for GPS range bias and range drift.

$$\mathbf{x}_k = [x_k, y_k, z_k, \dot{x_k}, \dot{y_k}, \dot{z_k}, c\Delta t, c\dot{\Delta t}] \tag{30}$$

where x points east, y points north and z in the attitude, $c\Delta t$, $c\dot{\Delta t}$ represent GPS range and drift states. Four pseudo range measurements are used as a measurement model of the Kalman filter.

$$Z_1 = \sqrt{(X_1-x)^2+(Y_1-y)^2+(Z_1-z)^2}+c\Delta t_1$$

$$Z_2 = \sqrt{(X_2-x)^2+(Y_2-y)^2+(Z_2-z)^2}+c\Delta t_2$$

$$Z_3 = \sqrt{(X_3-x)^2+(Y_3-y)^2+(Z_3-z)^2}+c\Delta t_3$$

$$Z_4 = \sqrt{(X_4-x)^2+(Y_4-y)^2+(Z_4-z)^2}+c\Delta t_4 \tag{31}$$

where (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), (X_3, Y_3, Z_3), (X_4, Y_4, Z_4) are the positions of the four GPS satellites respectively, and (x, y, z) are the position of the vehicle. The White noise with a standard deviation of 4 meters was applied to the GPS measurements. The mean values of INS are chosen as 2 meters, 2 meters, and 2 meters for the East (x), North (y), and Altitude (z) respectively. For the first part of the simulation, the UKF and EKF were used. The objective of this part was to investigate the sensitivity and robustness of EKF and UKF while the system is corrupted with non-white noise. Figures 4 and Figures 5 represent the corrected position and velocity errors of EKF and UKF. As it is demonstrated, the UKF shows better results in comparison to the EKF when both are corrupted with the non-white noise with mean values of 2 meters. The simulation was repeated for different mean values of input noise (mean values between 0 meter to 4 meters by steps of 0.1 meter). The simulation results shown that only for 0$\leq$ mean $\leq$ 0.2 the UKF is stable but it becomes diverge for mean values grater than 0.2 meter. Results also show that the EKF diverges for any value greater than zero.

In the second part of the simulation, the FAEKF and FAUKF were employed. Figures 6, and Figures 5, present the position and velocity errors of the FAEKF and FAUKF for non-white noise for mean value equal to 2 meters. As it is illustrated the fuzzy adaptive EKF and the fuzzy adaptive UKF clearly improved the performance of EKF and UKF. The position and velocity errors of FAEKF and FAUKF are much smaller than that of EKF and UKF. As simulation results indicated, the performance of the FAUKF is better in comparison with FAEKF. Innovation sequences of filters are shown in Figures 8. The innovation sequence is the difference between the actual measurement and the best measurement prediction based on the filter's internal mode, hence, it can be used to evaluate

TABLE I: Rule table of α for non-white noise.

α	Mean value		
	Z	S	L
Z	S	Z	Z
S	Z	L	M
L	L	M	Z

Z $-$ Zero; S $-$ Small; M$-$ Medium
L$-$ Large

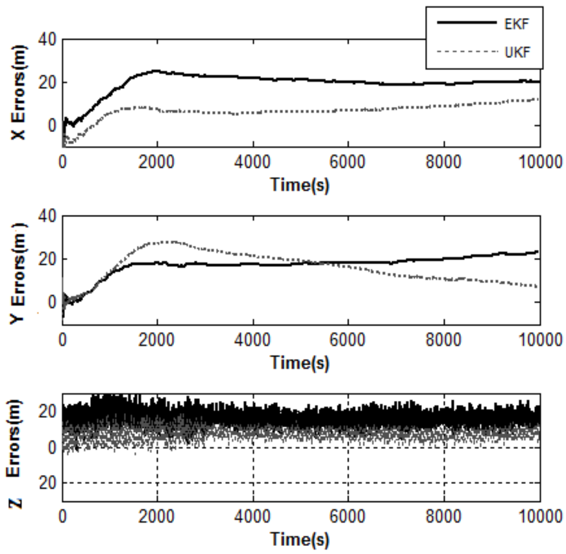

Fig. 4: The position error of EKF and UKF with non-white noise input.

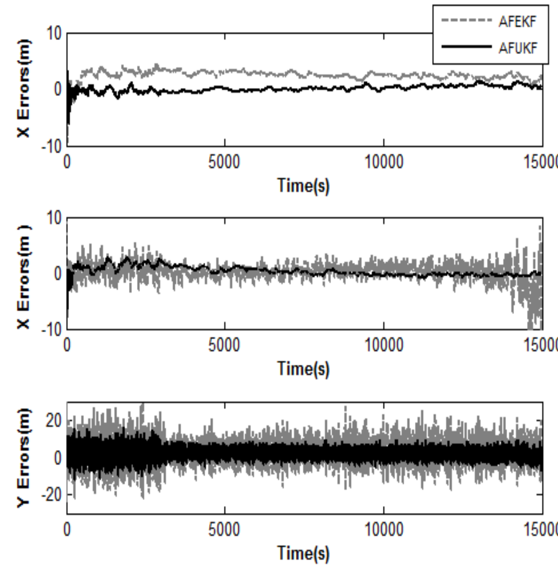

Fig. 6: The position error of FAEKF and FAUKF with non-white noise input.

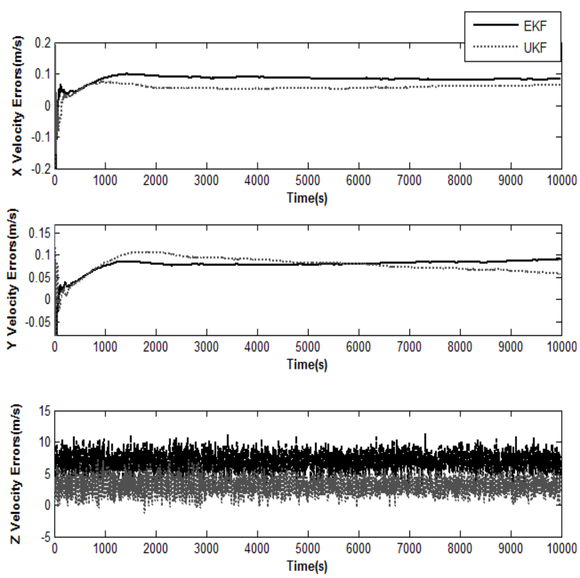

Fig. 5: The velocity error of EKF and UKF with nonwhite noise input.

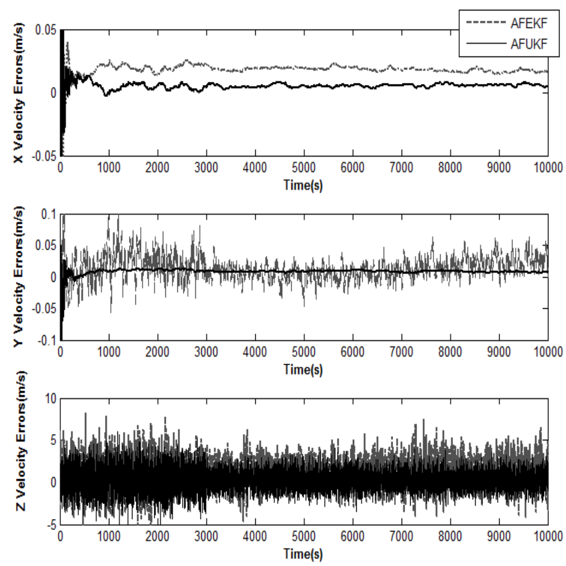

Fig. 7: The velocity error of FAEKF and FAUKF with non-white noise input.

filter's performance. From Figures 8 it could be noticed that the innovation sequence of EKF and UKF have a large drift, while the residual mean value of FAEKF and FAUKF are much smaller than that of EKF and UKF.

V. CONCLUSIONS

In this paper, a fuzzy logic adaptive system has been developed to improve the Unscented Kalman Filter (UKF) performance and prevent the filter from divergence. By monitoring the innovations sequences, the FLC can evaluate the performance of the filter. If the filter does not perform well, the controller would apply an appropriate weighting factor α to improve the accuracy of a UKF. Performance comparisons on UKF, EKF, FAEKF and FAUKF have been conducted. As the simulation results shown, the Fuzzy Adaptive UKF in comparison with other three filters has the best performance in estimation accuracy when dealing with non-white noise and has a very good potential as an alternative navigation state estimator.

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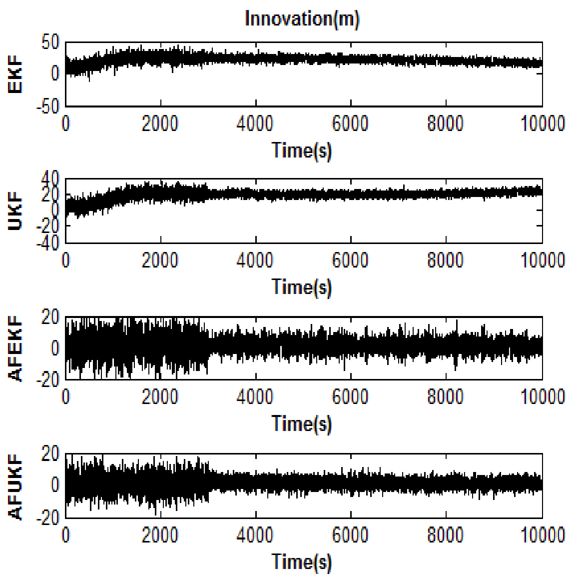

Fig. 8: Innovation sequences of EKF, UKF, FAEKF, and AFUKF for non-white noise.

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