Using relative distributions to investigate the Body Mass Index in England and Canada

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Abstract

In this paper we use relative distributions to examine changes in the distribution of the Body Mass Index (BMI) in England and Canada during the period 1994/5-2000/1. The use of relative distributions allows us to describe changes in the whole distribution of the BMI in a nonparametric fashion. While statistics analogous to the Gini Index can be constructed based on the relative distribution, important characteristics of changes in the distribution of the BMI such as changes in the proportions overweight and obese are more naturally handled using measures of relative polarization. Our results show that while BMI has increased in both countries, BMI in England has increased at a much faster rate than in Canada. Both groups show polarization over time towards both tails of the weight distribution, with the English polarizing towards the upper tail at a faster rate than Canadians.

JEL: I12, C14, C23

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1. Introduction

The prevalence of overweight and obesity, has recently been recognised as a serious health problem in some developed nations.\textsuperscript{1} The translation from increased overweightness and obesity to health problems is embedded in the WHO classifications: overweight individuals have increased relative risk (compared to those of ‘normal’ weight) for morbidity and mortality from particular diseases, while obese individuals have greatly increased risks. For example, the House of Commons Health Select Committee (2004) report relative risks greater than 3 for the obese compared to the non-obese for Type 2 diabetes, gallbladder disease, dyslipidaemia, Insulin Resistance, Breathlessness and Sleep apnea, and relative risks between 2 and 3 for Coronary Heart disease, Hypertension, Osteoarthritis and Gout. There are also increased relative risks (between 1 and 2) for a number of cancers.

The rise in obesity has been especially pronounced in English speaking countries since the early 1980’s (Cutler et al. 2003). Leicester and Windmeijer (2004) using the Health Survey for England report that the proportion of overweight or obese men between the ages of 16 and 64 increased from 45% in 1986-1987 to 64% in 2002. The same statistics calculated for women indicate a rise from 36% to 58%. In the United States, Obesity rates rose from 15% in 1980 to 31% in 2000 (Finkelstein et al. 2005). Interestingly, Canada does not appear to have experienced the same rises in overweight and obesity, particularly among women. Tremblay et al. (2002) estimate that the proportion of Canadian men overweight or obese increased from 48% to 57% between 1981 and 1996, while for women the comparable statistics suggested an increase from 30 to 35%. Obesity rates changed from 9 to 14% and 8 to 12% for men and women respectively.\textsuperscript{2}

In this paper we compare in detail the recent experiences of England and Canada by examining changes in the distribution of the BMI between 1994/5 and 2000/1. There are at least two reasons for considering the whole distribution of the BMI. Firstly, as explained above, while the cut-offs for BMI categories are related to different levels of risk across categories, to a greater or lesser extent these will misclassify some individuals to risk classes. This occurs as there will not generally be a discrete change in risk at the category thresholds. It is therefore important to conduct analyses, such as we present, which are sensitive to changes in the proportions of individuals around the thresholds not just across categories. Secondly, while analyses of obesity rates and the distribution of the BMI are related there is not a one to one mapping from differences in obesity rates to differences in BMI distributions. In particular while measures of levels and changes in obesity rates are derived from distributions of the BMI, many alternative BMI distributions and changes in them can generate the same obesity trends. In particular, two or more distributions are often compared by

\textsuperscript{1} Overweight is conventionally defined as having BMI greater than or equal to 25 and less than 30, while obese is defined as having BMI greater than or equal to 30. An individual’s BMI is calculated as their weight in kg divided by the square of their height in metres.

\textsuperscript{2} This may however be an underestimate as the change is estimated using different surveys, with the latter (the 1996 National Population Health Survey) using self-reports and the earlier survey (the 1981 Canada Fitness Survey) using anthropometric measurements. Cairney and Wade (1998) compare nine different surveys from 1978-1994/5 and estimate that self-reporting leads to 10% lower obesity levels than anthropometric measurements (based on a definition of BMI<=27).
analysing key summary statistics such as differences in means, variances, and the proportions above and below specified cut-offs, but these do not represent differences across the whole distribution unless the distributions are members of a known parametric family, as is assumed by linear regression approaches, for either the BMI or obesity indicators. For example, Leicester and Windmeijer (2004) report that while the proportion of men and women classified as overweight in England has stabilised between 1993 and 2002, the proportions classified as obese has continued to rise. While this can be explained by a simple mean shift, it requires strong restrictions on other characteristics of the distribution. A less restrictive approach is to initially consider the whole distribution and add structure by considering models which may be able to explain parsimoniously the distributional characteristics that are observed. The approach we take therefore allows us to see how obesity rates and other characteristics of the BMI distribution are changing rather than assuming they are due to location shifts.

The fundamental objects of interest for examining and predicting obesity trends both within and across countries are Relative Distributions of the BMI. The relative distribution is the set of percentile ranks that the observations from a (comparison) distribution would have if were they placed in another (reference) distribution. Relative Distributions allow comparisons across the whole distribution and have two important properties. Firstly, they are scale-invariant: Any monotonic transform will give identical results (unlike comparisons of Lorenz curves which are only invariant to proportional transformations). This implies that if health is a monotonic transform of BMI (which is approximately true above extreme underweight) and is the same across the groups we are comparing, we can translate results based on relative distributions for the BMI directly into results on the relative distribution of health. Similarly as long as (health-related) utility is a monotonic transformation of health, we can translate these results to (health-related) utility directly. Results based on Lorenz curves and Gini coefficients can only be translated if health (utility) is a multiplicative transform of BMI across groups without further information.

Furthermore, relative distribution methods will identify a difference in distributions if one distribution is rescaled by multiplying every observation by a given amount. This will not be identified by Lorenz curve comparisons as they are invariant to multiplicative transforms (whether identical or not). Thus Lorenz curve comparisons which find no difference between two distributions can only be used to conclude that the distributions are identical after having measured the means (and finding they are also identical). Secondly, the relative distribution is the ‘maximal invariant’: any comparison between two distributions which generates stronger conclusions than those that are obtainable from the relative distribution will not be scale invariant. Section 2 of this paper and Handcock and Morris (1999) describe relative distribution methods in more detail, including approaches to decomposition by location and other distributional characteristics and the measurement of polarization and other indices.

While we do not consider causal explanations in this paper, this discussion is also relevant to analyses attempting to measure the impact of determinants of BMI on changes in the full distribution. In particular, simply considering the conditional mean of the BMI or analysing a discrete measure of obesity may produce results which do not reflect the determinants of the BMI over any part of the distribution and do not reflect any implied effects of these determinants on inequalities in health.

While the extension of results may hold only approximately in our case due to a violation of monotonicity this is more likely to hold than when extrapolating from Lorenz or Gini comparisons of BMI to health or utility. In any case this extrapolation is not necessary for our results to be of interest.
based on the relative distribution.

We use data from the Canadian National Health Population Survey (1994/5) and Canadian Community Health Survey (2000/1) and the Health Survey for England (1994/5 and 2000/1) in order to compare two countries which appear to have experienced different trends in obesity. Section 3 describes these datasets in more detail. Section 4 contains our results which demonstrate the flexibility of relative distribution methods and highlights the impact of changes in the tails of the distribution and polarization in the distribution of BMI. Section 5 contains a short conclusion.

2. Relative Distribution Methods

Investigating inequalities generally revolves around the use of summary statistics to describe differences and changes. One of the most commonly used measures is the Gini coefficient, which is a member of the family of Lorenz curve measures. The Gini is often criticized for focusing on the middle of the distribution and only providing a summary measure of inequality. Relative distributions investigate differences in distributions (Handcock and Morris, 1999). When we complement displays of the PDF and CDF of the relative distribution with a range of entropy and polarization measures we can generate a detailed analysis of differences between any two distributions.

Consider a continuous variable \( Y \) which we observe for a baseline population, called the reference group. We observe the same variable for a different population, \( Y' \), called the comparison group. The cumulative distribution of \( Y \) is represented as \( F(y) \), while the CDF of \( Y' \) is \( F_o(y) \). The relative distribution can be defined as the distribution of the random variable, \( R \), where \( R = \frac{F_o(Y)}{F(y)} \). \( R \) indicates the percentile position of \( Y \) if it were placed in the distribution of \( Y_o \). A point on the relative CDF is then given by the proportion of the comparison distribution that falls below the \( r \)th percentile point in the reference distribution consistent with a given value of \( Y \). Performing this operation for all values of \( Y \) allows us to construct the CDF of \( R \) as:

\[
G(r) = F(F_o^{-1}(r)) = F(Q_o(r)) \quad 0 \leq r \leq 1
\]

where \( Q_o(r) \) is the quantile function of \( F_o \), and \( r \) indicates a point on the CDF of \( Y_o \). The PDF of \( R \), the relative density, is then the derivative of \( G(r) \) with respect to \( r \):

\[
g(r) = \frac{f(Q_o(r))}{f_o(Q_o(r))} \quad 0 \leq r \leq 1
\]

While this is a density ratio, it is also a proper PDF in that it integrates to 1 over the

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\( ^5 \) It is also hoped that our results may point to possible causal factors which have little or no variation within countries at a particular time point and have little change in aggregate over time and which cannot therefore be identified by examining one country in isolation.

\( ^6 \) This is analogous to the counterfactual analysis common in economics and which has been used in other forms to study wage inequality (Barsky et al (2002), DiNardo et al (1996), Lemieux (2002), Machado and Mata (2005)) and to study income related health inequality in health economics as described by O’Donnell et al. (2006).
unit interval. This is achieved by the rescaling through the quantile function. A density ratio over the scale of the original unit of measurement would not generally have this property; it is the use of the quantile function which makes the relative density a proper PDF by converting the denominator to a uniform density.

The primitive of relative distribution methods is the counterfactual percentile ranks that observations from one distribution (the comparison) would take in another (the reference). This leads to immediate comparison between two distributions rather than through the implicit and indirect route of comparison to equality of two distributions as is done in Lorenz comparisons.\footnote{The relative distribution can however also nest the Lorenz curve as a special case under certain conditions. The Lorenz curve is a relative distribution of the variable of interest when compared to a uniform distribution on the interval [0,1] which contains points containing the quantile ranks of the population of interest.}

The relative density provides a robust analysis of the differences between two distributions. It is also useful to examine the two basic components which form the differences: changes in location and changes in shape. If the comparison distribution is only different from the reference distribution because the distribution has simply shifted either additively or multiplicatively (additively on a natural log scale) then only a location change would be observed. For example, if it is found that a later (comparison) cohort has larger BMIs than the earlier (reference) cohort due to a shift in the distribution, then the location change would detect this. Any differences which remain after the location adjustment are due to changes in shape, which comprises scale, skew and other distributional characteristics.

In the context of the distribution of the BMI these changes are especially important. For example, comparing two distributions over time with the earlier distribution as the reference group, a simple location shift would indicate that everyone’s BMI is larger (or smaller) by the same amount (or percentage). Remaining shape effects are important for considering polarization effects, as have been considered in the context of the distribution of income (e.g. Anderson (2004), Levy and Murnane (1992)). In our context, this would be indicated by a tendency towards bimodality with some of those of normal weight shifting to the right (and left), while the tails may also become denser indicating increases in proportions in the extremely underweight and obese categories. While these simple decompositions are primarily descriptive tools, they provide an insight into more complex distributional changes that standard approaches miss. In addition they may suggest specific adjustments for covariates which vary between the comparison and reference populations.

The process of decomposition proceeds by first generating a counterfactual distribution $f_{0L}(y_r)$ which has the same location (median) as the comparison distribution $f(y_r)$ but the same shape as the reference distribution $f_0(y_r)$. This is done by adding the difference in the medians between the counterfactual and reference distributions to every observation in the reference distribution. By constructing a relative distribution using this new distribution, $f_{0L}(y_r)$ in relation to the reference distribution $f_0(y_r)$ we obtain a density ratio which will be uniform if the medians of the comparison and reference distributions are the same. The remaining element of the decomposition involves constructing the relative distribution of the
comparison distribution to the location shifted reference distribution $f_{0\ell}(y_r)$.

Departures from the uniform show differences in characteristics other than location between the comparison and reference distributions. This decomposition can be expressed as:

$$\frac{f(y_r)}{f_0(y_r)} = \frac{f_{0\ell}(y_r)}{f_0(y_r)} \cdot \frac{f(y_r)}{f_{0\ell}(y_r)}$$

The density ratio for the location effect is a proper density (it is always >=0 and it integrates to one over $r$). The density ratio for the shape effect is not a proper density because of the scale change imposed to ensure comparability of the magnitude of effects by using the scale of the reference distribution and examining the magnitudes of the location and shape effects at the same value of $y$ as $r$ varies between 0 and 1. Only in the situation where the comparison and reference distributions have the same location will the denominator of the scale component be uniformly distributed and the density ratio for the shape effect be a proper density.

A number of summary statistics can also be calculated to give a more digestible and objective picture of the relative distributions that we are studying. Two of the most important are entropy based measures and measures of polarization.

Entropy measures can be used to answer the questions: “How much does one distribution differ from another?” “How much does the location shift contribute to the overall difference between the two distributions?”. There are three entropy measures that we can calculate; (i) the overall entropy $D(F; F_0)$: the overall divergence between the comparison and reference groups; (ii) the divergence between the location-adjusted reference group and the reference group $D(F_{0\ell}; F_0)$ which measures the effect of the location shift on distributional divergence; (iii) $D(F; F_{0\ell})$ examines the divergence between the comparison distribution and the location-adjusted reference distribution therefore measuring the difference between the comparison and reference distributions which is due to shape differences. If the second measure is zero then it suggests that the divergence of the overall distributions is due to shape differences. If the entropy measure for the shape change is zero, then the change in location is causing the divergence in the distributions. All the entropy measures used here are based on the Kullback-Leibler divergence measure:

$$D(F; F_0) = \int_{-\infty}^{\infty} \log\left(\frac{f(x)}{f_0(x)}\right) dF(x) = \int_{0}^{1} \log(g(r)) g(r)dr .$$

While the decomposition is not directly additively decomposable due to the rescaling required to compute $D(F; F_{0\ell})$ (i.e. $D(F; F_0) \neq D(F_{0\ell}; F_0) + D(F; F_{0\ell})$), it is possible to construct an additive decomposition by changing the scale of the components.

If we have evidence of divergence between distributions due to changes in shape it is possible that we have polarization occurring. To investigate this possibility polarization measures can be used. We follow Handcock and Morris (1999) and use the median relative polarization index. This measure is particularly useful because it
is location adjusted, in this case for the median, which is an important link to the location and shape decompositions. Other measures of polarization are available but are not location adjusted making the interpretation of the measures harder. If there is no difference in shape between two distributions then the shape change should be represented by a uniform distribution. So a median-adjusted measure of polarization should be sensitive to relative polarization and reflect the difference of the median adjusted relative distribution from the uniform distribution – where the uniform represents median matched distributional equivalence. Handcock and Morris (1999) represent the median relative polarization index (MRP) as:

\[
MRF(F; F_o) = 4 \int_0^1 |r - \frac{1}{2}| g_{ad}(r) dr - 1
\]

It can be seen that those values further from the centre are given a greater weight with the weights increasing linearly with distance from the centre. The measure is re-scaled to provide an index ranging from -1 to 1; Negative values represent less polarization, such that the comparison distribution has more weight near the centre (less polarized) than the location adjusted reference distribution. Positive values represent increasing polarization such that the distribution has moved towards the tails and a value of 0 indicates no distributional difference in shape. This index is symmetric, it is invariant to monotonic transformations of the distributions, it can be interpreted in terms of a proportional shift of mass in the distribution from more central to less central values, and is additively decomposable allowing us to focus on the tails of the distribution to examine whether polarization is occurring because of movements towards the lower or the upper tail. The lower and upper relative polarization indices (LRP and URP respectively) are given by:

\[
LRP(F; F_o) = 8 \int_0^{1/2} |r - \frac{1}{2}| g_{ad}(r) dr - 1
\]

\[
URP(F; F_o) = 8 \int_{1/2}^1 |r - \frac{1}{2}| g_{ad}(r) dr - 1
\]

These are similar to the MRP. Using these we can see where the changes in the distribution are more pronounced. Handcock and Morris (1999) point out one important area of interpretation, because the two distributions have been median matched, the integrated density is made equal above and below the median. In which case, the measures here are not telling us whether median of Y has increased or decreased. The measures here are showing whether the polarizing shape change is larger above or below the median.

Handcock and Morris (1999) present methods for calculating standard errors for these measures. This allows us to test the hypothesis that the polarization is equal to zero. Confidence intervals for the relative distribution and polarization measures are constructed using normal approximations based on asymptotic distributions as described in Handcock and Morris (1999) chapters 9 and 10 respectively. The methods were implemented using R available from http://www.r-project.org/ in conjunction with specific software for relative distribution methods described and available at: http://www.csde.washington.edu/~handcock/RelDist/
3. Data

The National Population Health Survey (NPHS) is a nationally representative Canadian biennial panel which was first collected in 1994/5. It contains a range of health and socioeconomic variables. Although the data set is a genuine panel the public release file does not contain the panel identifier and in any case inference for relative distribution methods has been developed when the samples being compared are independent. If we used the later waves of the longitudinal NPHS this would include data on the same individuals (the NPHS comparisons) and thus there may be downward bias in measures of uncertainty such as standard errors and confidence intervals. In this paper we use the first cycle of the Canadian community Household Survey for 2000/1 which superseded the cross-sectional component of the NPHS and has the same sampling frame. The 1994/5 data set is used as the reference group, the 2000/1 data is used as the comparator group. For 1994/5 we have 12,298 observations on BMI, for 2000/1 we have 69,140. The later data set is much larger as the Canadian Community Health Survey cross-sectional sample is much larger than the NPHS - this also removes concerns of downward bias in the standard errors which would occur using longitudinal data. We use sample weights to ensure the results reflect the population in each cross-section.

The Health Survey of England (HSE) is a nationally representative data set, which was established to monitor the health of individuals in England. The HSE is a repeated cross section with different individuals surveyed each year; this means the data will be independent and measures of uncertainty will be correct. To maintain comparability with the NPHS cross-sections from 1994 and 1995 were combined as were data from 2000 and 2001. The 1994/5 combined data provided a sample of 20,049 individuals, the 2000/1 data provided 14,876. The HSE does not include sampling weights as it is self-weighting and thus representative of the population in each year (Prior et al. 2000).

The two important variables in this analysis are BMI and gender. Since the BMI can be an imperfect indicator of healthy weight, especially among the old, the young and pregnant women, we restricted our analysis to individuals aged between 20 and 64.

The summary statistics for BMI are presented in Table 1.

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8 While quantitative results will vary depending on which group is taken to be the reference and which the comparison group the comparator group for the decomposition analyses, qualitatively they will be identical.
Table 1: Summary statistics for BMI in Canada and England for 1994/5 and 2000/1

<table>
<thead>
<tr>
<th>Observation</th>
<th>Observation</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
<th>% Obese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada 1994/5: Women</td>
<td>6453</td>
<td>24.56</td>
<td>23.50</td>
<td>4.48</td>
<td>15.80</td>
<td>42.70</td>
<td>15%</td>
</tr>
<tr>
<td>Canada 1994/5: Men</td>
<td>5845</td>
<td>25.91</td>
<td>25.70</td>
<td>3.64</td>
<td>16.10</td>
<td>41.20</td>
<td>15%</td>
</tr>
<tr>
<td>Canada 2000/1: Women</td>
<td>36940</td>
<td>24.93</td>
<td>23.90</td>
<td>4.97</td>
<td>14.00</td>
<td>57.60</td>
<td>17%</td>
</tr>
<tr>
<td>Canada 2000/1: Men</td>
<td>32200</td>
<td>26.32</td>
<td>25.60</td>
<td>4.18</td>
<td>14.10</td>
<td>51.10</td>
<td>19%</td>
</tr>
<tr>
<td>England 1994/5: Women</td>
<td>10556</td>
<td>25.77</td>
<td>24.76</td>
<td>4.99</td>
<td>14.01</td>
<td>59.36</td>
<td>17%</td>
</tr>
<tr>
<td>England 1994/5: Men</td>
<td>9493</td>
<td>26.21</td>
<td>25.82</td>
<td>3.86</td>
<td>15.39</td>
<td>48.23</td>
<td>15%</td>
</tr>
<tr>
<td>England 2000/1: Women</td>
<td>7959</td>
<td>26.71</td>
<td>25.580</td>
<td>5.51</td>
<td>8.55</td>
<td>61.31</td>
<td>23%</td>
</tr>
<tr>
<td>England 2000/1: Men</td>
<td>6917</td>
<td>27.17</td>
<td>26.82</td>
<td>4.30</td>
<td>14.73</td>
<td>57.01</td>
<td>22%</td>
</tr>
</tbody>
</table>

The summary statistics clearly demonstrate the increases in BMI which are occurring over time. The mean and median levels of BMI are higher in the later time period for most groups, with England having a higher mean and median than Canada for each group. Over time the differences between the means and medians between England and Canada are getting larger, suggesting that BMIs in England are increasing more quickly. The median is always below the mean, suggesting that the distribution of BMI is right skewed with a long right hand tail. In 1994/5 the proportion of individuals classified as obese was similar for both Canada and England- by 2000/1 the proportion of obese individuals in England was considerably higher than for Canada, although Canada had seen an increase as well. For men and women in Canada the results are fairly similar, with men having a slightly higher average weight given height and a higher proportion of obesity in 2000/1. In England men have larger BMIs, on average, than women but there are a higher proportion of obese women in both time periods. For both countries average weight falls into the overweight category for both time periods.

The nonparametric density plots in Figures 1 and 2 show that the densities are right skewed with long right hand tails. The densities have moved to the right over time, with the modal point of the distribution being closer to 30 in 2000/1 than in 1994/5. There are also increases in the right hand tail. There is also some indication of an increase in the left hand tail for Canadian women. These changes at the tails may be the reason why the standard deviation has increased over time. Investigation of these hypotheses can be investigated using the relative distribution methods and related polarization measures.

4. Results

Initially we calculate Gini coefficients of inequality in BMI. This allows us to compare standard measures of inequality to see whether these measures change over time. We compare interpretations using standard summary statistics and Gini coefficients to those using the relative distribution and related entropy and polarization measures, demonstrating how the latter give a more complete and less restricted pattern of distributional comparisons using the BMI for men and women in Canada and England. Figures 3 to 10 present the relative distribution results for each of our categories. The polarization results are given in Table 2.
Using the NPHS and CCHS data, the Gini coefficients are 0.10 and 0.11 for women in 1994/5 and 2000/1 and 0.08 and 0.09 for men, demonstrating slight increases in inequality in the distribution of BMI over time. Figures 3, 4 and 5 show the comparisons between 1994/5 and 2000/1 for Canadian women and men separately with 1994/5 as the reference groups. In order to clarify the interpretation of the various results and measures we present we first consider the relative distribution and its decomposition in some detail.

The relative CDF is shown in Figure 3. The dotted (45°) line is the line of distributional equivalence. A curve below the 45° shows larger BMIs in the comparison distribution, while a curve above the 45° line indicates larger BMIs in the reference distribution – the curve can cross the 45° line. In Figure 3 the comparison group has slightly larger BMIs than the reference group, with only 47% of the comparison group having a lower BMI than the median of the reference group.

While relative CDFs are informative, the relative PDFs and their decomposition provide the most insights. Figure 4a) shows the overall relative pdf indicated by a solid line and the upper and lower limits of the 95% confidence interval for the relative pdf indicated by dashed lines. For values of r less than around 0.55 there is a greater frequency of observations in the reference distribution as(g(r)<1). For higher values we see that there is a greater frequency of observations in the comparison data, (g(r)>1, demonstrating an increase in weight at the top of the distribution and higher levels of obesity. Figure 4b) shows the relative distribution of the location shifted reference distribution to the reference distribution. We use an additive median location shift. The location shift shows how much of the difference in distributions is due to changes in the location of the distribution. If there is no shift in location the distribution should be uniform with relative density g(r) =1 for all r. If the divergence in the distributions arose because every BMI observation was increased by the same additive factor then we would expect all divergence between the two groups to be explained by a location shift to the right and an upward sloping location adjusted relative pdf. This is what we see in figure 4(b) as, for Canadian women, comparing a distribution with the same shape as the reference group and the location of the comparison group to the reference group leads to more observations in the reference group than the counterfactual comparison group at the lower end of the distribution, while the opposite is true at the top end of the distribution. This suggests an increase in median BMI between the two periods although for much of the distribution (0.2<r<0.6) we cannot reject the hypothesis that there was no change in density at that point ((g(r)=1) indicating that the location shift was relatively small. Any differences in the distributions left after adjusting for location shifts must, by definition, result from changes in shape. If the location shift is sufficient to explain all differences between the comparison and reference group, the relative density comparing the comparison density to the location matched reference density would be uniform. However, the distribution taking account of shape difference only, shown in Figure 4c), indicates evidence of polarization, with increases in both tails of the distribution away from the middle.(g(r) >1 for r<0.1 and r>0.9)

The entropy measures can be used to assess overall dispersion between the two distributions. The results demonstrate that the shape shift (Entropy=0.0077) has been
far more important in generating the differences between the two distributions (entropy for the location shift is approximately zero). Using the polarization measures we can get an idea of the extent of the changes. The overall polarization in Table 2 is 0.051, while the upper index is 0.015 and the lower index is 0.087. The overall measure shows the amount of polarization in the distribution, the lower (upper) index is the contribution to the median index of the relative distribution below (above) the median. The median and lower index measures are significant at the 5% level of significance and show that polarization is occurring, the majority of this in the lower tail.

While mean –variance comparisons suggest similar patterns for both sexes, with increasing weight and variance over the two periods the relative distributions and decompositions look somewhat different. Figure 5a) shows that for men for values of \( r \) less than around 0.8 there is a greater frequency of observations in the reference distribution rather than 0.55 for women suggesting a more rapid increase in weight for men. The additive median shift is close to zero for men as shown in figure 5b). However there is clear evidence of increasing polarization, particularly in the upper half of the distribution visible from both figure 5c) and the polarization measures. Not surprisingly given the above, the entropy measures suggest that the shape shift has again been most important.

**ENGLAND**

We now present an analogous set of results for England using the HSE data. These results are more striking. We also present the overall comparison for this case as it strikingly demonstrates the inadequacies of this standard measure int his context. The overall Gini coefficient for BMI in 1994/5 is 0.09 and in 2000/1 is 0.10, demonstrating a small increase in inequality. The relative distributions illustrate a considerable change in the overall distribution. As can be seen in figure 6(a) there is a higher frequency of observations in the upper half of the distribution than the lower half – the relative density demonstrates an approximately exponential increase with \( r \). A considerable amount of the change is from the median shift, as shown in figure 6(b) and the entropy measure. However, 6(c) shows that there has also been polarization, with individuals moving to the tails of the distribution. The Gini coefficient does not detect such movement. Looking at the entropy measures we see that both the median shift and the shape change contribute similar amounts to the overall difference between the two distributions. Examining the polarization measures in Table 2, we can see significant polarization in each tail. The Gini coefficient, because it focuses on the middle of the distribution, does not pick up these subtle yet important changes in the data. For women in 1994/5 and 2000/1 respectively the Gini coefficients were 0.10 and 0.11, while for men they were 0.08 and 0.09 respectively. However, while these are the same figures as observed for Canada the relative distributions are far from identical. Figures 7 and 8 demonstrate that for both women and men there are more observations in the right half of the distribution in 2000/1, with the relative density steeper for men. The location shift demonstrates that both women and men have experienced an increase in median BMI over the period but for men the location-shifted relative density is steeper and sharper than for women. There is evidence of polarization for both women and men. The entropy measures suggest that the divergence between the 1994/5 and 2000 distributions is greater for men than for
women (overall entropy of 0.030 for men compared to 0.016 for women). However, the biggest factor for men seems to be the median shift – BMI increasing over the period, with a relatively small effect of polarization. For women the polarization is the dominant factor in explaining the differences in the distributions. The polarization measures in Table 2 confirm these results. Both men and women experience significant polarization but for women the effect is much larger. The Median polarization statistic for men is 0.057, while for women it is 0.084, suggesting that English women are polarizing away from the median at a faster rate than English men.

ENGLAND v CANADA

For our final set of analyses we directly compared the distributions between Canada and England, using Canada as our reference population.

Making the comparisons separately for men and women we can see clear gender differences. For men there is little difference between the Canadians and the English in 1994/5, with the relative density quite flat, with not much effect for either the location or the shape shifts. However by 2000/1 (figure 9) more differences have emerged. However Figure 9(a) shows a much higher relative frequency of English men in the upper half of the Canadian distribution. The median shift in 9(b) shows that the distribution for English men is well described by a (large) additive shift relative to the Canadian distribution in 2000/1, while, somewhat interestingly, Table 2 shows that the English distribution is relatively more polarized below the median and less so above it.

Figures 10 shows differences between Canadian and English women which are relatively stable. There is a much lower frequency of observations for English women below the Canadian median in both periods, and conversely above it; in 2000/1 only around 40% of English women have lower BMIs than the median Canadian woman. There are almost no differences in the distributions conditional on an additive location shift equal to the difference in the medians of the distributions in 1994/5. The median shift drives nearly of the difference in 1994/5 while figures 10a) and 10b) show this is somewhat different in 2000/1 where the distribution for English women is also relatively polarized compared to Canadian women both above and below the median.

5. Conclusion

In this paper we have used relative distributions to examine changes in the distribution of the Body Mass Index (BMI) in England and Canada during the period 1994-2001. The use of relative distributions allows us to consider changes in the whole distribution of the BMI in a nonparametric fashion. While statistics analogous to the Gini Index can be constructed based on the relative distribution, important characteristics of changes in the distribution of the BMI such as changes in the proportions overweight and obese are more naturally handled using measures of relative polarization. Our results show that while BMI has increased in both countries, BMI in England has increased at a much faster rate than in Canada. In addition both groups show polarization over time towards both tails of the weight distribution, with the English polarizing towards the upper tail at a faster rate than Canadians. Future work will consider decomposing the distributions further by using covariate decompositions to explain the changes we have observed in the distributions in this
paper using education, income and lifestyle characteristics. Use of relative
distributions and associated methods allow this to be done non-parametrically
providing estimates of the impact of changes in socio-demographic composition of
the distributions while also isolating changes over time in the conditional distributions
for each socio-demographic group.
Figure 1: Nonparametric plots of BMI, Canada 1994/5 and 2000/1

Figure 2: Nonparametric plots of BMI, England 1994/5 and 2000/1
Figure 3: Canada 2000/1 to 1994/5: Women.

Figure 4: Canada 2000/1 to 1994/5: Women

a) Overall relative density  b) Location decomposition  c) Shape decomposition

entropy = 0.0041  entropy = 0.000  entropy = 0.0077
Figure 5: Canada 2000/1 to 1994/5: Men

a) Overall relative density  b) Location decomposition  c) Shape decomposition

Figure 6: England 2000/1 to 1994/5

a) Overall relative density  b) Location decomposition  c) Shape decomposition
Figure 7: England 2000/1 to 1994/5: Women

a) Overall relative density

\[ \text{entropy} = 0.017 \]

b) Location decomposition

\[ \text{entropy} = 0.0056 \]

c) Shape decomposition

\[ \text{entropy} = 0.011 \]

Figure 8: England 2000/1 to 1994/5: Men

a) Overall relative density

\[ \text{entropy} = 0.031 \]

b) Location decomposition

\[ \text{entropy} = 0.024 \]

c) Shape decomposition

\[ \text{entropy} = 0.0064 \]
Figure 9: England to Canada 2000/1: Men

a) Overall relative density
   \[ \text{entropy} = 0.026 \]

b) Location decomposition
   \[ \text{entropy} = 0.017 \]

c) Shape decomposition
   \[ \text{entropy} = 0.0074 \]

Figure 10: England to Canada 2000/1: Women

a) Overall relative density
   \[ \text{entropy} = 0.065 \]

b) Location decomposition
   \[ \text{entropy} = 0.050 \]

c) Shape decomposition
   \[ \text{entropy} = 0.0066 \]
## Table 2: Polarization Indices

### Canada

#### Women 2000/1 to 1994/5

<table>
<thead>
<tr>
<th></th>
<th>Lower CI</th>
<th>Estimate</th>
<th>Upper CI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
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#### Men 200/1 to 1994/5

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<th>Estimate</th>
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<th>p-value</th>
</tr>
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<td>0.032</td>
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### England

#### All 2000/1 to 1994/5

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<th>p-value</th>
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<tr>
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#### Women 2000/1 to 1994/5

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<th>Estimate</th>
<th>Upper CI</th>
<th>p-value</th>
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#### Men 2000/1 to 1994/5

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<th>Estimate</th>
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<th>p-value</th>
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<tr>
<td>Upper</td>
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</table>

### Cross country comparisons

#### England to Canada 2000/1: Men

<table>
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<td>Upper</td>
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<td>-0.057</td>
<td>-0.026</td>
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</table>

#### England to Canada 2000/1: Women

<table>
<thead>
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<th>Lower CI</th>
<th>Estimate</th>
<th>Upper CI</th>
<th>p-value</th>
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<tbody>
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<td>0.069</td>
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<tr>
<td>Lower</td>
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<td>0.071</td>
<td>0.099</td>
<td>0.000</td>
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<tr>
<td>Upper</td>
<td>0.011</td>
<td>0.039</td>
<td>0.068</td>
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</table>
References:


