Distance distributivity and the semantics of indefinite noun phrases

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May 6, 2016

1 Overview

• Binominal each: The students ate an apple each.

• New finding:
People prefer The students ate one apple each over The students ate an apple each.

• People prefer “numeral noun each” over “a/an noun each”, although both are generally fine (Hessel et al. 2015)

• Our claim: NPs with a/an N in object position can have an incorporation interpretation.

(1) read a book
(2) eat a cookie

• In other words, John read a book has two interpretations:

(i) There is a book such that John read it. (“normal” interpretation)
(ii) John engaged in book-reading. (incorporation interpretation)

• The incorporation interpretation is not compatible with a distributive interpretation.

• In an example like (3), there can be a slight garden path effect (you start with the incorporation reading) which means it is not as good as (4):

(3) The boys watched a movie each.
(4) The boys watched one movie each.

• need to provide: (i) the set of readings assigned to (3) without each, (5), and (ii) what binominal each contributes to the sentences it occurs in

(5) The boys watched a movie.

• it turns out that there are interesting generalizations relating (i) and (ii), which again lead us to the semantics of indefinites

• more general characterization of distribution/preferences of binominal each:

(6) Distribution:
(i) Binominal each must attach to an indefinite noun phrase;
(ii) Within this class, there is a preference to attach to NPs containing numerals than NPs containing the indefinite article.

• we will begin with (i), and use it to help make sense of the numeral vs. indefinite result

∗This talk builds on work done by and with our students Dejan Milačić, Ashley Sokalski, Kunio Hessel, and Lisa Sullivan. We thank Ash Asudeh, Lev Blumenfeld, Mary Dalrymple, Danny Fox, Jane Grimshaw, Martin Hackl, Irene Heim, Rob Truswell, and audiences at Sinn und Bedeutung, Carleton University, MIT, the University of Edinburgh, and the University of Oxford.
2 Binominal each

(7) The students read a book each.
(8) We’ll paint a few rooms each.
(9) Susan and Peter tricked two children each.

- Distance distributivity
  - Different from other uses of each \cite{SafirStowell1988, Zimmermann2002b, KobuchiPhilip2006, Milačić2014} and others

(10) binominal each: The boys carried a table each.
(11) floated each: The boys each carried a table.
(12) prenominal each: Each boy carried a table.

Binominal each does not occur with intransitives:

a. *The boys left each.

b. The boys each left.

c. Each boy left.

(14) Binominal each must occur with an indefinite NP:

a. The boys kicked \{a/one/the/every/no\} rock each.

b. The boys each kicked \{a/one/the/every/no\} rock.

c. Each boy kicked \{a/one/the/every/no\} rock.

- The subject is a plural (definite) NP; the object is an indefinite NP
- The “sorting key” (NP1) does not have to be a subject, and the “distributed share” (NP2) does not have to be an object (terms: \cite{Choe1987})

(15) Students have to sell a modest quota of three books each. (www)
(16) With one smile each, the three men disappears from the apartment, leaving the door open to reveal the mess after her fight to survive. (www)
(17) If u were going to a twin party then I would get the twins a present each. (www)

- in all languages we know of, markers of distance distributivity are restricted to marking indefinite NPs
- thus, always express: \( \forall x \exists y R(x, y) \)
- vary in how they mark the indefinite\(^1\)

(18) Variation

a. English: Add each to indefinite object


c. Hungarian: Reduplicate the numeral: the boys read one-one book \cite{Farkas1997}

Reduplication as a marker of distributivity occurs in a variety of languages and language families; for example, Finno-Ugric (e.g., Hungarian) Algonquian (e.g., Cree \cite{Junker2000}), Iroquoian (e.g., Lakhota \cite{RoodTaylor1996}), Salish (e.g., Lushootseed \cite{Urbanczyk2001}), St’at’imcets (Matthewson2000), Mayan (e.g., Chol \cite{Alvarez2011}), Dravidian (e.g., Kannada \cite{Anderson2012}) and Telugu (Balusri2006), Tibeto-Burman (e.g., Ao \cite{Gowda1975}), Bantu (e.g., Kanyok \cite{Mukash-Kalel1982}), Gil \cite{Gil1982, Gil2013} and Moravcsik \cite{Moravcsik1978} offer more examples.

\(^{1}\) They also vary in the domains that the universal quantifier can quantify over: just individuals (e.g., English), individuals and events/times/situations (e.g., German), or individuals, events/times/situations, and worlds (Russian). See Milačić et al. \cite{Milačić2015} for a more specific typological characterization.
Typological Generalization: Across languages from different language families: distance distributivity is expressed by adding morphology to indefinite NPs (NB: the added morphology is typically not identical with the language’s distributive quantifier)


- however, so far as we can tell, none of that literature derives (19)
- diagnosis: they focus on finding a semantics of binominal each as an operator
- we will suggest instead that each is a bound-variable
- our proposal builds on the observation that the sentence without each has a meaning that can be unambiguously expressed by adding each, namely, the distributive meaning
- suggestion: reuse all relevant semantic machinery needed to get the distributivity meaning without each; each just disambiguates by marking the underlying quantifier-dependence
- Binominal each seems to pair members of the subject set with members of the object set: capture this ‘pairing’ explicitly in the underlying semantic representation with appeal to General Skolem Functions

Marking quantifier-dependence

- for any choice of boy x, f maps x to a table f(x, table); the sentence is true so long as there is a mapping such that each boy lifted the table that they’re mapped to
- suggestion: the variable x in f(x, table) can be realized as ∅, in which case the boys lifted a table is ambiguous, or realized as each in English
- other languages: spelled out as reduplicated numeral, or as some other lexical item unrelated to distributive quantifier in the language (e.g., Slavic po)

Readings of the boys lifted a table

- we suggest that (21) has another reading: [∀x : boy(x)] [x engaged in table-lifting]

2In fact, there is evidence that each is subject to Condition A, e.g., *the boys believe Mary lifted a table each. See e.g., Hudson 1970, Kayne 1981, Burzio 1986.
An aside: The distributivity hierarchy

- We think that our Skolem analysis can make sense of another empirical generalization: speakers judge binominal *each* to be “more distributive” than other uses of *each*, and also *every* and *all* (Hessel et al. 2016 and also Tunstall 1998).

- Two on-line voluntary, anonymous questionnaire studies, forced choice.

- Questions:
  - In which case is *the boys fed the same dog* more likely to be true?
  - In which case is *the students ordered a pizza together* more likely to be true?

- Options, examples:
  - Every boy fed a dog.
  - The boys fed a dog each.

- Distributivity hierarchy, ranked from distributive to collective:
  - binominal each > prenominal each > every > floated each > all

- Our thoughts on why binominal *each* is the most distributive:
  - binominal *each* does not allow wide-scope of the indefinite, and this follows from our analysis.
3 Numerals vs. indefinite articles

- Speakers seem to prefer numerals (22) over an indefinite article (23) in the binominal each DP.

(22) a. The nurses treated one patient each.
    b. The chefs prepared seven dishes each.

(23) a. The nurses treated a patient each.
    b. The pilots flew a plane each

3.1 Our English study: Methods

- We conducted three different experiments (grammaticality questionnaires)

- The questionnaires had 28, 43, and 86 participants (actually more; we are only concerned with the native speakers of English here). The studies had different stimuli and slightly different designs, but the results were the same. We report only on the third study here.


- First question: Is English your first language?

- 14 target sentences, 30 fillers.

3.2 Our English study: Results

- 69 out of 86 participants (80%) preferred numerals over indefinites; 17 did not (7 had the opposite preference, and 10 had no preference).

- Binomial test for proportions significant at the 99% level, indicating a strong preference for numerals over indefinites.

(24) All examples with indefinites and all examples with numerals:

<table>
<thead>
<tr>
<th>Condition</th>
<th>$M(SD)$</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indef</td>
<td>5.18(1.68)</td>
<td>[4.82, 5.54]</td>
</tr>
<tr>
<td>NUM</td>
<td>5.86(1.42)</td>
<td>[5.56, 6.16]</td>
</tr>
</tbody>
</table>

The researchers published seven papers each; The nurses treated one patient each. M=6.21; SD=1.34
The pilots flew a plane each; The girls lifted a box each. M=5.05, SD=1.84
Welch t-test p<0.01
4 Indefinite objects may have incorporation semantics

- Proposal: John read a book has two interpretations:
  
  (i) There is a book such that John read it. (“normal” interpretation)
  
  (ii) John engaged in book-reading. (incorporation interpretation)

- Some examples of real incorporation (morphological incorporation):

  (25) pitstuvunga
  
  pitsi-tu-vunga
  
  dried.fish-consume-intrans.indic.1s ‘I’m eating dried fish.’

  (26) anin õdtiydaripten
  
  anin õd-tiy-dar-in-ten
  
  he not-give-rice-me-3past ‘He didn’t give me rice.’

  van Geenhoven (1998): in sentences like every man loves a woman, the narrow reading is a result of semantic incorporation

- there are also other suggestions in the literature that indefinites might have an incorporation semantics even in absence of morphosyntactic incorporation (e.g., Chung and Ladusaw 2004, Carlson 2006, López 2012)

- Why do people like read one book each better than read a book each?

  Our proposal: The incorporated reading is not compatible with binominal each, which requires the members of the set denoted by the object to be picked out individually. Reference to individuals (or sets of individuals) is not possible under the incorporation reading. If an indefinite NP receives an incorporated reading, listeners must repair when they get to each (garden path).

Carlson 2006:

- Carlson lists six + one stable properties of the semantics of incorporation (Carlson 2006).

- Most of them seem to hold for indefinite NPs as well.

- We refer to a/an NPs as a-NPs below to distinguish them from other indefinite NPs (some, one, any...)

- We go through Carlson’s criteria in sections 4.1–4.7

4.1 The incorporated nominal is interpreted as an indefinite

- “The incorporated nominal is interpreted as an indefinite, rather than as a definite or some quantified type of noun phrase.”

4.2 The indefinite is non-specific, rather than specific in import

- A-NPs can be interpreted as non-specific, and are interpreted as non-specific under the incorporated meaning.

4.3 The indefinite is interpreted as a narrow-scope indefinite only

- “The indefinite is interpreted as a narrow-scope indefinite only, showing no scoping interactions with other logical operators in the same sentence that is typical of syntactically-expressed indefinites.”

- A-NPs can be interpreted with narrow scope, and are interpreted this way under incorporation.

4.4 The incorporated nominal is interpreted as an existential

- “The incorporated nominal is interpreted as an existential, and not as a generic indefinite – it lacks the ‘universal’ flavor of true generics.”

- The normal interpretation of a-NPs is as existentials.
4.5 The verbs that allow for incorporation are stage-level verbs

- “The verbs that allow for incorporation are stage-level verbs, individual-level stative predicates like hate or know are systematically excluded.”
- stage-level: temporal stage, individual-level: generally true for the individual
- A-NPs can be part of stage-level predicates (he ate a burger, he imitated a lawyer); so far as we can tell, the intuition about an incorporated meaning for a-NPs only arises with stage-level predicates

4.6 The incorporated nominal is number-neutral in interpretation

- “The incorporated nominal is number-neutral in interpretation, though in most languages with a singular-plural distinction the count noun forms may easily be taken for singulars.”
- It seems like a-NPs are naturally interpreted as singulars.
- John read a book is often understood as asserting that John read exactly one book
- However, the singular interpretation can be overridden, or at least it’s not as important as when you say “one N”.
- It has been argued (e.g., [Heim 1991]) that the singular interpretation is merely an implicature (e.g., John read a book last week; in fact, he read two...).
- Difference between John read a book and John read one book: the exactly-one implicature is stronger with the numeral than with the indefinite
- It is well-known that the implicatures of numerals are more robust than other implicatures (e.g., see Chemla and Singh 2014a[b] for an overview)

(27) In the summers, I love lying on the beach reading a book.
(28) I hope I’ll make a friend today.

Contrast with:
(29) # In the summers, I love lying on the beach reading one book.
(30) # I hope I’ll make one friend today.

- In the examples with one, the exact quantity is more important, i.e., sentences containing numerals seem to be naturally used to answer a how many question
- This is especially clear in light verb constructions.

(31) Susie might have a drink tonight.
(32) I love taking a nap.
(33) I think I’ll have a swim in the pool.

- In light verb constructions, the V+a-NP can be replaced by a verb (to drink, to nap, to swim).
- It seems strange to argue with someone about the number of entities satisfying the a-NP in light verb constructions:

A: I had a drink last night.
B: # What are you talking about? You had three!
- B’s response would have been more reasonable had A said I had one drink last night (stronger implicature that it was only one)
- Also: Do you want to go for a drink? vs. Do you want to go for one drink? — The latter implies just one drink
4.7 The domain of the incorporated noun is usually restricted

- The restrictions can be grammatical: formal/syntactic restrictions on the noun or the verb
- or semantic: “some ‘familiar’ cultural significance be accorded the action whether there is semantic enrichment of not”
- Maybe something like read a book is more likely to take on an incorporation meaning than punch a lightbulb?

4.8 Summary

- It seems that a-NPs (e.g., a book) trivially fulfill many of the criteria of incorporation semantics, and can be argued to at least sometimes fulfill the remaining criteria.
5 Swedish

5.1 Binominal var in Swedish.

(34) Vi kontaktade en resebyrå var.
    we contacted a travel agency each
    ‘We contacted a travel agency each.’

(35) Dom köpte två hundar var.
    they bought two dogs each
    ‘They bought two dogs each.’

- The word var is equivalent to binominal each in (34–35).
- The indefinite article en/ett is the same as the number one in Swedish.
- Example (34) could also be translated as ‘We contacted one travel agency each.’

5.2 Swedish survey: Design

- Grammaticality questionnaire on SurveyMonkey.
- Instructions: rate examples according to naturalness, 1-5.
- 428 participants.
- Target sentences spread out over four surveys so that the same participant would not see the same lexical items in different frames.

5.3 Binominal var: singular and plural

- Like in English, speakers prefer numeral examples over indefinite article examples: they prefer (35) over (34).
  (Recall that en/ett is the indefinite article and the number one.)
- However, in addition to binominal var, Swedish has another equivalent of binominal each:

(36) Vi läste varsin bok.
    we read each POSS book
    ‘We read a book each.’

- The word varsin is (at least seemingly) composed of var ‘each’ and the third person reflexive possessor sin.
- Speakers prefer (36) over (34).
- The varsin form is only marginally acceptable in plurals.

- Overall results from our survey:

<table>
<thead>
<tr>
<th>example</th>
<th>average rating (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) varsin</td>
<td>Vi läste varsin bok.</td>
</tr>
<tr>
<td>(b) SG + var</td>
<td>Vi läste en bok var.</td>
</tr>
<tr>
<td>(c) PL + var</td>
<td>Vi läste två böcker var.</td>
</tr>
</tbody>
</table>

- the difference between (a) varsin and (b) SG + var is significant (p<.01)
- the difference between (b) SG + var and (c) PL + var is significant (p<.01)
- the difference between (a) varsin and (c) PL + var is not significant (p=.33)

So far: pretty much the same as English, except the indefinite article and the number one are homophonous in Swedish, and Swedish has the additional varsin form.
5.4 Pseudo-incorporation in Swedish

- Swedish allows what has been called ‘pseudo-incorporation’: a plain singular noun is allowed as an object, although singular nouns normally require a determiner.

(37) Dom har köpt ett hus.
    they have bought a house
    ‘They have bought a house.’

(38) Dom har köpt hus.
    they have bought house
    ‘They have bought a house.’ (Or “They have house-bought”)

- Pseudo-incorporation examples such as (38) have incorporation semantics, as outlined in Carlson (2006); see section 4 above.

- Not all verb+object combinations allow pseudo-incorporation in Swedish. Compare the examples in (39) to the examples in (40).

(39) a. Vi tog bil till festen.
    we took car to party
    ‘We took a car to the party.’

   b. Dom såg film tillsammans.
      They watched movie together
      ‘They watched a movie/movies together.’

(40) a. *Ni hittade ledträd.
       you.PL found clue
       (intended: ‘You found a clue’)

   b. *Hon dansade med kompis.
      she danced with friend
      (intended: ‘She danced with a friend.’)

- The judgements are Ida’s, supported by collocation searches on google. We also tested the following examples in the questionnaire:

(41) Maja och Kalle köpte en hund.
    Maja and Kalle bought a dog.
    ‘Maja and Kalle bought a dog.’

(mean: 4.90, median: 5)

(42) Sanna och Putte köpte hund.
    Sanna and Putte bought dog.
    ‘Sanna and Putte bought a dog.’

(mean: 4.56, median: 4)

(43) Familjen köpte bil.
    Family the bought car.
    ‘The family bought a car.’

(mean: 4.36, median: 5)

(44) *Hanna och Petter kritiserade bok.
    Hanna and Petter criticized book
    ‘Hanna and Petter criticized a book/books.’

(mean: 1.42, median: 1)
5.5 Swedish pseudo-incorporation and binominal var

- Examples that allow pseudo-incorporation receive significantly lower ratings for examples with singular NP + var than examples that do not allow pseudo-incorporation.

<table>
<thead>
<tr>
<th>each</th>
<th>allows incorporation</th>
<th>rating</th>
<th>does not allow incorporation</th>
<th>rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>varsin</td>
<td>ex. Vi köpte varsin hund.</td>
<td>4.73</td>
<td>ex. Vi kritiserade varsin bok.</td>
<td>4.66</td>
</tr>
<tr>
<td>SG + var</td>
<td>ex. Vi köpte en hund var.</td>
<td>3.93</td>
<td>ex. Vi kritiserade en bok var.</td>
<td>4.15</td>
</tr>
<tr>
<td>PL + var</td>
<td>ex. Vi köpte två hundar var.</td>
<td>4.71</td>
<td>ex. Vi kritiserade två böcker var.</td>
<td>4.59</td>
</tr>
</tbody>
</table>

- When pseudo-incorporation is possible, en X var ‘an X each’ is rated lower than when pseudo-incorporation is not possible.

- T-tests for incorporation VPs:

<table>
<thead>
<tr>
<th>INCORP.</th>
<th>averages</th>
<th>significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>varsin — SG + var</td>
<td>4.73 — 3.93</td>
<td>yes</td>
</tr>
<tr>
<td>varsin — PL + var</td>
<td>4.73 — 4.71</td>
<td>no</td>
</tr>
<tr>
<td>SG + var — PL + var</td>
<td>3.93 — 4.71</td>
<td>yes</td>
</tr>
</tbody>
</table>

- T-tests for non-incorporation VPs:

<table>
<thead>
<tr>
<th>NON-INCORP</th>
<th>averages</th>
<th>significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>varsin — SG + var</td>
<td>4.66 — 4.15</td>
<td>yes</td>
</tr>
<tr>
<td>varsin — PL + var</td>
<td>4.66 — 4.59</td>
<td>no</td>
</tr>
<tr>
<td>SG + var — PL + var</td>
<td>4.15 — 4.59</td>
<td>yes</td>
</tr>
</tbody>
</table>

- Caveat: We have not yet tried to replicate the Swedish study.

- The study included six incorporable and six non-incorporable V-NP combinations.

- Examples (45-46) were excluded from our analysis, because we were surprised by the very low rating for (46).

- However, note that the low rating for (46) is not problematic for our hypotheses.

(45) Ken och Sandra tog varsitt flyg hem. mean:4.59, median:5
Ken and Sandra took each their plane home ‘Ken and Sandra took a flight/plane each home.’

(46) Ken och Sandra tog ett flyg var hem. mean:1.84, median:1.5
Ken and Sandra took a plane each home ‘Ken and Sandra took a flight/plane each home.’

- The VPs that allow pseudo-incorporation differ from the verbs that don’t allow pseudo-incorporation, even though they pattern similarly in general (cf. the last two tables).

- The VPs that allow pseudo-incorporation do not work so well with SG + var (although they are grammatical). We argue that this is because such examples are especially likely to be interpreted with incorporation semantics (as evidenced by the fact that they allow pseudo-incorporation).

- In other words: “we house-bought” is closer at hand than “we book-criticized”, even when an indefinite article is present (when the object is not formally pseudo-incorporated)

- We assume that direct objects with indefinite articles can be semantically incorporated in Swedish as well as in English, even though Swedish also has the possibility of formal (morphological) pseudo-incorporation.
An aside: Some more stuff about Swedish universal quantifiers & distributivity

(47) Alla studenterna läste en bok.
    all students.the read a book.
    ‘All the students read a book.

(48) Varje student läste en bok.
    every/each student read a book
    ‘Every student read a book.’

(49) Varenda student läste en bok.
    every/each student read a book
    ‘Every student read a book.’

(50) Studenterna läste var och en en bok.
    students.the read each and one a book
    ‘The students each (and one) read a book.’

(51) Studenterna läste en bok var.
    students.the read one book each
    ‘The students read one book each.’

(52) Studenterna läste varsin bok.
    students.the read each.PESS book
    ‘The students read one book each.’

- Distributivity hierarchy (according to forced-choice survey, 150 respondents),
  ranked from more distributive to more collective:

  varsin > binominal var > \{ varje > floated var och en \} > alla
6 Summary

• We proposed that various puzzles in distance distributivity can be resolved by a better understanding of the semantics of indefinite noun phrases.

• We captured the restriction to indefinites by appealing to Skolemized Choice Functions.

• Within this class of acceptable hosts of binominal each, speakers prefer NPs with a numeral over NPs with an indefinite article with binominal each; in other words: *We ate three apples each* is preferred over *We ate an apple each*.

• We argue that this is because objects with indefinite articles (a/an) can be semantically incorporated in English.

• In a sentence like *they caught a fish each*, an incorporation reading is possible (though not necessary) up until *each*. If the listener gets the incorporation interpretation, they need to repair when they get to *each*.

• Objects with numerals are not compatible with incorporation semantics because they are not compatible with a number-neutral interpretation.

• van Geenhoven (1998) has already proposed (for different reasons) that indefinite objects in English can be semantically incorporated.

• verb + a/an-NPs (*read a book, eat a cookie*) are compatible with typical characteristics of incorporation Carlson (2006).

• Swedish allows pseudo-corporation for some verb-object combination; e.g., *köpa hus, köra bil* (*buy house, drive car*).

• Verb-object combinations that allow pseudo-incorporation are rated significantly lower with post-object var (∼ binominal each) than verb-object combinations that do not allow pseudo-incorporation.

• We propose that this is because verb-object combinations that allow pseudo-incorporation in general easily allow an incorporation reading.
References


Bauman, Carina, Nicole Holliday, Kuo-Chiao Lin, Nathan LaFave, Sean Martin, and Allison Shapp. 2012. Every sentence has how many quantifiers each? Binominal each in English. In Poster at Mid-Atlantic Colloquium of Studies in Meaning, University of Maryland.


Zimmermann, Malte. 2002a. *Boys buying two sausages each: On the syntax and semantics of distance-distributivity*. LOT.

A Distance distributivity meanings and their compositional derivation

(53) The boys lifted a table each


- that is, we argue that the meaning above can be captured if the logical form of 53 is: $\exists f[\forall x: \text{boy}(x)][\text{lifted}(x, f(x, \text{table}))]

- how is this derived?

- lift is a transitive predicate: $\text{lift}(x, y)$ (or, more accurately, $\lambda y.\lambda x.\text{lift}(x, y))$

- in 53 $y = f(x, \text{table})$ (this is the indefinite object a table each), and the variable in $f(x, \text{table})$, as well as the first argument of lift, is bound by the higher universal quantifier

- challenge: explain how the universal quantifier, and the complex second argument of lift, are derived

- consider the second argument $f(x, \text{table})$

- we want this to be the denotation of a table each

- $f$ is a variable over General Skolem Functions, which we assume here gets existentially closed in matrix position

- a General Skolem Function is a function which takes $k$ individual arguments $x_1, \ldots, x_n$, and a predicate $P$, and returns in individual in $P$: $f(x_1, \ldots, x_k, P) \in P$; in the above, $f(x, \text{table})$ maps individual $x$ to a table (e.g., Schlenker 2006)

- we assume that indefinites denote variables over $k$-arity General Skolem Functions, $k \geq 0$ (e.g., Winter 2004, Schlenker 2006, Steedman 2012)

- the universal quantifier $[\forall x: \text{boy}(x)]$ is generated by the subject the boys $D$, where $D$ is a ‘distributive’ operator; see e.g., the treatment of distributivity in Heim et al. (1991)

(54) Deriving the universal quantifier

a. $[[\text{boys}]] = \{b_1, b_2, b_3, b_1 \oplus b_2, b_1 \oplus b_3, b_2 \oplus b_3, b_1 \oplus b_2 \oplus b_3\}$

b. $[[\text{the boys}]] = b_1 \oplus b_2 \oplus b_3$ (the selects the maximal element in its input; e.g., Link 1983)

c. $[[\text{the boys }D]] = \lambda P, [\forall x: P(x)]$

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3See Milačić et al. (2015) for evidence that $k$ is upper-bounded by 1, and that the binder must be local.

4$[[D]] = \lambda x.e.\lambda P, [\forall x: X : P(x)]$, where $\bullet \subseteq$ is the ‘atomic part-of’ relation (see e.g., Heim et al. 1991, Milačić et al. 2015).