Lecture 2
Equilibrium from Chaos

Formal requirements for an equilibrium in pure strategies in a two-party electoral contest

- Each of two parties {i, o} chooses a vector of policies s^k from a *compact and convex* set to maximize its expected vote (or similar objective):

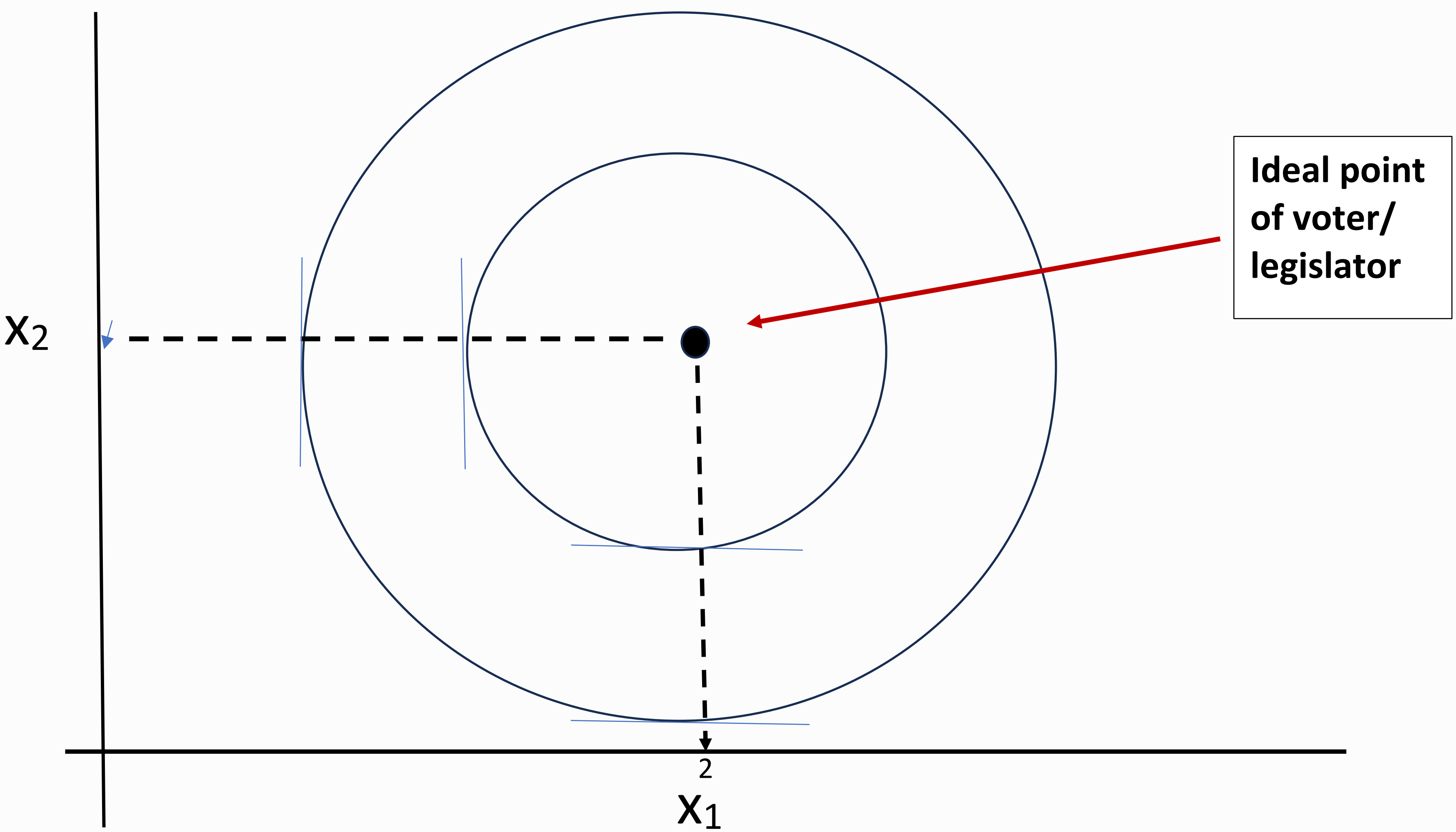
  If *s* ***concave*** *in* s^i *for fixed* s^o *and* $EV_o(s^i, s^o)$ *is convex in* s^o *for fixed* s^i*, and the ex*pected vote function are ***continuous***, then (see, e.g., Owen 1982, Thm IV 6.2, 80) an equilibrium in pure strategies exists.

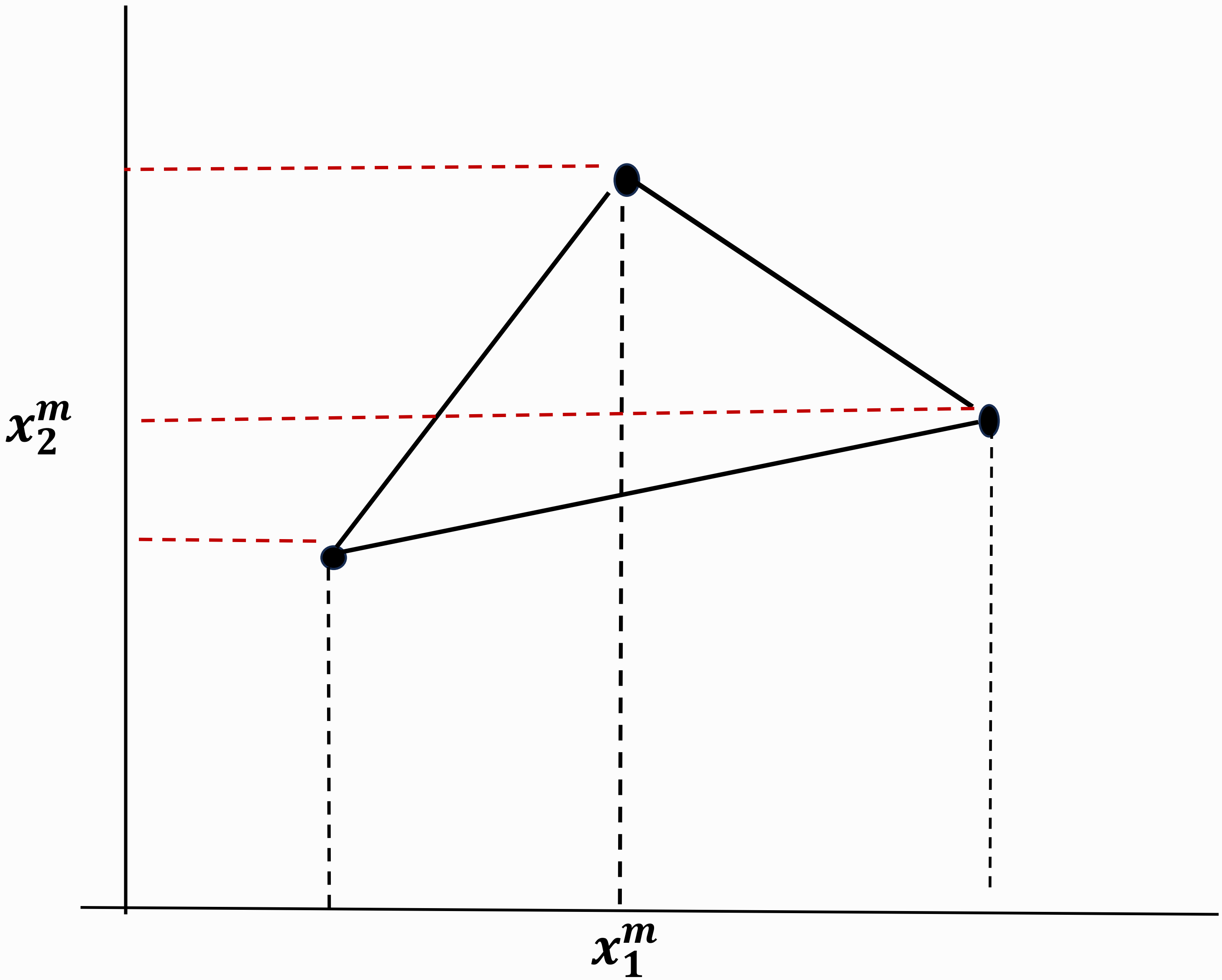
  If concavity/convexity is strict, the equilibrium is unique.

  ***Owen (1982). *Game Theory*. 3rd edition. Academic Press. A multi-party extension is by Wittman (1987).**

The ever-present threat of chaos under majority rule, and some institutional mechanisms that may attenuate the problem

- Setup for analysis of pure majority rule with Euclidean preferences





x_2^m
x_1^m

But, there is no equilibrium at the median of the medians under <u>pure</u> majority rule - i.e., no Condorcet winner – only chaos!

Using CyberSenate (J. Godfrey) to explore 'chaos'

- Basic ideas
- McKelvey's (1976) theorem
- the Plott (1967) condition

- Source of the chaos? .....

  Discontinuity of objective functions almost everywhere.

- Some (partial?) institutional solutions with deterministic voting:

  - *the uncovered set* (Miller 1980)
  - *agenda control* by an elected party 'dictator' in the legislature (Erikson and Ghitza 2016)
  - *structured induced equilibrium* (Shepsle 1979; Weingast and Marshall 1988)
  - *universalism* in response to risk aversion of legislators (Weingast, Shepsle and Johnsen 1981)

- What about the *inheritability problem* of cycling over institutional arrangements?

The probabilistic spatial voting model

- Basic ideas:
  - Parties only know what voters want up to a probability density. They optimize in the face of uncertainty, seeking to maximize expected votes, or their expected vote share, etc.
  - The stochastic nature of the objective insures that a small change in the party's platform will only lead to a small change in expected support **=>** continuity of the party's objective.
  - We also need concavity….

- **Further key aspects of the probabilistic voting framework**

i) There are two parties, each maximizing its expected vote, its expected vote share, its expected plurality, or the probability of winning (more or less equivalent objectives in large electorates). (Wittman 2007 generalizes to many parties).

ii) Policy platform of each party $k \in \{i,o\}$, s^k, is a multi-dimensional vector defined on a *compact and convex set*.

iii) There are *J* (homogeneous) groups of voters of size N_J, $\sum_J N_J = N$.

iv) Each voter's welfare is the sum of a policy-related, indirect utility component $v^J(s)$, assumed globally concave over the domain of s^{k}, and an individual stochastic, non-policy or valence component, ε_j, related to ideology, assessment of party productivity, or the personality and competence of the leader.

- Valence is crucial for the continuity of party objectives.....

  - If party platforms converge and the valence component is missing, any *slight* change in a platform will produce a *discrete* change in expected votes. Continuity of the party objective functions is then lost.

  - Valences that depend on party-specific political contributions can be used to construct a model in which party platforms do not converge.

v) Define the stochastic non-policy bias for the opposition versus the incumbent of voter *j* in group *J*: $\beta_j = \varepsilon_j^o - \varepsilon_j^i$.

Then the probability that citizen *j* in group *J* will support the incumbent is

$$\boldsymbol{\pi_j^i} = \begin{cases} \mathbf{1} & \boldsymbol{if\ v^J(s^i) - v^J(s^o) > \beta_j} \\ \mathbf{0} & \textbf{otherwise} \end{cases} \tag{1}$$

with $\pi_j^o = 1\text{-}\ \pi_j^i$.

vi) Let $\boldsymbol{d^J = v^J(s^i) - v^J(s^o)}$. If F_J is the distribution function of β_j in (1), $F_J(d^J)$ is the share of votes from group *J* expected by the incumbent party.

vii) Using group population weights, and summing over the F_J yields the total expected vote share for the incumbent and opposition:

$$\boldsymbol{E(s^i,s^o) = \sum_J \left(\frac{N_J}{N}\right) \cdot F_J} \quad \text{and} \quad \boldsymbol{E\,(s^o,s^i\,) = 1 - E}$$

viii) There is a tradeoff between uncertainty and risk aversion in determining the concavity of party objectives

A heuristic argument due to Enelow and Hinich (1989)....

Start with $\boldsymbol{E(s^i,s^o)} = \sum_J \left(\frac{N_J}{N}\right) \cdot \boldsymbol{F_J}$. As the variance of the bias β_j increases, F_J flattens out and becomes a linear function of d^J.

Suppose each term in the sum is written as $c + a \cdot d$ **=>**

$$\boldsymbol{E = C + a \{ \sum_J \left(\frac{N_J}{N}\right) \cdot d^J \}.}$$

Since d is concave for given s^o, because *v* is, the sum *E* must be concave.

More generally, the more strictly concave each v^J is, the less each F_J has to approach linearity for *E* to be concave; that is, the less uncertainty is required.

=> Underlying the concavity of $E(s^i,s^o)$ is a trade-off between risk aversion of voters - the degree of concavity of v^J - and the extent of uncertainty - the variance of the distribution of the expected vote density.

- An argument in favor of concavity: Assuming *E* is concave is equivalent to assuming that each party - surely more instrumental than are voters - is able to devise what it thinks is an optimal electoral strategy.

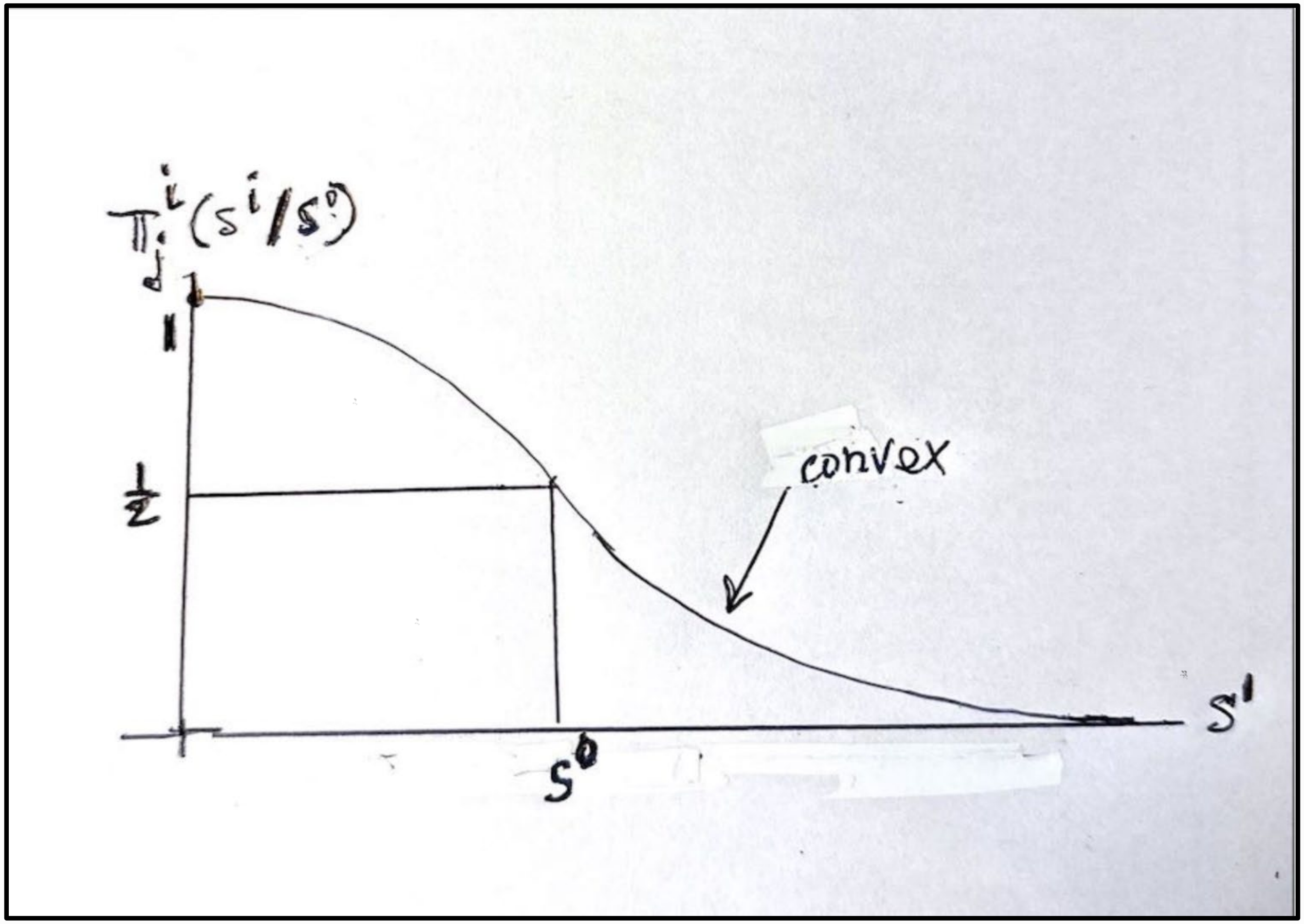
- A fall back: Treat equilibrium as local – concavity applies around the equilibrium, but not necessarily if we contemplate moves far away from it. (Schofield 2007 uses this approach.)

ix) Probabilistic voting does not banish the chaos of the vote cycle, which continues to lurk beneath us:

1 If the chance that a voter will vote for a party is very low, because the proposed platform will impose a large loss on this voter, moves still further away may have only small, and increasingly smaller, effects on the voter's decision.

This means that over some part of the domain of *s*, expected vote functions may be convex (Kirschgaessner 1998).....

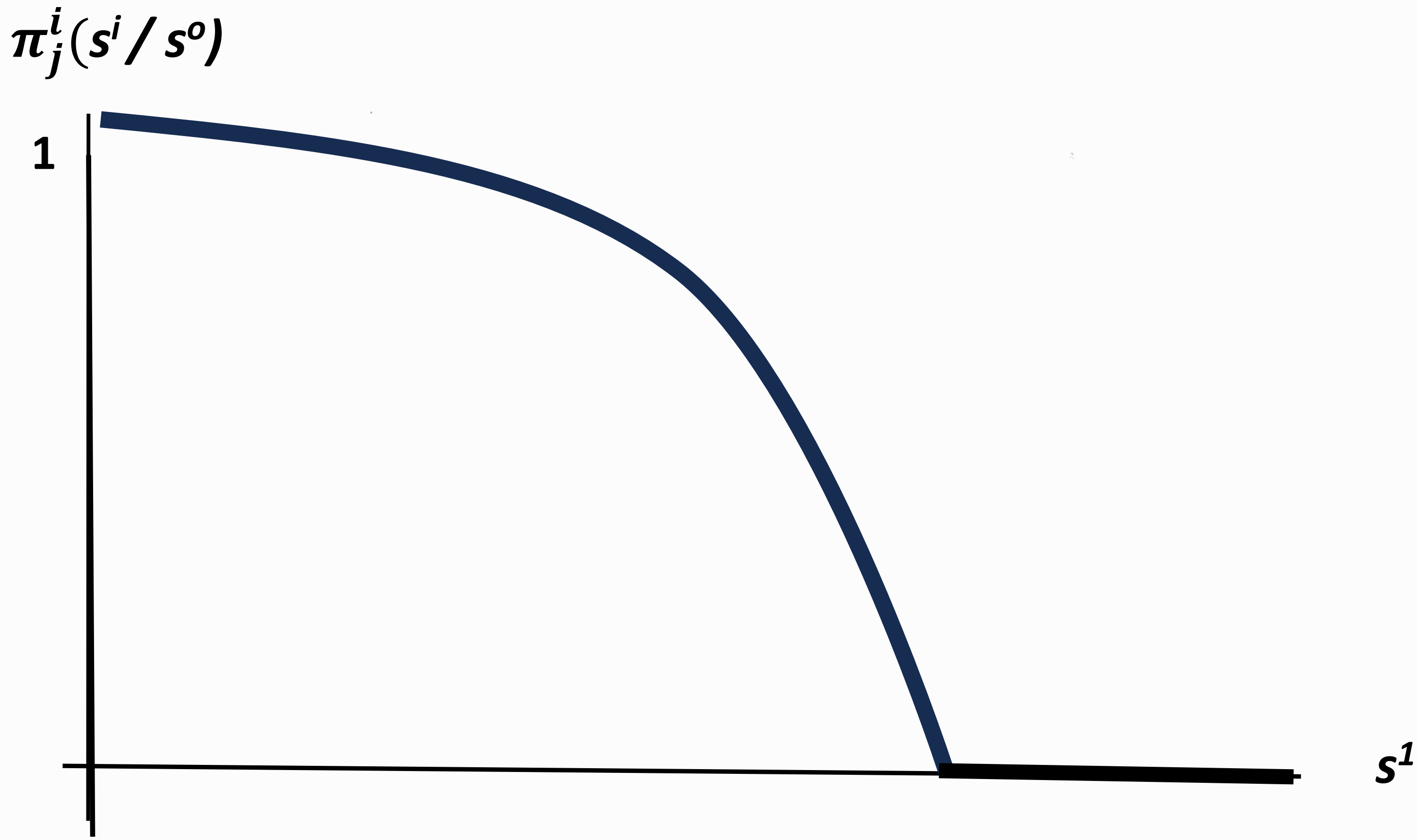
Convexity of the expected vote function may be normal over some part of the policy space.



2 Another problem is polarization: e.g., the probability of a pro-choice voter supporting a pro-choice candidate is zero (Usher 1994).

This means that the total expected vote function may not be globally concave.....

The expected vote function may not be globally concave



Highly contentious issues leading to non-concavity and 'chaos' are sometimes (usually?) left out of the political arena, and either ignored entirely or left to the courts to decide.

- I note that there is no abortion law in Canada of any kind, while in the U.S. it is dealt with mostly in the courts.
- Other examples?

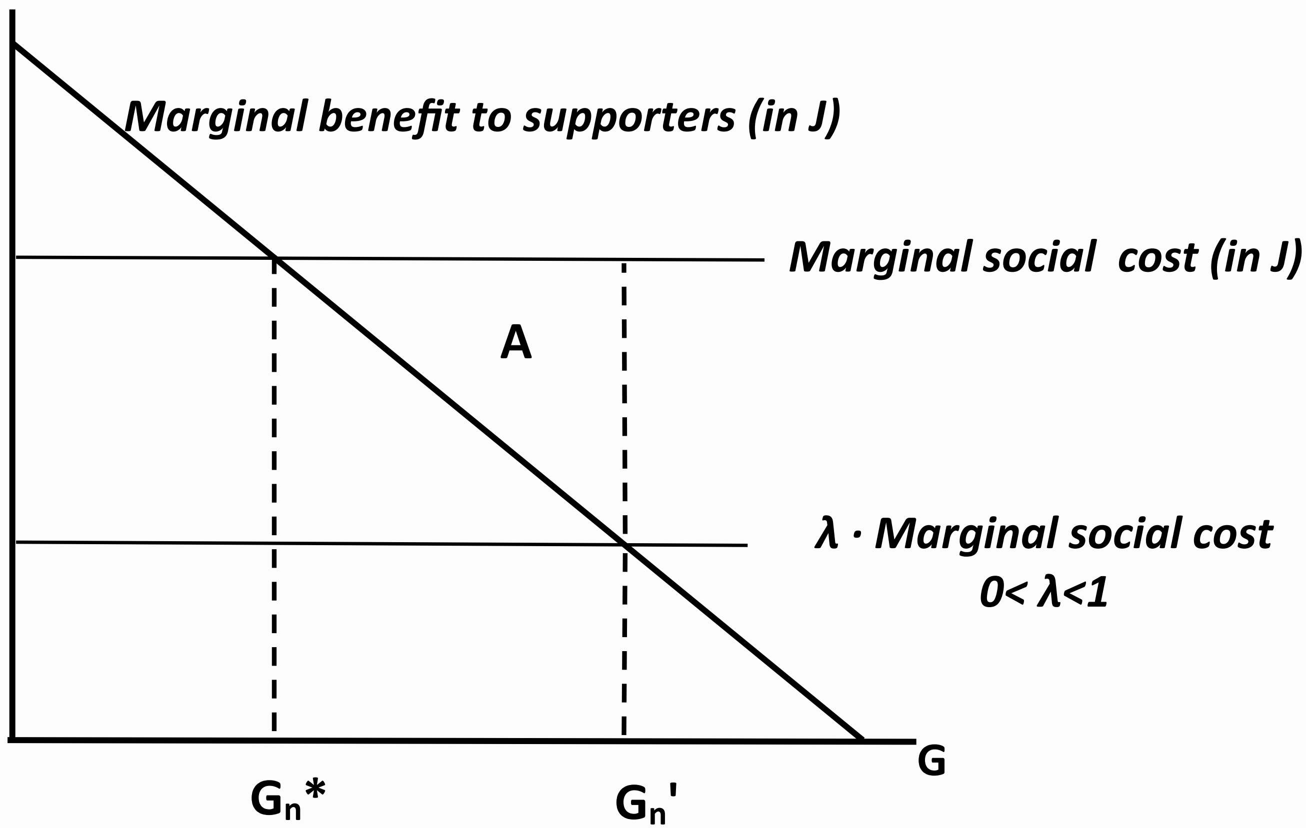
A second example of the third step in the neo-classical political economy approach: the common pool problem:

- The duality of public goods and common pool problems:

  - A public goods problem involves benefits that are non-excludable and so cannot be supplied in a market, while the costs of the good are privately born. The common pool problem has the opposite structure: benefits are private to the voter, while the costs are imposed on everyone.

  - In the public goods case, there is a need for more action. With a common pool, there is a need for *inaction*.

The fiscal common pool problem



- Why does a fiscal common pool arise?

- *Because it is often possible for some voters (and their elected representatives) to use the fiscal system to spend other people's money.*

- It is possible to do so because it is costly for the individuals who lose out to resist by acting collectively, or by 'leaving'.

  (The idea that citizens can leave to avoid fiscal coercion leads to the use of incentive compatibility constraints in mechanism design)

- We could also say that the problem stems from the existence of *decision externalities* – a situation in which decision makers do not have incentives to fully internalize all costs and benefits of their actions

We model the equilibrium directly rather than use a representation theorem (Persson, Roland and Tabellini 2007):

- There are 4 parties *J={1,2,3,4}*, each with equal nos. of supporters. Parties maximize expected votes or seat shares (equivalent under PR).

- The policy vector ***s*** consists of a *local* public good for each group $\{g^J\}$ and a uniform lump sum tax τ, the same for each group. The constant marginal cost of g_J is 1.

- If it is a formal coalition, each party in government controls only its own g^J, e.g., via a separate ministry.

- Voter utility: $\boldsymbol{v^J}\ (\boldsymbol{g^J}) = \mathbf{1} - \boldsymbol{\tau} + \boldsymbol{H}(\boldsymbol{g^J})$

- The government budget constraint: $\mathbf{4}\,\boldsymbol{\tau} = \sum_J \boldsymbol{g^J}$.

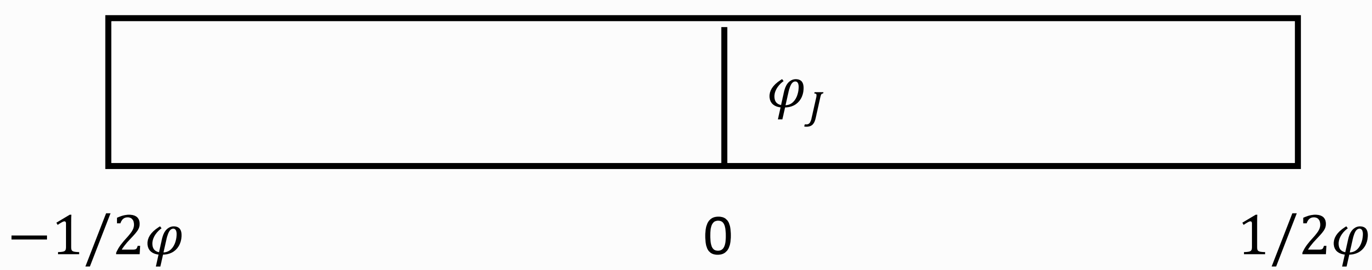
- Voters in group *J* vote for party *J* if

$$\boldsymbol{v^J(s) - v^{*J} \geq \beta_J,}$$

and for the opposition otherwise, where v^{*J} is what voters expect in the future, fixed independently of current policy (retrospective voting).

- Assume the bias term has a uniform distribution over the interval $[-1/2\varphi, 1/2\varphi]$ such that

$$\boldsymbol{-1/2\varphi < v^J(s^i) - v^J(s^o) < 1/2\varphi}. \quad \textit{(Why this matters?)}$$



- If F_J is the cumulative distribution of the party bias for group *J* , the proportion of group J voting for the incumbent is

$$\boldsymbol{F_J}[\boldsymbol{v^J}(\boldsymbol{s^i}) - \boldsymbol{v^{J*}}] = \boldsymbol{\varphi}\{(\boldsymbol{v^J}(\boldsymbol{s^i}) - \boldsymbol{v^{J*}} + \mathbf{1/2}\boldsymbol{\varphi}\} \qquad (2)$$

where $\varphi = \partial \mathrm{F}/\partial v^J$ is the 'sensitivity' of a voter in group *J* .

Note: The derivatives of F are the influence weights used in the first lecture. These weights vary with uncertainty about the party bias: the smaller the variance of the bias, which varies with $1/\varphi$ in the uniform distribution case, the greater will be the representative voter in group *J's* influence.

- **Single party government under PR: one government authority choosing all of the policy instruments**

  - Say a party combining groups 1 and 2 (before the election) is in power, and a coalition of 3 and 4 is in opposition.

  - The Lagrangean for the expected vote share (and seat share) of the coalition in power is:

$$\mathcal{L} = \frac{1}{4}\ \{\sum_{J=1}^{4} F[\ v^J(g^J) - v^{J*}]\} + \lambda[4\ \tau - \sum_J g^J]$$

$$= \frac{\varphi}{4}\ \{\sum_{J=1}^{4}[(1 - \tau - H(g^J) - v^{*J}] + 1/2\varphi\} + \lambda[4\ \tau - \sum_J g^J]. \quad (3)$$

Note: 1/4 of the electorate comes from each group of party supporters.

- Solving for g^J yields the Pareto-efficient levels:

$$\boldsymbol{H_g\left(g^J\right) = 1} \quad \text{or} \quad \boldsymbol{g^J = H_g^{-1}(1)}$$

=> no common pool problem.

- **Coalition government under PR**

  - Now suppose 1 and 2 form a winning coalition **after** the election. Each party *p = {1,2}* in the coalition maximizes the following expected vote (and seat) share:

$$\mathcal{L}^p = \frac{1}{4}\,\{F[\,v^p(g^P) - v^{p*}] + \frac{1}{2}\sum_{J=3}^{4} F_J[\,v^J\left(s^i\right) - v^{J*}]\} + \lambda[4\,\tau - \textstyle\sum_J g^J]. \quad (4)$$

Note (g^P): It is assumed that voters support only the party in the coalition they favor (if they think it is doing a good job). No satisfied supporters of the other coalition partner vote for its partner **=>** each coalition member has an incentive to tax ***its partner's supporters*** to pay for benefits to ***its*** supporters.

Note (1/2): Here it is also assumed that supporters of out of government parties split their vote among the governing coalition partners if they think the coalition is doing a good job **=>** the second term in square brackets is multiplied by 1/2.

- Solution for $\boldsymbol{g^J}$:

$$\boldsymbol{H_g\left(g^J\right) = 1/2} \quad \text{if } J \in \{1,2\}$$

$$\boldsymbol{H_g\left(g^J\right) = 1} \quad \text{if } J \in \{3,4\}$$

=> a common pool problem *within* the governing coalition.

- **P/R/T show that the type of electoral system (PR vs SMP) does not affect overspending via the common pool problem: it is the presence or absence of coalition government.**
  - But consider again the common pool diagram.....
  - And consider methods to deal with the *decision externalities* that underlie the common pool problem in all democratic political systems (e.g., internalization vs. bounding the loss).

A perspective on the relationship between exchange-contractarianism, social planning and neo-classical normative political economy.

- Finally, I return to the difference between the normative view just explored, social planning, and the exchange-contractarianism of Wicksell (1896), Lindahl (1919), and Buchanan and Tullock (1962).

- Concern with the coercive power of the state has a long history, going back to Jean-Jacques Rousseau's *Social Contract* (Book IV, 1762):

When the state is instituted, residence constitutes consent; to dwell within its territory is to submit to the Sovereign. Apart from this primitive contract, the vote of the majority always binds all the rest. This follows from the contract itself. But it is asked how a man can be both free and forced to conform to wills that are not his own. How are the opponents at once free and subject to laws they have not agreed to? I report that the question is wrongly put. The citizen gives his consent to all the laws, including those which are passed in spite of his opposition, and even those which punish him when he dares to break any of them.

- The problems underneath Rousseau's view of assent to coercion by the state are at least twofold:

  i) Individuals do not agree on social objectives, so their participation in communal affairs is predicated on the preservation of individual rights limiting the scope of collective action

  ii) Assent to coercion is conditional: there must be limits to the agency of politicians, bureaucrats and the military, and limits to the ability of groups of citizens to take advantage of others using the collective choice process (e.g. via fiscal coercion).

  This is not utilitarian reasoning.

- **What is fiscal coercion?**

  Fiscal coercion = the difference between a citizen's utility under what they regard as appropriate treatment by the public sector, and the utility that they actually enjoy as a result of its operation.

- Formally,

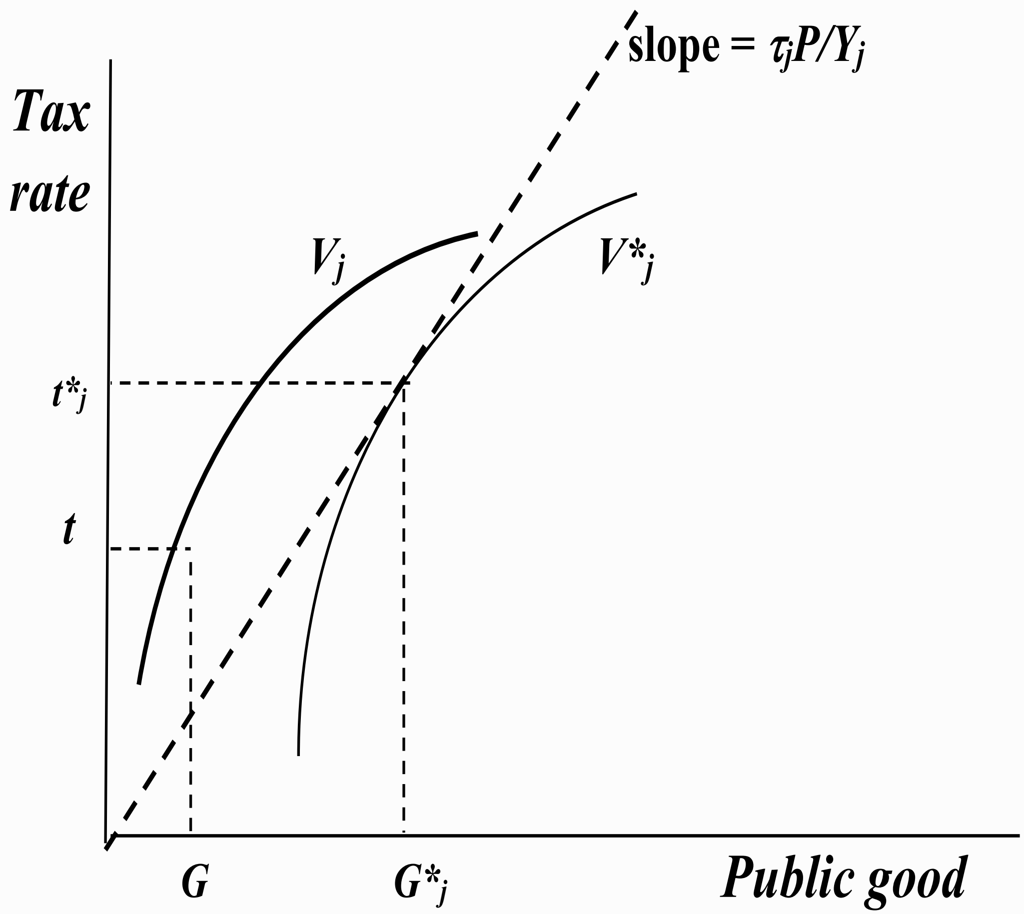
  - Let $\tau_j = (T_j /PG)$ be the individual actual tax-share, where $T_j = t_j Y_j$ is the total tax payment given proportional rate t_j on income Y_j , with P the constant supply price of the public good G.

  - *A*ssume individuals believe they would pay this tax share if quantity adjustment were possible (the *'individual-in-society'* approach of Buchanan 1976 and Breton 1996)

  - Let V_j be the actual indirect utility, and V^*_j their maximum utility when free to choose G_j^* at the individual tax price $\tau_j P$.

  - Then fiscal coercion of an individual can be defined as

$$[\boldsymbol{V_j^*}(\boldsymbol{G_j^*}) - \boldsymbol{V_j}\,(\boldsymbol{G_j})] \ \ \textbf{\textit{where}} \ \ \boldsymbol{G_j^*} = \boldsymbol{argmax}_{\{\boldsymbol{G}\}}\,\boldsymbol{V_j} \quad (5)$$



Coercion Using the Individual-in-Society Counterfactual

(Winer, Tridimas and Hettich 2014, Fig. 7.1)

- To combine concerns with social welfare and with fiscal coercion, we could choose fiscal structure to maximize social welfare subject to aggregate coercion being no more than some level *K*.

- If social welfare is $S=\sum_j V_j$, this involves

$$\boldsymbol{L = \sum_j V_j + \mu\,[\,\sum_j t_j Y_j - PG] + \kappa\,[\,K - \sum_j (V^*_j - V_j)]} \qquad (6)$$

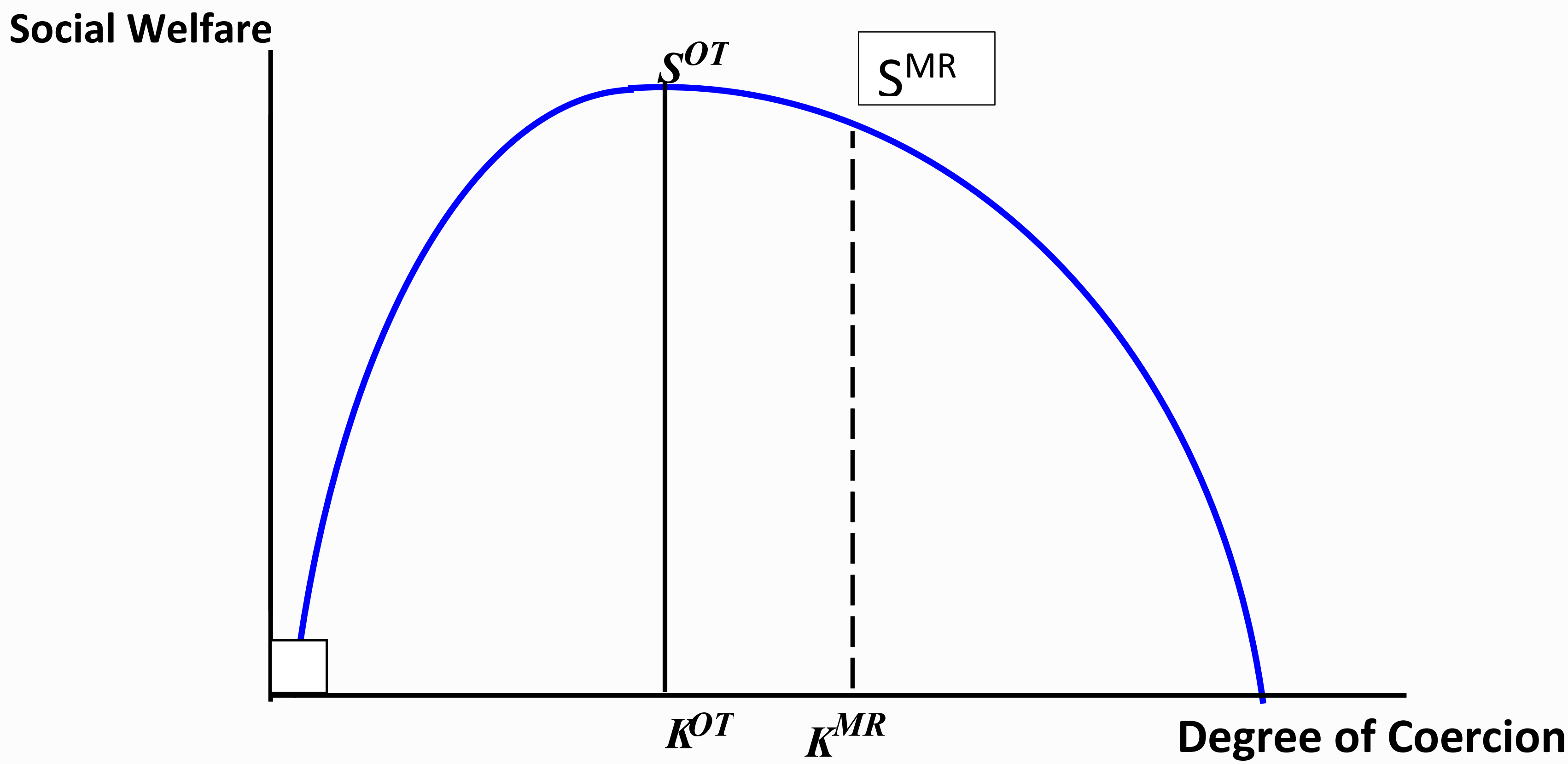
  where *the counterfactual* V^*_j and the shadow price of coercion κ are determined simultaneously with fiscal instruments.

- Determination of K is required to close the model. *It is not obvious how to do this.* Wicksell (1896) advocates approximate unanimity to make collective decisions – this would probably severely restrict social welfare.

- It is of interest to note the following relationships between social planning, majority rule and the extent of coercion (Winer, Tridimas and Hettich 2014):

  i) Coercion in standard social planning, $\boldsymbol{\kappa}^{OT}$, will be a maximum, since the planner is allowed to coerce anyone to any extent as long as social welfare increases (as a matter of social solidarity?).

  ii) Coercion under majority rule - of any kind - will *exceed* that imposed by the social planner, and social welfare (the unweighted sum of utilities) will be smaller, since collective choice introduces discrimination according to political influence, in addition to that according to narrowly defined individual preferences.



The Welfare Coercion Trade-off

- Buchanan and Tullock (1962) minimize the sum of coercion costs - which they do not explicitly define - and the costs of making collective decisions to choose an optimal collective decision rule (which is not unanimity).

- **Some partial solutions to the problem of coercion of one by the group, and of all by the state, that have been suggested:**

  - Adopt a broad base income tax to limit the ability of government to interfere in private lives - i.e., 'to dip deeply into great incomes with a sieve' (Simons 1938)

  - Adopt a simple proportional tax system without a demogrant (Buchanan and Congleton 1998)

A tentative comparison of approaches to normative analysis (tentative)

Social planning:

Max SW(s) s.t. R(s) = 0 => s*

Domain of s unrestricted.

- *SW(s)* is a social welfare function based on normative theorizing;
- Only preferences, technology and endowments included in *R(s)*;
- No allowance for endogeneity of *s* in full g.e. structure, $\mathcal{R}(s)$, via collective choice, p-a problems, etc;
- **Policy recommendations***:* Changes in *s* that bring it 'closer' to *s**.

Wicksellian exchange-contractariansim:

Max SW(s) s.t $\mathcal{R}$(s) = 0 and s.t. $\sum_j (V^*_j - V_j) \leq K$ => s**

Domain of s may be restricted according to non-utilitarian objectives.

- **Policy recommendations**: Institutional changes in actual g.e. structure $\mathcal{R}$(s) that bring full equilibrium *s* 'closer' to *s***. May include reforms in (constraints on) *s* directly.

(Neo-classical) normative political economy

Max S(s) s.t. $\mathcal{R}^*(s) = 0$ => s^{***} .

Domain of s may be restricted according to non-utilitarian objectives.

- *S* is a political support function in an competitive, well functioning liberal democracy $\mathcal{R}^*(s)$ **=>** s^{***} is an ideal political equilibrium.
- **Policy recommendations***:* Institutional changes in actual g.e. structure $\mathcal{R}(s)$ that bring equilibrium *s* 'closer' to s^{***}.
  May include reforms in (or constraints on) *s* directly, taking guestimate of consistency with political equilibrium ($\mathcal{R}(s)$) into account.